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Klein-Gordon, Dirac and Schrodinger Equations, Matter and Wave Function Collapses, Photon Momenta and the Gamma Ray Burst based on the 4-D Euclidean Space

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Abstract;
The negative energy solutions of the Klein-Gordon and Dirac equations have been used to define the antiparticle with the positive energy from the Feynman - Stueckelberg interpretation in the standard model based on the 4-D Minkowski space with the relative time. The negative energy solutions of the Klein-Gordon and Dirac equations are used to define the antiparticles with the negative energy in the present 3-D quantized space model. Note that all the present results are based on the 4-D Euclidean space with the absolute time (t) and relative time (t_l). Then the particle with the positive energy and antiparticle with the negative energy have the 4-D P_4 = TCP = T_cP symmetry. The continuity equations are derived from the Klein-Gordon, Dirac and Schrodinger equations. The Klein-Gordon and Schrodinger equations with the potential barriers are solved. The Klein paradox is explained by using the pair production of the electron with the positive energy and positron with the negative energy under the $P_4 = T_cP$ symmetry, The effective velocity ($c_{eff} = \sqrt{2}$ c) and effective energy ($E_{tl} = \sqrt{2}E$) along the relative time axis (ct_l) give the relative time momentum of $p_{tl} = \frac{E_{tl}}{c_{eff}} = \frac{\sqrt{2}E}{\sqrt{2}c} = \frac{E}{c} = p_t = p_x$. This indicates that the photons with the energy of E have the constant photon velocity of c and the constant photon momentum of $p_{tl} = \frac{E_{tl}}{c_{eff}} = \frac{\sqrt{2}E}{\sqrt{2}c} = \frac{E}{c} = p_t = p_x$. For the negative energy particles, the space momentum directions are opposite to the particle velocity directions because the energy is negative. And the matter collapse to the photons at the very high particle velocity is discussed from the length expansion. It is, for the first time, proposed that the gamma ray burst is originated from the matter collapse to the photons near the black hole. Also, the wave function collapse is explained from the length expansion. The wave function collapse takes place when the measurement makes the particle velocity to be zero. It is concluded that the negative energy solutions of the Klein-Gordon and Dirac equations support the existence of the partner antimatter universe with the negative energy and negative time direction. Our matter universe has the positive energy and the positive time direction.

Key words; $P_4 = T_cP$ symmetry; Matter collapse; Wave function collapse; Klein paradox; Continuity equations; Negative energy solutions; Gamma ray burst; Photon momenta

Contents

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1. Introduction

The pure absolute negative energy and the negative time direction are not allowed in the standard model (SM) based on the 4-D Minkowski space. Here the time (t) is the relative time and only the relative negative energy has been used. And the Klein-Gordon equation [1 – 5] gives the negative energy solution of $E=-(E_0^2+p_x^2c^2)^{0.5} < 0$ with the negative particle energy density of $\rho = E|\Psi|^2$. Because of these two problems, the Klein-Gordon equation has been discarded. Then the Dirac equation [6 – 10] was introduced for the electron by including the spin terms. The Dirac equation gives the negative energy solutions with $E=-(E_0^2+p_x^2c^2)^{0.5} < 0$, and positive particle probability density of $\rho = \overline{\Psi}\gamma^0\Psi = |\Psi_1|^2 + |\Psi_2|^2 + |\Psi_3|^2 + |\Psi_4|^2 > 0$. And the negative energy solutions of the electron obtained from the Dirac equation have been reinterpreted to define the positron with the positive energy because the negative energy solution is not allowed in the standard model. In other words, it is proposed from the Feynman - Stueckelberg interpretation [11,12] that the positron with the positive energy (E>0) and forward time direction (t>0) is the same as the electron with the

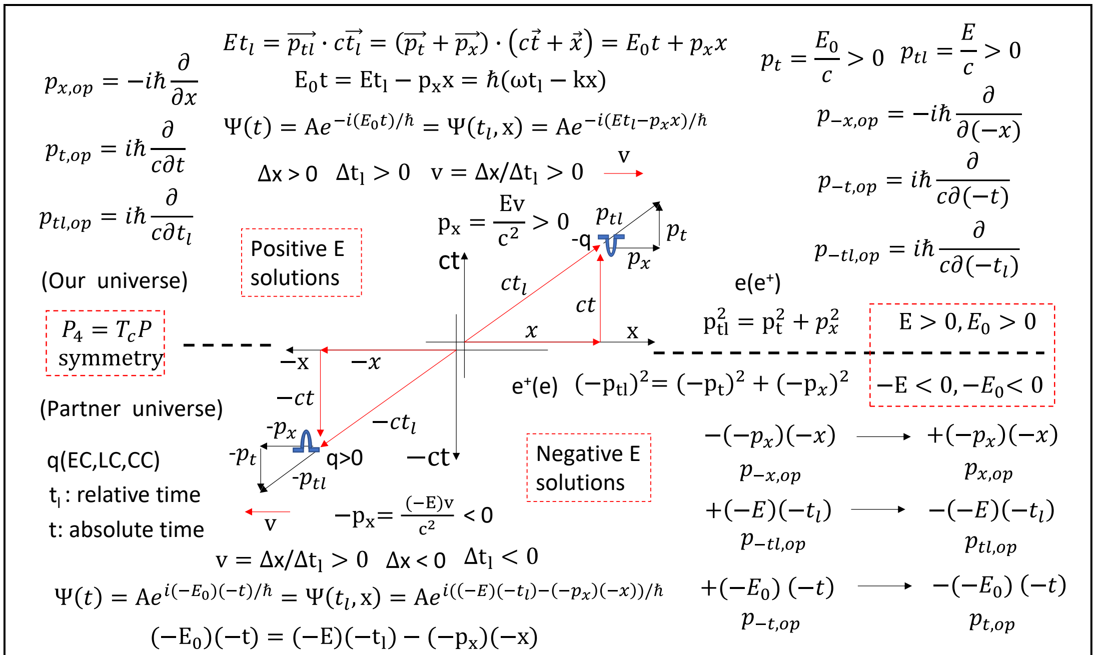

Fig. 1. Positive energy solutions and negative energy solutions have the 4-D P_4 = TCP = T_cP symmetry. The corresponding plane wave functions are shown.

