Article

A Tentative Correction to the Inappropriate Application of Mathematical Theory to Physics — and A Correction to the Assumptions underlying Probability Theory

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Abstract: For a long time, physicists always directly use mathematical tools to deal with physical problems, and few people pay attention to the difference between mathematical theory and physical theory. Just like the dilemma that physicists once faced when dealing with the problem of blackbody radiation function.

By analyzing the difference between the theoretical basis of mathematics and the theoretical basis of physics, this paper draws the following conclusions: (1) The theoretical basis of mathematics and the theoretical basis of physics are different, so when we use mathematical tools for physics research, we need to be very careful. (2) Finiteness and discreteness should be the basis of the whole physical theory; This paper points out that it is not advisable to use infinite "∞" and infinitesimal "0" without restriction and demonstration in physics, as well as the continuity of functions, which will bring a lot of trouble to physical theory.

At the same time, through the analysis of Banach-Tarski paradox and Bertrand paradox, this paper proposes that if we revise the basic assumptions of probability theory: assuming that "points" have quantized sizes, and "lines" also have quantized widths. After the correction, we can not only avoid the troubles caused by Bertrand paradox, but also make probability theory better for practical application.

Keywords: infinity; infinitesimal; continuity; finiteness; discreteness; Banach-Tarski paradox; Bertrand paradox

1. Introduction

For a long time, physicists have been directly using mathematical tools to deal with physical problems, and acquiesced in the rationality of mathematical tools in dealing with physical problems; at the same time, mathematicians have also directly applied mathematics to explain many practical problems, and acquiesce in the rationality of this way.

Few people notice that in fact the theoretical foundations of mathematics and physics are different, for example:

1) In mathematics, we define a "point" as having no size. In geometry, what is mapped to a straight line is the set of real numbers " R". This mapping relationship is self-consistent and reasonable in mathematical theory. Otherwise, the mapping relationship between geometry and coordinates will be confused.

However, theoretical applications based on this assumption can be problematic in practical applications in physics or other fields. Because we have not yet experimentally confirmed the existence of matter without size, at least the current physical theory limits the length to the "Planck length".

Therefore, in fact, the assumption basis of mathematical theory is very different from that of physical theory. When we use mathematical tools to deal with physical problems, we must be careful, otherwise we will make mistakes.

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2) In geometry, functions $y = f(x)$ tend to be continuous; however, we know that in the physical world, we face almost all discrete cases.

The foundation of the entire quantum mechanics is both quantized and discretized; the laws of physics are built on a discrete basis.

If we do not limit the mathematical tools, there will be many problems in the application, just as the physicists faced the dilemma of the black body radiation distribution function in those days [1-3].

This paper hopes to avoid problems in the application of mathematical theory to physics by strictly defining and distinguishing the theoretical foundations of mathematics and physics.

2. The boundaries of mathematics in physics

2.1. Application Boundaries of Infinite "∞ " and Infinitesimal "0"

1) In mathematical theory, the use of "∞ " and "0" is natural, whether it is infinity or infinitesimal, it is an important symbol and variable in the field of mathematics. Moreover, the approximation of functions is often used to discuss and compare infinitely large and infinitesimal series problems.

However, when we introduced " ∞" and "0" into the field of physics, we did not notice the essential difference in the meanings of the symbols " ∞" and "0" in mathematics and physics. The foundation of the birth of " ∞" in the field of mathematics is our definition of "number", which is based on the high degree of abstraction of human thinking and the reasoning process of abstract objects; however, in the field of physics, no one has ever proved it. The real meaning and existence evidence of " ∞" and "0", for example, no one has proved that the energy of our universe is infinite, and no one has ever proved that our universe is infinite; our current physical theory also has strict restrictions on infinitesimals, for example, restrictions on Planck length, energy, time, etc.

2) The physical world is limited, there is no infinite " ∞"

Taking energy conservation and momentum conservation as examples, in fact all forms of "conservation" are meaningful only under the premise of "limitation". This means that our universe is a universe with a finite structure, and it should also be a closed universe under the evolution of time.

If the universe is infinite, or the direction of evolution tends to be infinite, then our laws of conservation of energy and momentum will become meaningless.

For example: if a beam of light leaves an object, travels over time to infinity in the universe and never comes back, it makes no sense to talk about conservation of energy, which is somewhat similar to the second law of thermodynamics. For a universe whose evolution direction tends to be "infinity", when energy is dispersed and wants to gather again, a part of the energy must be consumed so that the energy of the universe can be gathered again. The law of conservation of energy is like the second law of thermodynamics at this time. If it is not in an "isolated reversible system", there will be consumption, and energy will not be truly conserved for the universe as a whole.

