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Article

Correction of Sea Brightness Coefficient Satellite Data in the Presence of Dust

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Abstract: Satellite measurements are one of the main sources of data on the state of the marine environment. To obtain information about the sea brightness coefficient , it is needed to correctly carry out atmospheric correction. In the presence of dust aerosol over the Black Sea, physically incorrect values of the spectral brightness coefficient often occur, and specifically negative values in the IR region of the spectrum**.** The main objective of the study is to evaluate the influence of dust aerosol on the spectral dependence of sea brightness, based on analytical calculations from the transfer theory using the principle of plane-parallel layers and results of validation of AERONET-OC field and remote sensing data. The work analyzes spectral dependence of the first error eigenvector of the standard atmospheric correction in the presence of dust aerosol. As result it is given that with an absorbing aerosol, the atmospheric correction error is described by the spectral course of molecular scattering, i.e. close to λ^{-4}.

Keywords: sea brightness coefficient; optical characteristics; ocean color; dust; atmospheric correction

1. Introduction

Optical characteristics of sea water data is contained in the water-leaving radiance that emerged from the water column Lw(λ) [1].Remote sensing instruments, such as MODIS Aqua (Moderate Resolution Imaging Spectroradiometer), measure the spectral radiance exiting the top of the atmosphere (L_{TOA}), which consists of several components. The "atmospheric correction" procedure consists in excluding the contributions of all atmospheric and sea surface components from the L_{TOA} value. Sea brightness coefficient Rrs(λ) is a standard product of the atmospheric correction of satellite data [2].

The results of satellite algorithms are regularly calibrated using innovative methodologies [3], but despite this, a number of errors of standard algorithms were noted. One of these errors is negative values of sea brightness coefficient in the short-wavelength region at 412 nm and 443 nm in the presence of dust [4].

Since absorbing aerosol worsens the quality of satellite products, the identification of dust and the determination of its optical properties is a complex problem relevant for the Black Sea. Large errors in the case of using basic algorithms can be avoided only when the real vertical structure of the absorbing aerosol is close to the candidate model, for which the entire aerosol is located in a thin layer of the atmosphere.

At the moment, there are two algorithms that can take into account the presence of At present, none of the proposed methods has found wide application for automated atmospheric correction in the presence of dust aerosol, since they require additional information on aerosol stratification [5-7]. The main objective of the study is to evaluate the influence of dust aerosol on the spectral dependence of sea brightness, based on the results of validation of AERONET-OC field and remote sensing data. Spectral dependence of the first error eigenvector of the standard atmospheric correction in the presence of dust aerosol.

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2. Materials

One of the most effective instruments of studying the characteristics of atmospheric aerosol, as well as field measurements of Ocean color, is the global network of observation ground automated stations (platforms) AERONET (Aerosol ROboties NETwork). The advantage of this network is the use of the same type of automatic photometers (Cimel-318) and standardized procedures for calibration and processing of the received data. For the entire period of operation within the framework of the AERONET network, the Black Sea region was represented by 4 regularly measuring stations: Sevastopol (44.616N, 33.517E), Gloria (44.600N, 29.360E), Galata_Platform (43.045N, 28.193E) and Eforie (44.075N, 28.632 E). However, now not all stations continued to operate within this network: the Sevastopol station ceased to function in 2015(Figure 1).

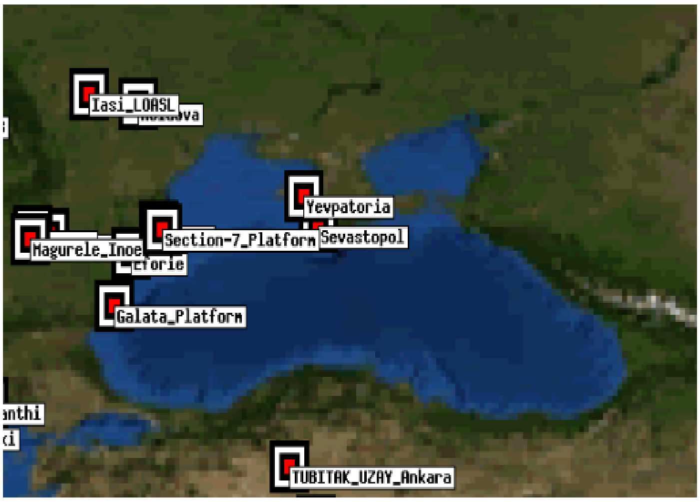

Figure 1. Placement of AERONET stations for Black Sea region.