negative energy (-E) and backward time direction (-t) because of the (-E)(-t) = (E)(t) relation. Often this concept of the antiparticle like the positron has been applied in the Feynman diagram. In fact, the positron that was predicted in the Dirac equation was experimentally discovered. However, it is clear that the negative energy (-E) and backward time (-t) are different from the positive energy (E) and forward time (t), respectively even though (-E)(-t) is the same as (E)(t) mathematically. Also, the reason why the charge sign of the particle with the negative energy and backward time is changed to the different charge sign of the antiparticle with the positive energy and forward time is not explained. Therefore, it is thought that the definition of the antiparticle like the positron obtained from the Feynman - Stueckelberg interpretation is not well justified. The Schrodinger

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equation is the non-relativistic version of the Klein-Gordon and Dirac equations. In the Schrodinger equation, the total energy is the sum of the kinetic energy and potential energy. The

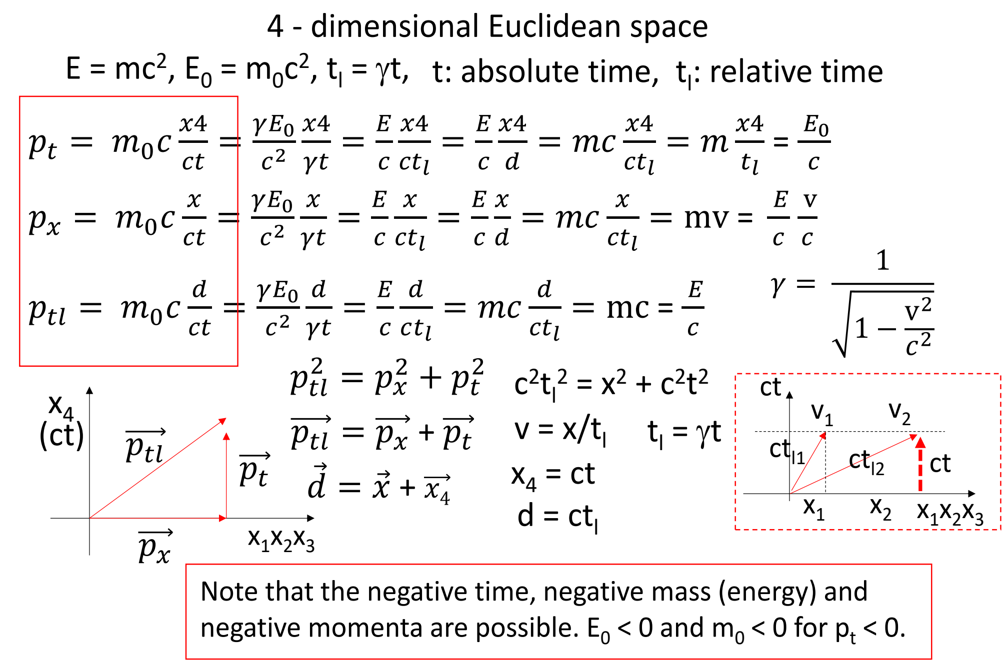
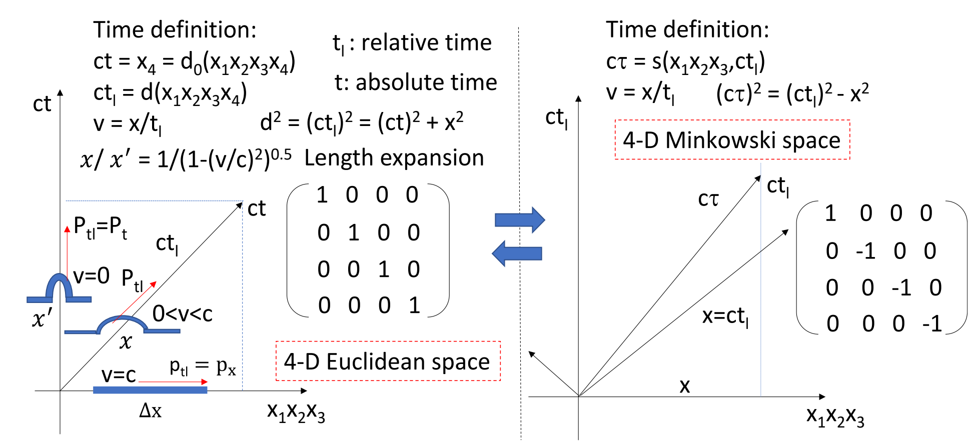


Fig. 2. The 4-D momenta based on the 4-D Euclidean space are shown.



The photons are the flat space. $P_{tl}=P_x$ is the observed momentum for the photon with Δx. Then, $E = \hbar\omega = P_xc$. And the photon has the finite length (Δx) and finite energy (E) because of the matter collapse at the high velocity of v.

Fig. 3. The 4-D Euclidean and 4-D Minkowski space are compared.

rest mass energy is omitted in the Schrodinger equation in the standard model.

In the present 3-D quantized space model (TQSM) based on the 4-D Euclidean space, the absolute negative energy and negative time direction are allowed in Figs. 1, 2 and 3. The 4-D Minkowski space and 4-D Euclidean space are compared in Fig. 3. Then the time (t) along the time axis is the absolute time. The time (t_l) along the variable 4-D distance axis is the relative time. Therefore, the