Therefore, our universe should be a closed universe, and the direction of evolution is also closed, not open.

3) There is no infinitesimal "0" in the physical world

For mathematics, infinitesimal is a concept based on mathematical theory, and it is indispensable to the entire mathematical system, so it is also a reasonable conclusion in line with the mathematical system.

But for the real physical world, before we strictly prove the existence of infinitesimal "0", we should strictly distinguish the difference between infinitesimals in mathematics and physics.

However, When we apply mathematical tools to physical systems, without restrictions, problems arise:

For example: Newton's gravitational potential energy formula (1) and electric field potential energy formula (2) are as follows:

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$$E_p = -\frac{GMm}{r} \quad (1)$$

$$E_q = -\frac{kQq}{r} \quad (2)$$

Where G is the gravitational constant, $k = \frac{1}{4\pi\varepsilon_0}$. According to current theory, when $r \to 0$ the potential energy can be infinite if it tends to be infinitely small. Based on the gravitational potential energy formula (or general relativity), we deduce the existence of black holes as singularities in the universe [4-5]. Regrettably, however, in the above black hole theory of gravity, the derivation process of the entire theory we use is based on mathematics, not physics.

So far, we have never strictly experimentally proved the specific value E_p of the gravitational potential energy between two static masses M and objects m(of course, we are constantly trying to prove that there is a minimum distance for gravitational action between objects, but in fact this are two completely different problems, and it is not directly equivalent to the magnitude of the gravitational potential energy between two objects), and has never been strictly experimentally proved whether there really exists matter and gravitational effects under infinitely small radii.

All our theoretical derivations are based on mathematics, not physics. What is worrying, however, is that few people have clearly pointed out that **there is actually a huge difference between the foundations of mathematical theories and the foundations of physical theories** .

We cannot arbitrarily use the tools of mathematics to deal with problems in physics unless based on rigorous experimental verification.

As the author pointed out in another article: Our real physical world seems to be more inclined to " The gravitational potential energy E_p and the electric field potential energy E_q between any two substances are finite", that is, the minimum radius of action of all fields such as electric field and gravitational field r, is a finite value, not an infinitesimal value [6].

Therefore, it is problematic to directly deduce the "black hole theory" based on the gravitational potential energy formula. At least the basis of this theory is currently a "mathematical" black hole theory; rather than a "black hole theory" based on physical theory in the true sense.

From this, we lead to the first theorem of the application of mathematical theory in physics:

Theorem 1: The research object of physical theory is limited. In physical theory, there are no two limit cases of the infinite "∞ " and the infinitesimal " 0 ".

2.2. Application Boundaries of "Continuity of Mathematical Functions"

1) In mathematics, functions $f(x)$ tend to be continuous. The theoretical starting point of the continuity theorem is our definition of "numbers". When all the "numbers" we define have no size, then we can define sets, mapping relationships, limits, Using the infinite approximation method, we finally rely on a set of set theory to define the continuity of the function.

Based on the continuity of functions and our definition of the limits of functions, we apply this theory to various fields such as calculus, geometry, topology, etc. The proof of the entire mathematical theory is flawless because our original definition of logarithms determines the integrity of the edifice of mathematical theory and the self-consistency of the theory.

2) However, in the practical application of physics, the problems we encounter are often not like this.

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The foundations of our physics edifice: time, length, energy, and mass all have Planck minimum unit constraints.

Time itself may be continuous (we have no way of knowing this at present), but all the physical phenomena we study, and the instruments we use to define time, are defined based on the "period T" in which matter operates , the most accurate timer we have invented today is only a very high periodic frequency. Periodicity means "discreteness".

The same is true for length. When we try to understand the size of microscopic matter (such as the size of an atom, the size of a nucleus, and the size of an electron), we still rely on the wavelength formula determined by the de Broglie wave function. Therefore, length is also discrete to the physical world.

And energy, mass, through quantum mechanics, we found that when the depth of research is close to the physical limit, the basis of almost all physical units we face is almost discrete rather than continuous.

3) Therefore, we have to be very careful not to make mistakes when we use the mathematical tools that have been developed in succession. Just as the blackbody radiation brought trouble to the entire physics community in the past [1-3], if we hadn't gradually deepened our understanding of the concept of quantization, there would not be today's quantum mechanics and related theories of statistical physics based on quantization.

Although at present, the quantization processing method has become common in the field of physics, but I think it will become more in-depth in the development of physics in the future. The basis of "discrete" has become the theoretical basis of the whole physics.