For the western part of the Black Sea, data on the water-leaving radiance (L_w) are regularly provided, as well as the normalized water-leaving radiance (L_{WN}) calculated by the method proposed by Zibordi et al. [8-9] to remove the dependence on survey geometry and bidirectional effects in L_w. It is worth noting, since in the future satellite and field measurements of the water-leaving radiance will be validated and all values of $L_{WN}(\lambda)$ will subsequently be converted to $Rrs(\lambda)$ by dividing by the solar constant $S_0(\lambda)$.

The source of satellite measurements of $Rrs(\lambda)$ was the results of MODIS-Aqua radiometer measurements. MODIS-Aqua has 36 spectral channels, but only 9 of them were originally related to the ocean color (including the 673-683 nm channel, designed to detect chlorophyll fluorescence excited by solar radiation. MODIS has radiometric sensitivity in 36 spectral channels in the spectral range from 0.4 to 14.4 μm. For monitoring the state of the ocean, the most interesting are the aerosol optical depth, the optical thickness and height of clouds, the concentration of chlorophyll, the concentration of suspended particles and the dispersion index of marine suspension, the absorption index of sea water, as well as day and night temperature of the ocean surface.

3. Results

In this study, we propose to use an analytical method for accounting for aerosol stratification in the problem of radiation propagation in plane-parallel layers.To describe the effect of absorbing aerosol on radiation transfer, it is proposed to use the principle of interaction of plane-parallel layers, which works with the transmittance (T) and reflection coefficients of a plane-parallel layer (R) [10-12]. The reflectance as long as transmittance is expressed through upward and downward radiance and can be calculated as:

$$R,T = \frac{\pi \cdot L}{\mu_0 F_0}$$

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Where L is upward or downward radiance at the boundary of the layer, μ_0 – cosine of the Sun's zenith angle.$F_0(\lambda)$ – irradiance of perpendicular surface. For generality, the transmission function includes direct light attenuated according to Bouguer's law, i.e.

$$T = \frac{\pi \cdot L_{sc}}{\mu_0 F_0} + \frac{\pi}{\mu_0}\exp[-\tau / \mu_0]\cdot\delta(\mu_0 - \mu)\cdot\delta(\varphi_0 - \varphi)$$

where L_{sc} is scattered radiance; τ – optical thickness of the layer; $\delta(x_0 - x)$ – Dirac delta-function; μ – cosine of observation zenith angle; φ_0, φ – azimuth angles. According to the theory both R and T is considered as operator X working with arbitrary radiance field $L_0(\mu_0, \varphi_0)$:

$$\hat{X}\cdot L_0 = \frac{1}{\pi}\int_0^{2\cdot\pi}\int_0^1 X(\mu,\varphi,\mu_0,\varphi_0)\cdot L_0(\mu_0,\varphi_0)\mu_0 d\mu_0 d\varphi_0$$

For inhomogeneous layers, one should distinguish between reflection and transmission of the layer when light falls from above and below. For an optical system consisting of two layers, the corresponding operators are determined from the solution of the system of linear equations. In a case when the second layer is optically thin we do not need to take into account the re-reflection between layers. Then the total reflectance will be following:

$$\widehat{R} = \widehat{R}_1 + \widehat{T}_1^u \cdot \widehat{R}_2 \cdot \widehat{T}_1^d \tag{1}$$

where lower index means number of layer, while upper does direction of propagation for the radiance. For an approximate estimate, let's move from (1) to the scalar form:

$$R = R_1 + T^u T^d R_2 \tag{2}$$

In the present case, the layer is added from below and the vertical axis is directed downwards. Since the reverse direction of the vertical axis is accepted in the optics of the atmosphere, then (instead of the dependences of optical characteristics on height) we will assume that the scattering and absorption indices depend on the ratio of atmospheric pressure at a given height $P(h)$ to atmospheric pressure at sea surface level P_0. Thus, the dimensionless depth value can be calculated as $z = P(h)/P_0,\ z\in[0,1]$. Consequently, $d\tau_m/dz = const = \tau_m^0$, where τ_m – is the optical thickness of the upper layer of the molecular atmosphere at a specific height *(h)*, τ_m^0 – is the total optical thickness of the molecular atmosphere. In turn, the optical characteristics of an aerosol depend on its stratification. Since the second layer is thin, there is an analytical expression for the reflectance:

$$R_2 = \frac{p(\cos\gamma)b(z)dz}{4\mu\ \mu_0} \tag{3}$$

Where $b(z)$ is the total scattering (aerosol+Rayleigh) at the depth z, $p(\cos\gamma)$ is the phase function depending on scattering angle γ, $\cos\gamma = \sqrt{1-\mu^2}\sqrt{1-\mu_0^2}\cos(\varphi-\varphi_0) - \mu\cdot\mu_0$.