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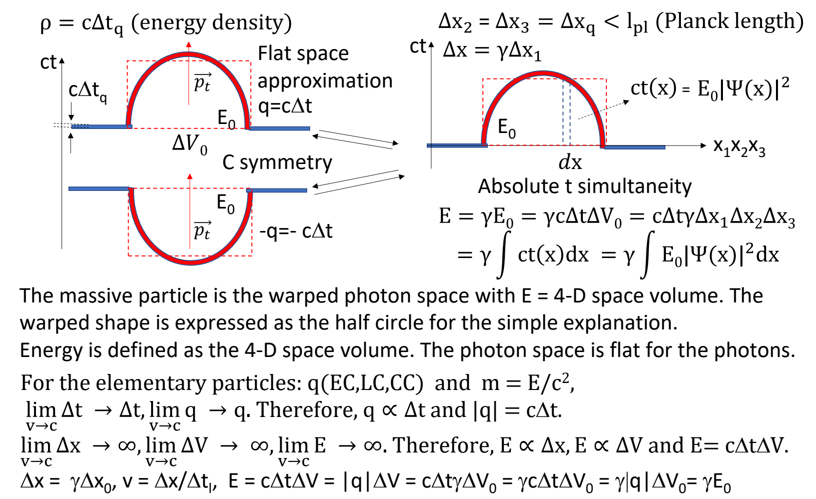

Fig. 4. The quantum wave function and the warped photon space are defined on the 4-D Euclidean space.

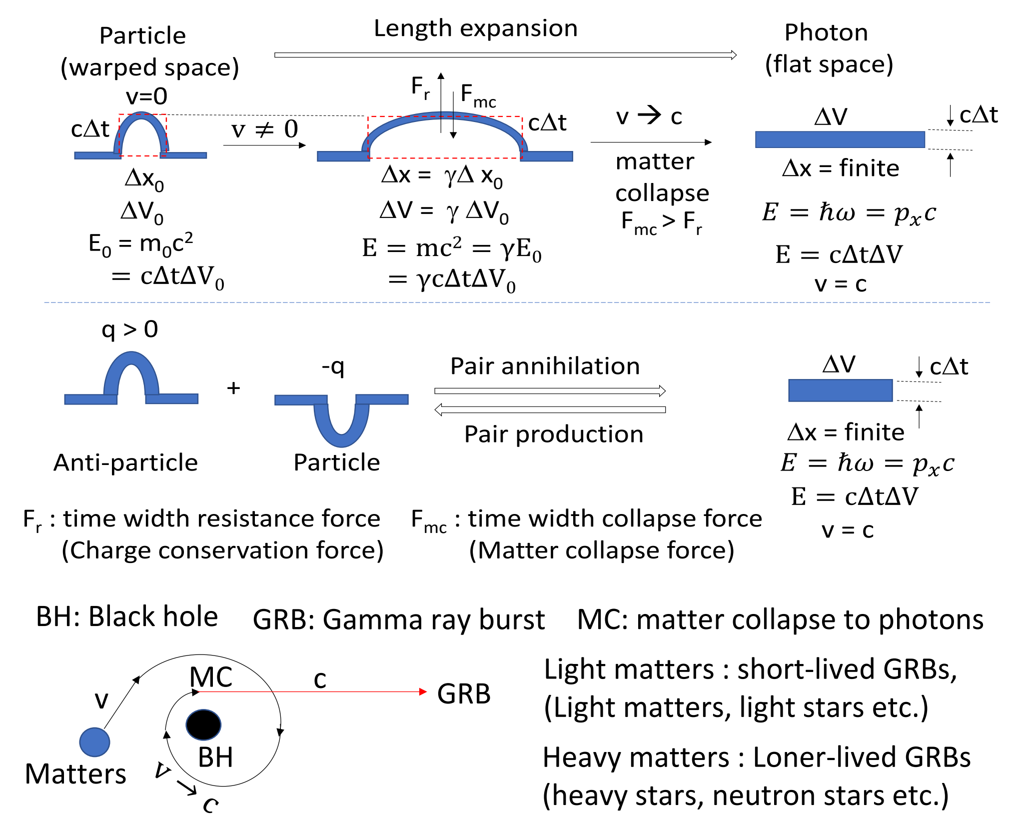

Fig. 5. The matter collapse to the photons at the high velocity is proposed. The gamma ray burst is originated from the matter collapse to the photons.

relative time (t) in the standard model corresponds to the relative time (t_l) in the TQSM model in Fig. 3. Therefore, based on the two times of relative time (t_l) and absolute time (t), the Klein-

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Gordon, Dirac and Schrodinger equations are reinterpreted by including the rest particle masses in the present work. Here, the 4-D momenta based on the 4-D Euclidean space are derived as shown in Fig. 2. Then the negative energy coordinates and positive energy coordinates are related by the 4-D symmetry of $P_4 = TCP = T_cP$ in Fig. 1. The positive energy solutions of the Klein-Gordon and Dirac equations describe the particles within the positive energy coordinates with E>0 and t>0 (forward time) and the negative energy solutions of the Klein-Gordon and Dirac equations describe the antiparticles within the negative energy coordinates with the -E<0 and -t (backward time). In this P_4 symmetry, the charge sign of the particle is changed to the charge sign of the antiparticle. This means that the charge C symmetry is closely related to the time symmetry. Therefore, because the time symmetry should include the C symmetry, the time symmetry is defined as $T_c = TC$ symmetry like the space P symmetry includes the handedness H symmetry. Because the massive particle is defined as the warped photon space in the present TQSM model, the upward warping and downward warping of the particle are defined as the positive charge and negative charge, respectively. The time T_c symmetry changes the warping direction along the absolute time (ct) axis of the particle. Therefore, the negative energy solutions are explained by the 4-D $P_4 = T_cP$ symmetry in Fig. 1. The pair production of the electron with the positive energy and the positron