Theorem 2: The object of physics research is discrete, not continuous. Thus, discreteness is the basis of the entire theory of physics, as discovered by the theory of quantum mechanics .

3. Modifications to the Assumptions of Mathematical Probability Theory

We know that in general, the system of mathematical theory is self-consistent. Although Gödel's incomplete theorem points out the dilemma faced by the completeness of mathematical theory, no major problems have been found in the application of basic disciplines such as number theory and geometry.

However, with the development of mathematics itself and the expansion of application fields, the connection between mathematics and physics has become more and more closely, the application of mathematical theory has become more and more practical, and the dependence on the physical world has become higher and higher. Among them, the most obvious belongs to probability theory.

The theoretical basis of probability theory still uses the basic assumptions of mathematics, such as: the definition of point, the definition of randomness, set theory and so on. As a result, we found many paradoxes in practical applications, such as:

1) Bertrand paradox

2) Banach -Tarski paradox (in mathematics it does not belong to the category of probability theory, but its essence is the same, so this article will discuss it here)

Below, we will discuss the above two paradoxes in detail and find solutions to them fundamentally:

3.1. Bertrand paradox

Bertrand Paradox: "What is the probability that a chord in a circle is arbitrarily chosen, and the chord length is greater than the side length of the inscribed equilateral triangle of the circle?"

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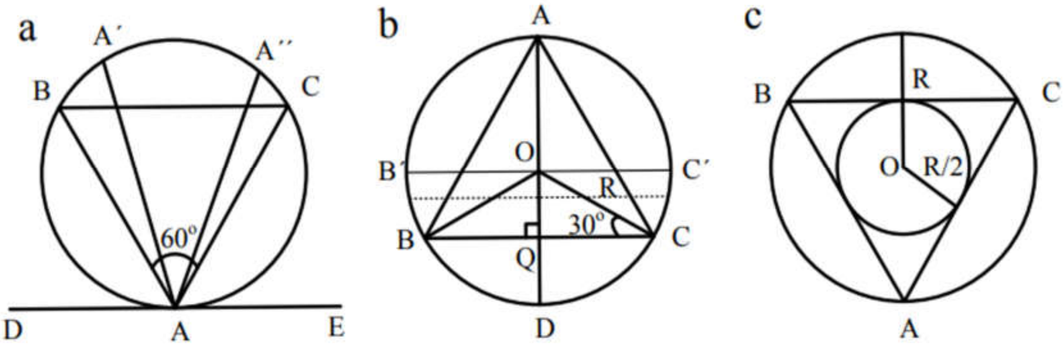

Figure 1. Bertrand paradox.

There are three solutions as follows, and get different probabilities respectively:

a) A chord inside a circle must have two intersections with the circle. We take any vertex of the inscribed equilateral triangle as point A. Make the chords in the circle all start from point A. The tangent DE of the circle through point A intersects the circle at point A. We set the angle between the chord through point A and the tangent AD to be θ, as shown in Fig. 1(a), if the chord length is longer than the chord length of the equilateral triangle, there must be $\angle A'AD \in (60°, 120°)$. ie $60° < \theta < 120°$. That is to say, among the chords starting from A, the chords whose chord length is longer than the side length of the equilateral triangle all fall within ∠BAC. The chords from point A that fall outside this area are all shorter than the sides of an equilateral triangle. So we apply the principle of indifference, which is uniformly distributed over the θ interval [0°, 180°]. So we can get:

$$P = \frac{\frac{\pi}{3}}{\pi} = \frac{1}{3}$$

b) Pick a point at random on the diameter perpendicular to any side of the triangle, and make a chord perpendicular to the diameter through this point, as shown in Fig. 1(b), from the properties of a circle inscribed in an equilateral triangle, at this point the length of the chord is equal to the length of the side of the triangle when it is at the midpoint of the radius, and greater than the length of the side of the triangle when the point is less than the distance from the center of the circle $\frac{R}{2}$. So the probability is:

$$P = \frac{1}{2}$$

c) We draw another inscribed circle of the triangle inside the inscribed equilateral triangle of the great circle, which intersects the three sides of the triangle respectively. As shown in Fig. 1(c), the radius of the small circle is R/2. The chord length of the random chord is longer than the side length of the equilateral triangle when the midpoint of the random chord is inside the small circle. We have no reason to say that the midpoint of the chord falls on a certain point within the small circle rather than other points, so according to the principle of indifference, it is considered to be uniformly distributed, then we can get:

$$P = \frac{\pi \cdot \left(\frac{R}{2}\right)^2}{\pi \cdot R^2} = \frac{1}{4}$$

The research papers on the interpretation and solution of Bertrand paradox mainly focus on the following two ways:

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1) Clarify the "method of randomized trials", e.g. L. Marinoff (1994) [7] and G. D' Agostini [8], Diederik Aerts1 and Massimiliano Sassoli de Bianchi (2014) [9], most of the authors have noticed the huge difference between Bertrand paradox between theory and practice.

