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On a 2D variant of Newton's gravitational law and the MOND theory.

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Abstract

The efficiency of the MOND theory of Mordehai Milgrom might be unrelated to distance, but instead connected to flatness of galaxies and the fact that the large scale distribution of matter is essentially two-dimensional. A combination of the pushing gravity paradigm with essential flatness of large luminous matter clusters leads to an approximate MOND theory which would allow to avoid the dark matter hypothesis.

Key words: gravitation, MOND theory, galactic velocity curves.

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1 Introduction

While preparing a pedagogical document to explain the pushing gravity of Fatio-Lesage for a general public, all of a sudden, the author wondered what could be a 2 dimensional version of Newton's law of gravitation. A possible connection then appeared between the laminar distribution of matter at high scale and the slow decay of gravitational attractive forces between very distant objects proposed by Mordehai Milgrom in his MOND model. This may lead to a new explanation of the strange velocity curves far from the center of galaxies, as well as Zwicky's paradox in the Coma galaxy cluster. In this case, there would be no need for dark matter to explain the divergence from Newtonian attraction law. The inverse square law may turn out to be an effect of spatial dimension three. And in 2D , the inverse square would be simply replaced by...the inverse law without square! After recalling briefly the missing mass enigma and the principle of the Fatio-Lesage theory, we explain how it may lead to a variant of Milgrom's MOND hypothesis. Moreover, although the effect of laminar distribution leads naturally to overcoming the paradox in the framework of the Fatio-Lesage theory, it might even appear natural without making any assumption on the cause of gravity. Our conclusion is that the strange velocity curves observed in spiral galaxies may be a consequence of their flatness, both globally and locally near the boundary.

2 Recalling the missing mass enigma.

The missing mass problem originated in very interesting observations made by Fritz Zwicky around 1930 and reported in the two basic papers [13, 14]. While examining the Coma galaxy cluster in 1933, Zwicky discovered the existence of a gravitational anomaly, consisting in an excessive rotational velocity of the luminous matter compared to the calculated gravitational attraction within the cluster. He calculated the dynamical gravitational mass of the galaxies within the cluster from the observed rotational velocities and obtained a value much larger than expected from the total luminosity of the cluster. In [5], we pointed out the accumulation of circumstances which led to an exageration of the discrepancy, but the discordance remains and presently many specialists, following an hypothesis made by Zwicky himself, are looking for unseen "dark matter", possibly of non-baryonic nature. This track became popular after the discovery of the so-called flat galactic rotation curves by Vera Rubin (cf. [10, 11]), also called the galaxy rotation problem.

3 The MOND theory.

In 1983, in [9], M. Milgrom made the hypothesis that Newton's law might not be correct for very remote objects and in this case the inverse square law should be replaced by an inverse law. It seems that his idea allows to understand the motion of the most external parts of galaxies and clusters and might give an alternative to the dark matter hypothesis. However, it is not easy to find a physical interpretation of this divergence from Newton's law. Then a modified-inertia MOND approach was proposed as a change in Newton's second law at small accelerations. This questions the fondations of dynamics, as a matter of fact even the classical Newton's second law is difficult to understand in a completely empty space, cf. [7] .

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4 The Fatio de Duillier - Lesage theory.

According to Wikipedia: "Le Sage's theory of gravitation is a kinetic theory of gravity originally proposed by Nicolas Fatio de Duillier in 1690 and later by Georges-Louis Le Sage in 1748. The theory proposed a mechanical explanation for Newton's gravitational force in terms of streams of tiny unseen particles (which Le Sage called ultra-mundane corpuscles) impacting all material objects from all directions. According to this model, any two material bodies partially shield each other from the impinging corpuscles, resulting in a net imbalance in the pressure exerted by the impact of corpuscles on the bodies, tending to drive the bodies together." In [5], we describe this paradigm in a rather detailed manner and we recall the main usual objections against the Lesage theory, On the other hand, we observe that the pushing gravity model opens the door to a possible variability of the "gravitational constant" G at very large spatial (or time) scale. And we point out that different local gravitational constants might fill the gap in Zwicky's estimate.

5 Newton's law in 1D and 2D.

5.1 Starting point

In the previous preprint [6], the author tried to understand the simplest case of Lesage's pushing gravity, namely the mutual attraction of two nucleons. He found out that the theory, contrary to what was claimed until now, can work even with purely elastic shocks, because the gravitons transfer a part of their kinetic energy even in the elastic case. The argument according to which rebounding gravitons can cancel the effect of incoming ones is also answered by this "toy model" since rebounding gravitons have less kinetic energies than directly incoming corpuscles.

5.2 A peculiar situation

After that study, the author tried to imagine how to recover Newton's law of gravitation for massive objects by summing the vector fields corresponding to atoms. But he readily realized that starting from punctual corpuscules makes it impossible to take account of the distance! Because in the calculations, the distance of the nucleons has no effect. Which means that in 1D, the inverse square factor just disappears.

5.3 What happens in 2D?

It is only when trying to picture out the situation for a teaching purpose that the author realized something: the inverse square law in Newton's formula is related to dimension 3. It comes from the fact that the proportion of gravitons eclipsed by one body seen from a distant point at distance d is proportional to the solid angle, varying like $1/d^2$. In one dimension, the distance has no effect, and in 2D, the angle of vision is proportional to $1/d$. Therefore in 2 dimensions, Newton's law should become

$$F = -k\frac{mm'u}{||u||^2}$$

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where u is the vector difference of positions between two quasi-punctual flat coplanar objects. In other terms, in the case of small 2D masses confined in a plane the force is radial directed towards the attracting object, with norm

$$||F|| = k\frac{mm'}{d}$$

where $d = ||u||$. Here the gravitational potential becomes logarithmic, which may look counterintuitive, but it was already the case for the MOND model.

5.4 Does it really depend on Lesage's paradigm?

What is involved here is geometry, and this seems to be rather independent of the hypotheses made on the cause of gravity. Newton did not need that to write his formula in 3D. Hence the formula in 2D might finally be more intrinsic than initially imagined.

6 Recovering a MOND like effect in the case of galaxies.

It is interesting to try to compare the effect of gravity for the rotation of stars which are close to the center and those who are far from it. In the first case, the gravitational effect is basically 3D, since nearby central stars, the thickness is comparable to the distance from the center, so that the most important contribution follows the 3D Newton law. On the other hand, far from the center, the global contribution is more and more comparable to 2D gravity because the thickness becomes small compared with the average distance of attracting stars. This way, we should recover a velocity curve very similar to what can be computed using the MOND paradigm. This is all qualitative, but we must also point out that since thickness tends to 0 near the boundary of the galaxy, the predominance of 2D effects is even reinforced for the most external part of the galaxy.

7 Conclusion.

The situation of the matter in galaxies is close to a 2D setting. It might be the same for galaxy clusters, contrary to the assumption made by Zwicky at a time when serious studies about accretion disks did not start. It is very strange that professionals do not seem to have suspected that flatness of galaxies could play an important role in the overall gravitational effect. Large structures, for a reason which is not clarified yet, tend to organize in laminar structures, as shown by the discoveries of Lapparent & alt [8]. This might be the reason why the MOND model has some success not only in predicting the velocity curve in galaxies near the boundary, but also to solve Zwicky's paradox. Finally, we understand that it could be difficult to justify this at the quantitative level by calculations involving classical methods such as the virial theorem. A limiting continuous model does not seem much easier to handle, and passing to the limit when the thickness tends to zero seems to be a non-obvious mathematical challenge requiring the invention of new methods.

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