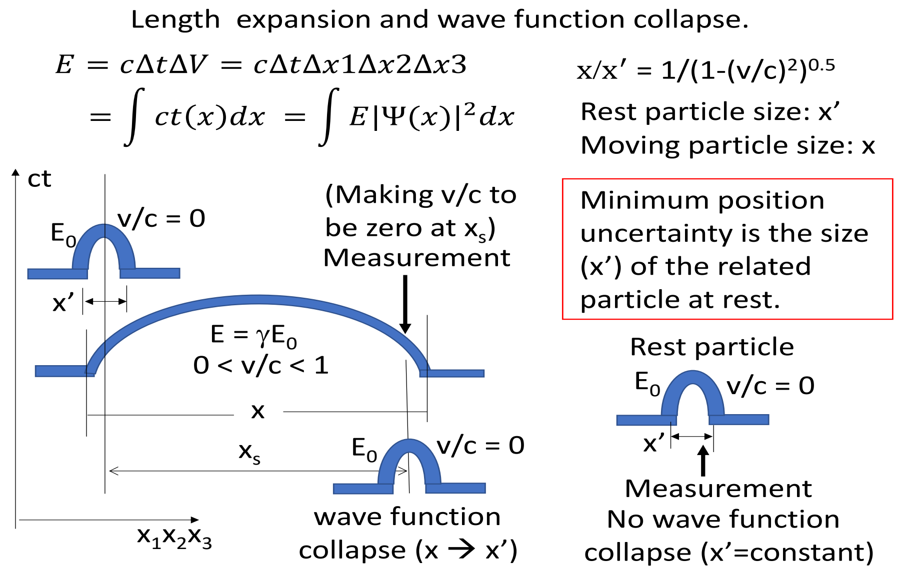

Fig. 6. The wave function collapse takes place from the measurement which makes the particle velocity to be zero.

with the negative energy by the $P_4 = T_cP$ symmetry is applied to explain the Klein paradox. The continuity equations are also derived from the Klein-Gordon, Dirac and Schrodinger equations based on the 4-D Euclidean space in terms of the TQSM model. Also, the plane wave solutions of the Klein-Gordon and Schrodinger equations including the potential barriers [13 – 19] are derived based on the 4-D Euclidean space. The Klein paradox [15 – 19] is explained by the $P_4 = T_cP$ symmetry. The wave function collapse by the measurement and matter collapse at the high velocity are explained by using the relativistic length expansion and relativistic energy increase. It is concluded that the gamma ray burst [20 – 27] is originated from the matter collapse to the photons near the black hole. The photons with the energy of E have the constant photon velocity and the constant photon momentum on the 4-D Euclidean space. For the negative energy particles, note that the space momentum directions are opposite to the particle velocity directions.

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2. Photons, matter collapse, gamma ray burst, $P_4 = T_cP$ symmetry and plane wave functions

At the 4-D $P_4 = T_cP$ symmetry, the space P symmetry includes the handedness H symmetry of the left handedness and right handedness. The time T_c symmetry includes the time direction symmetry of the charge C symmetry. The 4-D shapes of the particles change the time direction of the warping under the time T_c symmetry and space direction under the space P symmetry. The particle and its antiparticle have the CH symmetry relation. The downward time (t) warping of the particle is defined as the negative charge and the upward time (t) warping of the particle is defined as the positive charge in Fig. 4. In Fig. 3, the 4-D Minkowski space and 4-D Euclidean space are compared. The photons are the flat space. $p_{tl}=p_x$ is the observed momentum for the photon with Δx. Note the length expansion of the particle given as the function of the velocity in Figs. 3, 4, and 5. Then, $E = \hbar\omega = p_x c$. And the photon has the finite length (Δx) and finite energy (E) because of the matter collapse to the photon at the high velocity of v in Fig. 5. In Fig. 4, the massive particle is defined as the warped photon space. The energy of the particle is defined as the 4-D volume of

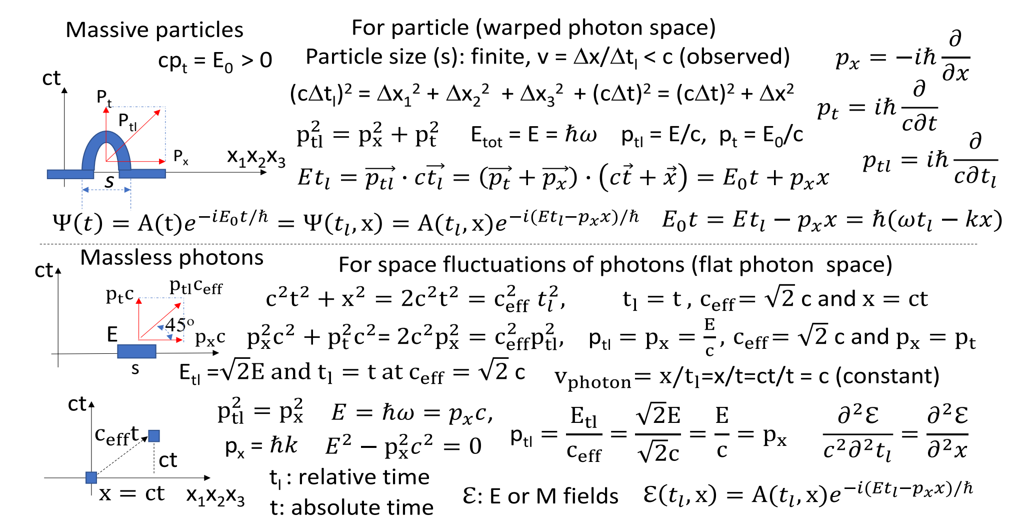

Fig. 7. The photons and massive particles are compared.

the warped photon space in Fig. 4. The charge and energy (mass) of the particle are defined in the present work. The matter collapse from the massive particle to the massless photons is described as the result of the length expansion at the high velocity of the particle in Fig. 5. The pair creation from the photon and pair annihilation to the photon of the particle and antiparticle are compared with the matter collapse in Fig. 5. The reason why the gamma ray burst (GRB) takes place has not been well understood [24 – 27]. The origin of the gamma ray burst is the challenging research topic in the theoretical point of view. In the present work, for the first time it is proposed that the gamma ray burst is originated from the matter collapse to the photons near the black hole in Fig. 5. The matters are falling and swirling toward the black hole by the strong gravitational force as shown in Fig. 5. Then the velocity of the matters is very rapidly increasing. Then, the velocity of the matters is close to the photon velocity near the black hole. Then at the very high velocity, the matters are collapsing to the photons without the gravitational force with the black hole as shown in Fig. 5. Then the strong gamma rays are emitted to make the gamma ray burst. The gamma ray burst takes place for the very short period because the whole matters are rapidly collapsed to the photons for the very short time. In other words, this successful explanation of the gamma ray burst supports the matter collapse to the photons. And in Fig. 6, the wave function collapse is explained as the result of the length expansion and measurement of the particle. The measurement of the particle means making the velocity (v) of the particle to be zero at the measuring space position of x_s in Fig, 6. In Figs. 7 and 8, Massive particle and massless photon are compared for the 4-D momenta. The 4-D momenta are defined in Fig. 2. In Figs. 7 and 8, the massless photon is the flat space and massive particle is the warped photon space. The massive particle has the variable