Substituting (3) into (2) we get:

$$\frac{dR}{dz} = T^u(z)\cdot T^d(z)\cdot\frac{p(\cos\gamma)\cdot b(z)}{4\mu_0\mu} \tag{4}$$

We consider this mechanism from the point of view that the scattered radiance passes through an absorbing aerosol. That is why we should estimate T functions. In a single-order approximation T functions account attenuation due to scattering and absorption. Indeed, the attenuation due to scattering is compensated by a multiple scattering, especially for anisotropic phase function, i.e. $p(\cos\gamma) < 1$. If T1 and T2 are considered equal to 1, then this will be Gordon's linear approximation. In this case, we

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propose to take into account the absorption in the transmission functions by introducing the vertical profile of the aerosol absorption as $a(z)=\frac{d\tau_a(z)}{dz}\cdot(1-\Lambda(z))$, where $\tau_a(z)$ is aerosol optical depth from the top of atmosphere up to depth z, $\Lambda(z)$ – single scattering albedo. In the case of exponential height dependences of aerosol and molecular scattering, the aerosol stratification function has the form of a power function $g(z)=\frac{1}{\tau_a^0}\frac{d\tau_a(z)}{dz}=\frac{h_m}{h_a}z^{\frac{h_m-h_a}{h_a}}$, where τ_a^0 – AOT and $h_m\approx 8\,km, h_a\approx 1.2\,km$ – the height of the equivalent homogeneous atmosphere for air molecules and aerosol particles. We propose to take into account the absorption in the transmission functions by introducing a(x) - the variability of the absorption with respect to height:

$$T^u(z)=\exp\left[-\frac{1}{\mu}\int_0^z a(x)dx\right], T^d(z)=\exp\left[-\frac{1}{\mu_0}\int_0^z a(x)dx\right] \quad (5)$$

Formula (4) can be represented as a sum with the substitution of the expression for T.

$$\frac{dR}{dz}=\frac{p_m(\cos\gamma)\cdot b_m(z)}{4\mu_0\mu}\exp\left[-\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\cdot\int_0^z a(x)dx\right]+\frac{p_a(\cos\gamma)\cdot b_a(z)}{4\mu_0\mu}\exp\left[-\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\cdot\int_0^z a(x)dx\right]$$

where the subscripts *m* and *a*– refer to molecular and aerosol scattering, respectively.

In the present work, we are not interested in solving the indicated approximate equation, but in an analytical estimate of the error in the effects of absorption by aerosol, i.e., difference of two solutions of the equation, at $a(z)=0$ and at $a(z)\neq 0$.

$$r=R(a(z)=0)-R(a(z)\neq 0)$$

The second term refers only to the optical properties of the aerosol and can formally be considered as part of the aerosol model, the choice of which is based on signal values in the near-IR range. The first term describes the decrease in the contribution of molecular scattering and, therefore, significantly affects the atmospheric correction error in the short-wavelength part of the visible range. Referring to this, formula (4) can be written as:

$$\frac{dr}{dz}=\frac{p_m(\cos\gamma)\cdot\tau_m^0}{4\mu_0\mu}\left(1-\exp\left[-\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\cdot a_0\int_0^z g(x)dx\right]\right) \quad (6)$$

Where $a_0=(1-\Lambda)\tau_a^0$ – optical thickness of aerosol absorption.

It should be noted that the value of the integral $\int_0^z g(x)dx<1$ due to stratification and $(1-\Lambda)<<1$. The value of AOT multiplied by the geometric factor is limited by the possibility of observing the sea color, then the exponent indicator is small. Then the expression for the atmospheric correction error due to the stratification of the absorbing aerosol has the form

$$r=\frac{p_m(\cos\gamma)\cdot\tau_m^0(\lambda)}{4\mu_0\mu}a_0\cdot\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\int_0^1\int_0^z g(x)dx\cdot dz \quad (7)$$

where the factor a_0 takes into account the absorbing properties of the aerosol, observation geometry is taken into account in $\frac{p_m(\cos\gamma)}{\mu_0\mu}\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)$, while the double integral does not depend on the wavelength and takes into account the stratification of the absorbing aerosol with respect to air molecules. According to Rayleigh's law, we know that $\tau_m^0\approx\lambda^{-4}$. Therefore, with an absorbing aerosol, the atmospheric correction error is described by the spectral course of molecular scattering, i.e. close to λ^{-4}.