Mathematical theories can be extremely abstract, but when we introduce mathematics into probability theory, a theory so closely related to applications, we see the difference. Bertrand paradox is a typical representative, and we find that the above three probability models cannot be designed in the real world.

Therefore, this has also become the main way for scholars to solve Bertrand paradox.

2) Some scholars believe that Bertrand itself does not want to raise a mathematical theoretical problem that bothers everyone, but expects to point out the importance of specifying "random selection method" in probability theory through Bertrand paradox itself. For example Dominic Klyve (2013) [10].

Of course, some scholars believe that we have not yet found a real solution to Bertrand paradox, such as Darrell P. Rowbottom (2013). [11]

This article agrees with the solutions 1) and 2) above because:

First, probability theory is an applied discipline, not a purely mathematical theoretical discipline. It has gradually got rid of the theoretical framework of pure mathematics and involved in practical application fields. Therefore, since probability theory is faced with practical applications, in any actual random experiment, we must first specify the "mode of random experiment".

But unlike the above solutions, this article expects to explicitly point out the following two points:

I. The cause of Bertrand paradox is the mathematical theory itself. The assumption in mathematics that "points" have no size and "lines" have no width is the root cause of the paradox .

II. Probability theory is an applied subject, so its theory is mainly applicable to applied fields rather than pure mathematical theoretical research. Therefore, it is necessary for us to revise its theoretical basis: it is stipulated that "points" have sizes (preferably to match the physical theory, defined as quantized), and "lines" have widths .

Any theory, the theoretical assumption basis of its construction, should match the application category corresponding to its theory, otherwise the theory will lose its own meaning.

Therefore, the purpose of our construction of probability theory is to carry out practical application in our real physical world, not to carry out pure theoretical research. Under such a premise, we need to carefully examine whether the theoretical basis of probability theory matches its application scope.

When the probability theory was constructed, it continued the relevant theoretical foundations in the field of mathematics: the definition of numbers, the definition of sets, the definition of elements, the definition of points, the definition of straight lines, etc., all directly followed the traditional theory of mathematics. But what mathematicians don't find is that probability theory has extended from the purely theoretical realm of mathematics to practical applications, where our assumptions are no longer applicable and need to be revised.

After modification, we will find that the theory of probability theory will become more self-consistent and more in line with its practical application. We still use Bertrand paradox as an example, but we make some modifications to it. As shown in the two cases b) and c) in Fig. 1:

We abstract the problem again as: throw random "points" one by one into a circle, and ask the probability that these points are less than $R/2$ from the center of the circle.

The first solution: If we assume that the "points" have no size, then the result will be consistent with b) and c) in the traditional Bertrand paradox , resulting in two different probabilities $P = 1/4$ and $P = 1/2$. When we use the area method to solve the problem, as in c) , we get a probability of $P = 1/4$; and when we solve the problem with the probability of passing a point on the diameter of the center of the circle, as in b) Again , we would get a probability of $P = 1/2$.

The second solution: the solution after assuming that the "point" has a size (quantization) and the "line" has a width.

When we assume that the points have sizes, then we find that the probabilities of b) and c) above will be uniformly $P = 1/4$. Because the diameter passing through the center of the circle is no longer an ideal mathematical situation: "never coincide", but presents the property that the closer to the center of the circle, the easier it is to coincide. In such a case, if we assume that the probability of quantized "points" within any area unit is equally random, then the probability of "points" distributed on the diameter will no longer be equal, but close to the center of the circle has a lower probability (because of the coincidence), while regions close to the circumference have a higher probability.

In this way, we revise Bertrand paradox of probability, and at the same time, revise the inappropriate assumptions of probability theory.

(Note: We do not discuss the case of a in Fig. 1 because it is difficult to implement this theoretical model in real experiments)

3.2. Banach -Tarski paradox (abbreviated as: Ball sharing paradox)

In 1924, Stefan Banach and Alfred Tarski first proposed this theorem. This theorem states that, given the axiom of choice, a solid three-dimensional sphere can be divided into finite (unmeasurable) parts, and then reassembled elsewhere simply by rotation and translation, to form two complete radii with the same original ball. (We will not repeat the derivation process of the whole theorem here)

For Banach-Tarski paradox, almost the entire mathematical community would not consider it a paradox, but a well-known mathematical theorem, e.g. Robert M. French (1988) [12], Tom Weston (2003) [13], Wagon , S. (2010) [14], Francis Edward Su (1990) [15].