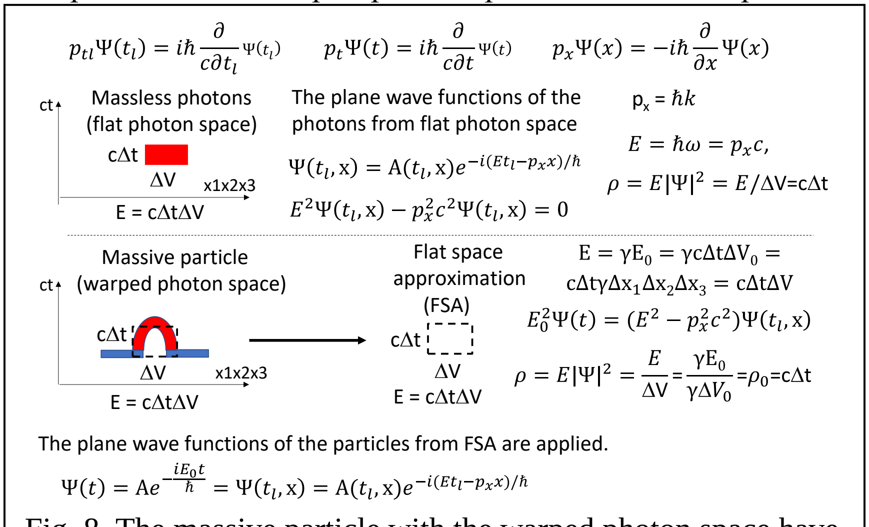

Fig. 8. The massive particle with the warped photon space have the flat space approximation using the plane wave functions.

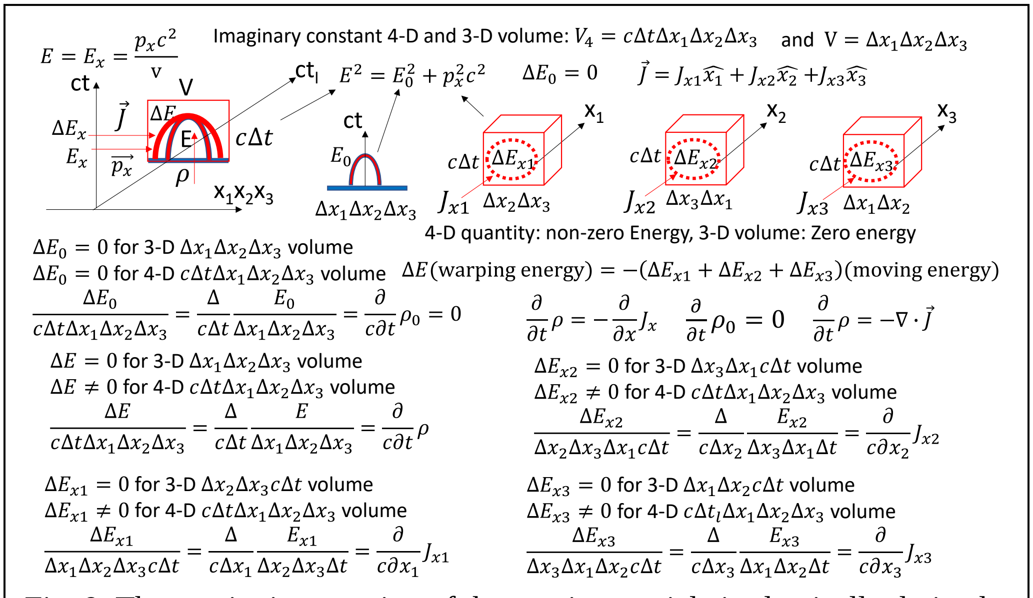

Fig. 9. The continuity equation of the moving particle is classically derived.

velocity of v and the photon has the constant velocity of c. Therefore, the particle and photon have the different analysis of the 4-D momenta. First the massive particle has the 4-D momentum relation of $p_{tl}^2 = p_x^2 + p_t^2$ in Figs. 2 and 7. Then $Et_l = \overrightarrow{p_{tl}}\cdot\overrightarrow{ct_l} = (\overrightarrow{p_t} + \overrightarrow{p_x})\cdot(\overrightarrow{ct} + \vec{x}) = E_0t + p_x x$. The warped shape of the massive particle can be approximated to the flat space as shown in Fig. 8. Then the plane wave function can be used as the first approximation for the massive particle. The applied plane wave function is $\Psi(t) = Ae^{-\frac{iE_0t}{\hbar}} = \Psi(t_l, x) = A(t_l, x)e^{-i(Et_l - p_x x)/\hbar}$ from the relation of $E_0t = Et_l -$

$p_x x = \hbar(\omega t_l - kx)$. The relation between the wave function and warped shape of the massive particle is explained in Fig. 4.