Next, an experimental verification of this theoretical conclusion is carried out. We analyzed 49 cases of dust plums over the Black Sea region and 133 days with a clean atmosphere (no clouds, smog or haze). All selected cases were supported by visual analysis of satellite imagery, as well as analysis of 7-day return trajectories calculated daily by the NASA Goddard Space Flight Center (GSFC) [13]. Additionally, for the selected dates, an analysis was made of the optical characteristics of the atmosphere (aerosol optical thickness (AOD) and the Angstrom parameter (AE)) using AERONET network data for Gloria and Galata stations (Table 1). The comparative analysis touched not only on the validation of the spectral radiance of the sea, but also on the aerosol optical depth, where in situ AOD measurements were taken as reference values. Thus, AOT at 862 nm according to the results of MODIS-Aqua and AERONET correlate by 72% for the Black Sea. It should be noted that in the presence of dust aerosol, the correlation of AOT values drops to 37% for the Black Sea (systematic underestimation of satellite AOT).

Table 1. Optical characteristics of atmospheric aerosol under different conditions of the atmosphere, under Black Sea taking into account RMS.

	Clean Atm.	**Dust**
	AOT	
1020 нм	0,039±0,005	0,072±0,01
870 нм	0,051±0,007	0,091±0,02
667 нм	0,077±0,01	0,133±0,03
551 нм	0,110±0,01	0,184±0,05
532 нм	0,117±0,03	0,195±0,06
490 нм	0,132±0,02	0,217±0,08
443 нм	0,157±0,02	0,256±0,08
412 нм	0,175±0,03	0,283±0,10
	AE	
440–870	1,688	1,499
440–865	1,775	1,535

As can be seen from Table1, dust aerosol has the highest AOT, exceeding more than 2 times the average AOT for a clean atmosphere. The AE parameter decreases on average by 20–25% in the presence of dust and haze. In general, we obtained similar AOT spectra as for [14], which additionally confirms the correctness of the selection.

The principal component method (PCA method) was chosen as the mathematical basis for the variation of satellite and field measurements. Often, for such tasks of evaluating the performance of an algorithm, least squares metrics are used, in particular, regression mean square error (RMSE), coefficient of determination (R^2) and regression slope. The regression mean squared error provides a suitable metric for testing exercises when the error distributions are Gaussian and when the goal of the study is to detect sensitivity to outliers (which is possible when testing the model). However, non-outlier Gaussian datasets are not ubiquitous for all ocean color data, which sometimes makes these metrics more informative than metrics without as much sensitivity to outliers and non-Gaussian distributions. Therefore, in this paper, an alternative mathematical analysis was proposed to estimate the spectral features of the change in Rrs(λ) under various external conditions, the PCA method for selected cases with an estimate of the contribution of the first eigenvector. The covariance matrix of a random vector is a characteristic of its distribution, more information about this method is contained in [15-16]. For a specific problem, the difference field values for AERONET stations and the corresponding satellite values from MODIS-Aqua satellite observations were taken as a conditional mathematical expectation. The first eigenvector describes the main variability of the reconstructed brightness coefficient due to errors in the atmospheric correction procedure, as well as changes in the optical properties of the sea during the transition

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from the coastal part of the Black Sea to its central part or in the case of spatial brightness inhomogeneities during phytoplankton blooms. Earlier in [17], this approach was already used to study the influence of dust aerosol, but the mathematical expectations of the covariance matrix were calculated based on the values of Rrs(λ) averaged over the pixels of the study area, i.e. only the results of remote sensing were taken into account without reference to field measurements. Even then, it was found that in the single cases considered for a clean atmosphere, the first eigenvector had minimal atmospheric correction errors, while for dust, the first eigenvector had the form of a power function. In an extended problem, where long-term statistics for specific pixels from AERONET stations are analyzed, a new approach was chosen using PCA analysis with reference to field measurements. The results of the covariance analysis of the comparison of satellite and field measurements for the Black Sea according to the MODIS-Aqua measurements are presented in Figure 2.