However, when we analyze the nature of the Banach-Tarski paradox and **Bertrand paradox, we will find that the origin of these two problems is exactly the same: our** definition of "point" and "line" in mathematical theory, and the basic theory of set theory.

For example, we can simplify the Banach - Tarski paradox problem by considering the following two functions:

$$y(x) = x \tag{3}$$

$$g(x) = 2x \tag{4}$$

Where, $x \in [0,1]$, according to the definition of the continuity of mathematical functions, we know that $y(x)$and $g(x)$are continuous, and a one-to-one mapping relationship can be established between the two functions:

$$y(x) \to g(x)$$
$$f: x \to 2x, \ x \in [0,1]$$

Mapping relationship of the above functions: $y(x) \to g(x)$, It is equivalent to dividing a line segment of length 1 into two line segments of length 1, and it can be repeated indefinitely:

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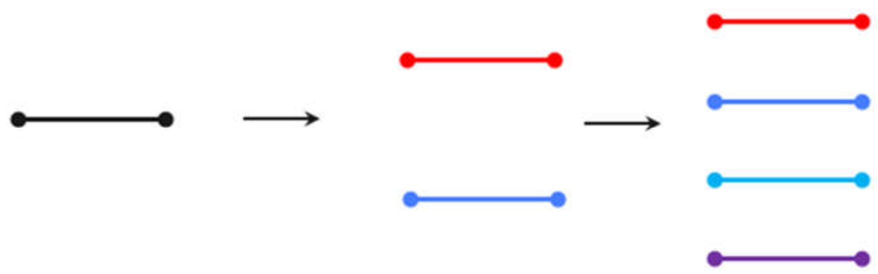

Figure 2. Simplified graph paradox.

This is superficially different from the Banach -Tarski paradox, but there is no essential difference in mathematics, and its origins are derived from the basic definition of "number" in our entire mathematical theory. In infinite sets, we have many ways to implement the Banach-Tarski paradox mapping relationship, which is very common in discussions of mathematical set theory. The reason why Banach - Tarski paradox is different is that it visualizes the conclusions of set theory through operations such as "dividing the sphere" in geometry, which surprises many people who are not familiar with mathematical set theory. It's like turning "mathematical magic" into reality.

When we have defined from the beginning that "numbers" have no size, points have no size, defined mapping relationships, and defined the continuity of functions, then Banach - Tarski paradox and Bertrand paradox will be the direct conclusions of mathematical theory.

Only difference: we put Banach -Tarski paradox is only used in the field of mathematical theory, and has never tried to prove it in practice, so few people doubt its rationality in mathematical theory.

On the contrary, Bertrand paradox is a basic conclusion in probability theory, and we regard probability theory as a practical discipline, so it is easy to connect it with practical application, which has led to it being widely regarded as a paradox Argument. If not for the purpose of application, Bertrand paradox itself is not surprising in the field of mathematical theory, but it is normal.

3.3. Amendments to the theoretical basis of probability theory

We make the following two requirements for any theory:

1) The theory itself is self-consistent, and there is no logical contradiction;

2) The theory itself needs to be adapted to its application category, otherwise it will be out of the meaning of constructing the theory.

Then, for probability theory, because it has been out of the category of pure theoretical mathematics, it should be actually applied to the real physical world. Therefore, we need to revise its theoretical basis accordingly:

Modified basic definition of probability theory: "points" have size, "lines" have width, and they are all quantized.

After the revision, the entire theoretical system of probability theory will become complete and in line with the application purpose of its theory.

4. Conclusion

Through the analysis of the article, we have drawn the following important conclusions:

1) The theoretical basis of mathematics is different from the theoretical basis of physics, so we need to be very careful when using mathematical theoretical tools for physics research, otherwise it will be easy to make mistakes;

2) Finiteness and discreteness should be used as the basis of the entire physical theory; this paper points out that the use of infinite " ∞" and infinite small " 0 " and the continuity of functions without restrictions and demonstrations in physics are not desirable. It will bring a lot of trouble to the theory of physics.

3) Through the analysis of Banach-Tarski paradox and Bertrand paradox, this paper proposes that we should revise the basic assumption of probability theory: it is assumed that "points" have quantized sizes, and "lines" also have quantized widths.

After the correction, we can not only avoid the troubles caused by Bertrand paradox, but also make probability theory better for practical application.

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