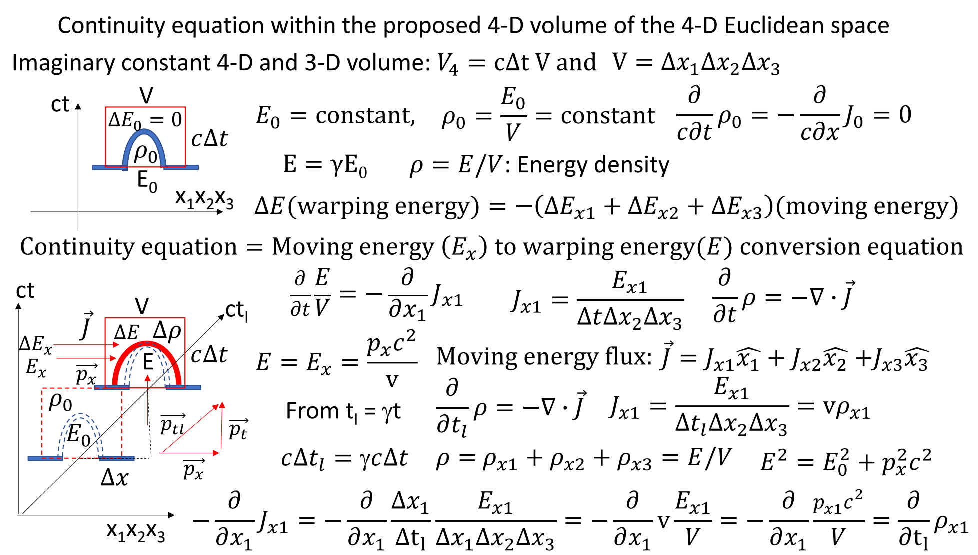

Fig. 10. The continuity equation of the moving particle is classically derived. The moving energy due to the space momentum is converted to the warping energy.

For the plane wave functions of particles,

$$\Psi(t) = Ae^{-\frac{iE_0 t}{\hbar}} = \Psi(t_l, x) = A(t_l, x)e^{-i(Et_l - p_x x)/\hbar}$$

$$\Psi(t_l) = Ae^{-iEt_l/\hbar} \qquad \Psi(x) = Ae^{ip_x x/\hbar} \qquad E = \gamma E_0 \qquad E^2 = E_0^2 + p_x^2c^2$$

$$p_{tl}\Psi(t_l) = i\hbar\frac{\partial}{c\partial t_l}\Psi(t_l) \qquad p_t\Psi(t) = i\hbar\frac{\partial}{c\partial t}\Psi(t) \qquad p_x\Psi(x) = -i\hbar\frac{\partial}{\partial x}\Psi(x)$$

Relativistic Klein-Gordon equation

$$\Psi^*(t)p_t^2\Psi(t) = \Psi^*(t_l, x)(p_{tl}^2 - p_x^2)\Psi(t_l, x) \quad (1)$$

$$\Psi(t)p_t^2\Psi^*(t) = \Psi(t_l, x)(p_{tl}^2 - p_x^2)\Psi^*(t_l, x) \quad (2)$$

$$p_t^2\Psi(t) = (p_{tl}^2 - p_x^2)\Psi(t_l, x)$$

$$(p_{tl}^2 - p_x^2 - \frac{E_0^2}{c^2})\Psi(t_l, x) = 0$$

For the energy density,

$\frac{\partial}{\partial t}\Psi^*(t)\Psi(t) = \frac{\partial}{\partial t}\rho_0 = 0$ because E_0 is constant and $\frac{\partial}{\partial t}A = 0$.

From (2)/2 – (1)/2, and $\frac{\partial}{\partial t_l}\rho = -\nabla \cdot \vec{J} \qquad \vec{J} = J_x\hat{x}$

$$\rho = \left(\Psi^* i\hbar\frac{\partial}{\partial t_l}\Psi - \Psi i\hbar\frac{\partial}{\partial t_l}\Psi^*\right)/2 \qquad \rho = E|\Psi|^2$$

$$J_x = -i\hbar c^2(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)/2 \qquad J_x = p_x c^2|\Psi|^2$$

$\frac{\partial}{\partial t_l}A \neq 0 \quad E + \Delta E$

$\frac{\partial}{\partial x}A \neq 0 \quad p_x + \Delta p_x$

$E^2 = E_0^2 + p_x^2c^2$

$\frac{\partial}{\partial t_l}\rho = -\nabla \cdot \vec{J}$

Fig. 11. The continuity equation is derived from the Klein-Gordon equation.

The massless photon has the constant velocity of c along the space axis in Fig. 3 and 7. $v_{photon} = x/t_l = x/t = ct/t = c$ (constant). And photon has the constant velocity of c along the absolute time (ct) axis. Then the photon moves along the relative time axis with the constant effective velocity of $c_{eff} = \sqrt{2}\,c$ in Fig. 7. The relative time axis of the photon has the 45° angle with the space axes. And $E_{tl} = \sqrt{2}E$ and $t_l = t$ at $c_{eff} = \sqrt{2}\,c$ and $p_{tl} = \frac{E_{tl}}{c_{eff}} = \frac{\sqrt{2}E}{\sqrt{2}c} = \frac{E}{c} = p_x$. This indicates that the photons with the energy of E have the constant photon velocity of c and the constant photon

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Relativistic Klein-Gordon equation

For the probability density,

From (2)/(2E) – (1)/(2E), and $\frac{\partial}{\partial t_l}\rho = -\nabla\cdot\vec{J}$

$\frac{\partial}{\partial t}\Psi^*(t)\Psi(t) = \frac{\partial}{\partial t}\rho_0 = 0$ because E_0 is constant and $\frac{\partial}{\partial t}A = 0$.

$$\rho = \frac{1}{2E}\left(\Psi^* i\hbar\frac{\partial}{\partial t_l}\Psi - \Psi i\hbar\frac{\partial}{\partial t_l}\Psi^*\right) \qquad J = \frac{-i\hbar c^2}{2E}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$$

$\frac{\partial}{\partial t_l}A \neq 0 \quad E + \Delta E$

$\frac{\partial}{\partial x}A \neq 0 \quad p_x + \Delta p_x$

$$\rho = \frac{2E}{2E}|\Psi|^2 = |\Psi|^2 > 0 \qquad J_x = \frac{2p_x c^2|\Psi|^2}{2E} = \frac{p_x c^2|\Psi|^2}{E}$$