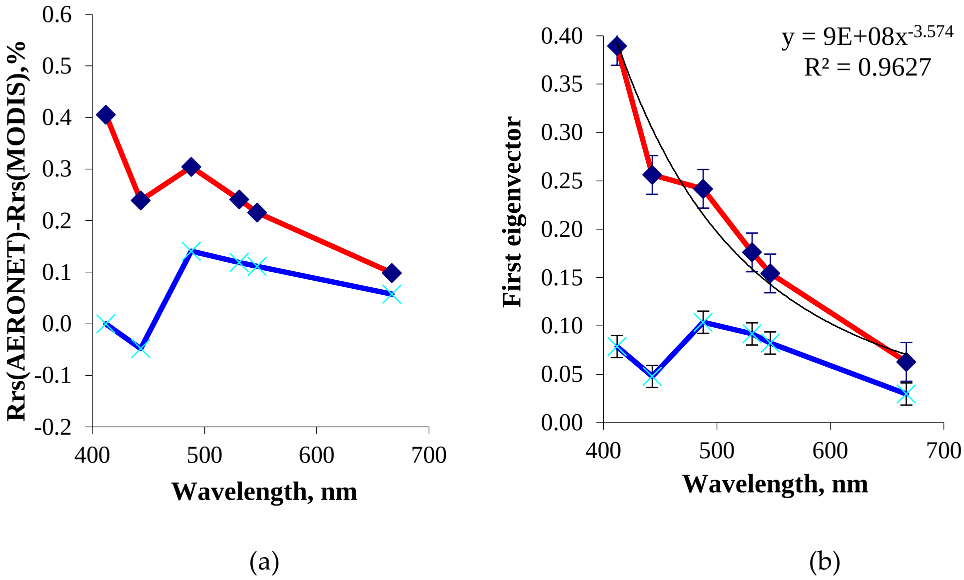

Figure 2. Validation error for Rrs(between MODIS Aqua satellite values and AERONET field measurements) for the Black Sea and the first eigenvector of the covariance matrix.

The contribution of the first eigenvector (Fig. 2(b)) in the presence of dust aerosol was 85%. When it was approximated, we got the expression $y = 9E{+}08\ \lambda^{-3.574}$, where the coefficient of determination was $R^2 = 0.96$. The results of data validation in dusty conditions confirmed the analytical conclusion that, in the presence of absorbing aerosol, the spectral law of atmospheric correction errors is close to the -4 function. This effect is explained by the fact that dust aerosol is determined by the methods of remote sensing using the Gordon and Wang algorithms using the infrared channel, but the arid aerosol has the main effect on the ratio of the aerosol and molecular components.

4. Discussion

To develop the algorithm, it was necessary to provide analytical estimates to take into account aerosol stratification in the radiative transfer equation, and to prove that dust aerosol affects the value of the atmospheric correction error. In this paper, we propose an approach to describing the effect of stratification of an absorbing aerosol. The factors affecting the difference between the reflection coefficients of the atmosphere with non-absorbing and absorbing aerosol are identified. The spectral dependence of molecular scattering, the spectral properties of its absorption by aerosol, stratification, and the geometric factor are included in the composition as factors. First of all, we are interested in the form of the spectral function in order to use it as an interpolation function for atmospheric corrections errors. In this study, it is shown that, with an absorbing aerosol, the atmospheric correction error is described by the spectral course of molecular scattering, i.e. close to λ^{-4}. This is due to the absorption of the aerosol component of the molecular component. Analytical conclusions were confirmed during the validation of satellite and field measurements. To analyze the characteristic trends in the absolute error of satellite and field Rrs(λ), the principal component method (PCA) was used for selected dates with an estimate of the contribution of the first eigenvector of the covariance matrix. As a result, it was found that the largest difference between satellite and field measurements is present in the case of dust aerosol, since the average difference in the sea brightness coefficient is maximum. In the presence of an absorbing aerosol, an explicit systematics is observed, namely, when approximating the first eigenvector for MODIS, we obtained $y = 9E{+}08\ \lambda^{-3.574}$. The spectral course of the first vector in cases of dust shows a tendency to increase in the short-wavelength region with an intermediate local maximum of about 500 nm and a sharp decrease in values in the long-wavelength region of the spectrum, this effect is explained by the fact that dust aerosol is determined by remote sensing methods using the Gordon and Wang algorithms using the infrared channel , however, the arid aerosol has the main influence on the ratio of the aerosol and molecular components. In the future, this information can serve as a basis for creating a regional algorithm for additional data correction.

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Data Availability Statement: The data presented in this study are available on request from the corresponding author.

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Conflicts of Interest: The authors declare no conflict of interest.

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