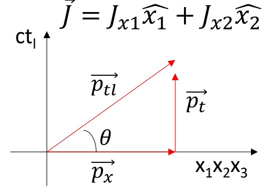
$$E^2 = E_0^2 + p_x^2c^2$$

$$p_{tl}^2 = p_t^2 + p_x^2$$

$$\frac{\partial}{\partial t_l}\rho = -\nabla\cdot\vec{J}$$

$$v = \frac{x}{t_l} = c\frac{x}{ct_l} = c\frac{p_x}{p_{tl}} = c\frac{p_x}{E/c} = \frac{p_x c^2}{E}$$

$$\vec{J} = J_{x1}\widehat{x_1} + J_{x2}\widehat{x_2} + J_{x3}\widehat{x_3}$$



Continuity equation,

If $\vec{J} = J_x\hat{x}$, $-\frac{\partial}{\partial x}J_x = -\frac{\partial}{\partial x}v|\Psi|^2 = -\frac{\partial}{\partial x}\frac{p_x c^2}{E}|\Psi|^2 = \frac{\partial}{\partial t_l}|\Psi|^2 = \frac{\partial}{\partial t_l}\rho$

Fig. 12. The continuity equation is derived from the Klein-Gordon equation.

At v/c <<1, $E = E_0 + \frac{p_x^2}{2m_0}$ $\qquad \Psi(t) = Ae^{-\frac{iE_0t}{\hbar}} = \Psi(t_l, x) = A(t_l, x)e^{-i(Et_l - p_x x)/\hbar}$

$$\Psi^*(t)i\hbar\frac{\partial}{\partial t}\Psi(t) = \Psi^*(t_l,x)\left(i\hbar\frac{\partial}{\partial t_l} + \frac{\hbar^2}{2m_0}\nabla^2\right)\Psi(t_l,x) \qquad (1)$$

$$-\Psi(t)i\hbar\frac{\partial}{\partial t}\Psi^*(t) = \Psi(t_l,x)\left(-i\hbar\frac{\partial}{\partial t_l} + \frac{\hbar^2}{2m_0}\nabla^2\right)\Psi^*(t_l,x) \qquad (2)$$

$\frac{\partial}{\partial t_l}A \neq 0 \quad E + \Delta E \qquad \frac{\partial}{\partial x}A \neq 0 \quad p_x + \Delta p_x$

From (2) – (1), and $\frac{\partial}{\partial t_l}\rho = -\nabla\cdot\vec{J}$

$\frac{\partial}{\partial t}\Psi^*(t)\Psi(t) = \frac{\partial}{\partial t}\rho_0 = 0$ because E_0 is constant and $\frac{\partial}{\partial t}A = 0$.

For the probability density, $\rho = \Psi^*\Psi = |\Psi|^2 > 0 \qquad \vec{J} = J_x\hat{x}$

$$J = \frac{-i\hbar c^2}{2m_0c^2}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) = \lim_{\frac{v}{c}\ll 1}\frac{-i\hbar c^2}{2mc^2}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) = \lim_{\frac{v}{c}\ll 1}\frac{p_x c^2|\Psi|^2}{E}$$

Schrödinger equations: At v/c <<1, $E_s = E - E_0 = \frac{p_x^2}{2m_0}$ and $t_l = \gamma t = (1 + v^2/(2c^2)\ldots)t = t$

$$\Psi(t_l, t) = Ae^{-i(Et_l - E_0t)/\hbar} = Ae^{-i(Et_l - E_0t_l)/\hbar} = Ae^{-i(E-E_0)t_l/\hbar} = Ae^{-iE_st_l/\hbar} = \Psi(t_l)$$

$$\Psi(t_l, x) = Ae^{-i(E_st_l - p_xx)/\hbar} \qquad H_s\Psi(t_l,x) = i\hbar\frac{\partial}{\partial t_l}\Psi(t_l,x) \qquad E_s\Psi(t_l,x) = \frac{p_x^2}{2m_0}\Psi(t_l,x)$$

$H_s\Psi(t_l,x) = E_s\Psi(t_l,x) = (E - E_0)\Psi(t_l,x) = \left(\frac{p_x^2}{2m_0} + V(x)\right)\Psi(t_l,x)$ with V(x)

$ip_x = -i\hbar\frac{\partial}{\partial x} = \hbar\frac{\partial}{i\partial x} \qquad \Psi(x) = Ae^{-p_xx/\hbar} = Ae^{(ip_x)(ix)/\hbar}$ Imaginary x axis (ix) within V.

Fig. 13. The continuity equation is derived from the Schrodinger equation.

momentum of $p_{tl} = \frac{E_{tl}}{c_{eff}} = \frac{\sqrt{2}E}{\sqrt{2}c} = \frac{E}{c} = p_t = p_x$ on the 4-D Euclidean space. The obtained wave equation is $\frac{\partial^2\mathcal{E}}{c^2\partial^2 t_l} = \frac{\partial^2\mathcal{E}}{\partial^2 x}$. The plane wave function of the photon is $\mathcal{E}(t_l, x) = A(t_l, x)e^{-i(Et_l - p_xx)/\hbar}$ for the electric and magnetic fields in Figs. 7 and 8. The energy densities of the photon and massive particle are shown in Fig. 8. The continuity equations of the probability density and current are explained and derived in terms of the Klein-Gordon, Dirac and Schrodinger equations based on the 4-D Euclidean space in the next section.

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3. Continuity equations of the Klein-Gordon, Dirac and Schrodinger equations

The current and probability density have the close relation as shown in the continuity equation of $\frac{\partial}{\partial t}\rho = -\nabla \cdot \vec{J}$. In Figs. 9 and 10, the continuity equation is derived from the classical view of the relativistic equation of $E^2 = E_0^2 + p_x^2c^2$ based on the 4-D Euclidean space. The energy current of J comes from the space momentum term of $p_x^2c^2$. The energy probability density comes from the energy term of E^2 in Fig. 9. The current term of $\nabla \cdot \vec{J}$ is decreased when the density term of $\frac{\partial}{\partial t}\rho$ is increased. In other words, the moving energy of the particle is transferred to the warping energy of the particle in Figs. 9 and 10. Here, note that the particle energy is defined as the 4-D volume of $E = \gamma E_0 = \gamma c\Delta t\Delta V_0 = c\Delta t\gamma\Delta x_1\Delta x2\Delta x3 = c\Delta t\Delta V$ in Figs. 4, 5, 6, and 8. And the rest mass energy of the particle is constant. Therefore, $\Delta E_0 = 0$ always. The continuity equation of the particle for the rest mass energy is always $\frac{\partial}{\partial t}\rho_0 = -\nabla \cdot \vec{J_0} = 0$ in Figs. 9 and 10.

In Fig. 11, the relativistic Klein-Gordon equation of $p_t^2\Psi(t) = (p_{tl}^2 - p_x^2)\Psi(t_l, x)$ is shown. The plane wave function is $\Psi(t) = Ae^{-\frac{iE_0t}{\hbar}} = \Psi(t_l, x) = A(t_l, x)e^{-i(Et_l - p_xx)/\hbar}$. The applied time and space momentum operators are $p_{tl}\Psi(t_l) = i\hbar\frac{\partial}{c\partial t_l}\Psi(t_l)$, $p_t\Psi(t) = i\hbar\frac{\partial}{c\partial t}\Psi(t)$ and $p_x\Psi(x) = -i\hbar\frac{\partial}{\partial x}\Psi(x)$. In Fig. 11, the energy density and current are $\rho = E|\Psi|^2$ and $J_x = p_xc^2|\Psi|^2$, respectively. The negative energy and negative energy density are obtained from $E^2 = E_0^2 + p_x^2c^2$. This negative energy solution indicates that the negative energy universe exists as shown in Fig. 1. The positive particle probability density $\rho = \frac{2E}{2E}|\Psi|^2 = |\Psi|^2 > 0$ can be obtained in Fig. 12. In Fig. 13, the continuity equation of the Schrodinger equation is derived by including the rest mass.

Dirac equation with the constant rest mass energy term of E_0

$(i\gamma^0\frac{\partial}{\partial t_l} + i\vec{\gamma}\cdot\vec{\nabla} - m_0)\Psi(t_l, x) = 0$ $\quad E^2 = E_0^2 + p_x^2c^2$ $\quad \vec{J} = \Psi^+\vec{\alpha}\Psi$

$\rho = \overline{\Psi}\gamma^0\Psi = |\Psi_1|^2 + |\Psi_2|^2 + |\Psi_3|^2 + |\Psi_4|^2 > 0$ For the particle probability density

Two solutions of Klein-Gordon and Dirac equations in terms of the TQSM model

Particle with the charge of -q in the positive E universe

$E = mc^2 > 0, E_0 = m_0c^2 > 0, t > 0, t_l > 0, \rho > 0$

Anti-particle with the charge of q in the negative E universe

$-E = -mc^2 < 0, -E_0 = -m_0c^2 < 0, -t < 0, -t_l < 0, \rho > 0$

$P_4 = T_cP$ = TCP symmetry (-E)(-t) = (E)(t)

Two solutions of Dirac equations in terms of the SM model ($t \rightarrow t_l$)

Electron $E > 0, t_l > 0$

Electron $-E < 0, -t_l < 0$ $\stackrel{?}{=}$ Positron $E > 0, t_l > 0$

Anti-particle Definition (?) $(-E)(-t_l) = (E)(t_l)$

Fig. 14. The negative energy solutions of the Klein-Gordon and Dirac equations are interpreted by P_4 symmetry in terms of the TQSM model [28].

The Schrodinger equation is the non-relativistic version of the Klein-Gordon equation. Therefore, the continuity equation of the Schrodinger equation is obtained from the continuity equation of Klein-Gordon equation at the low velocity approximation of $v \ll c$.

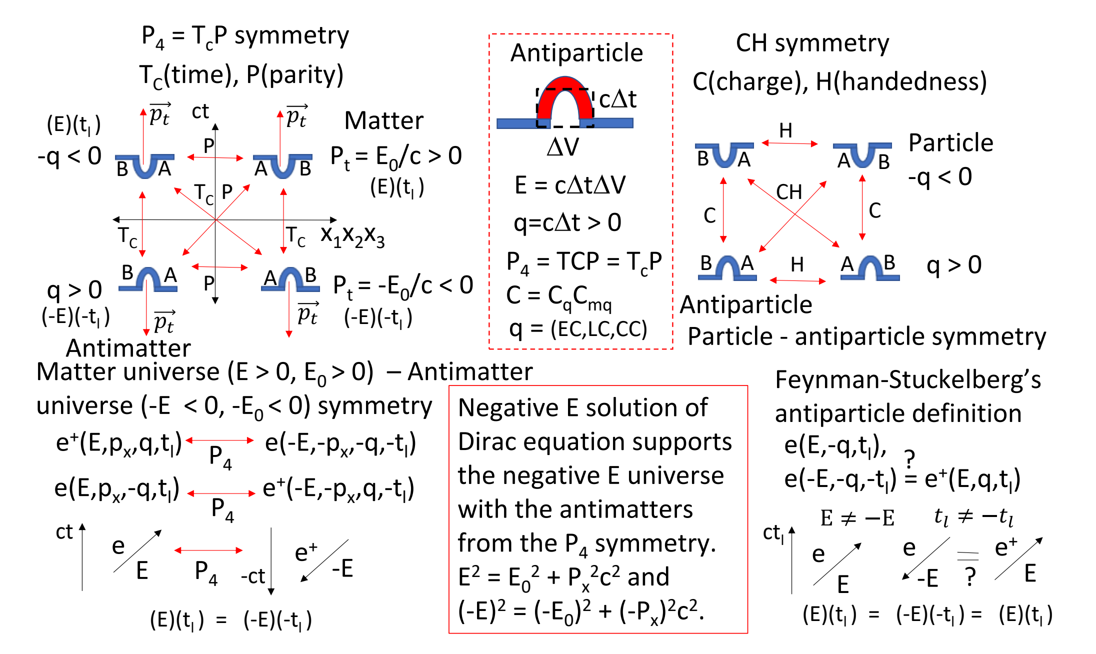

Fig. 15. The P4 symmetry and CH symmetry are explained.

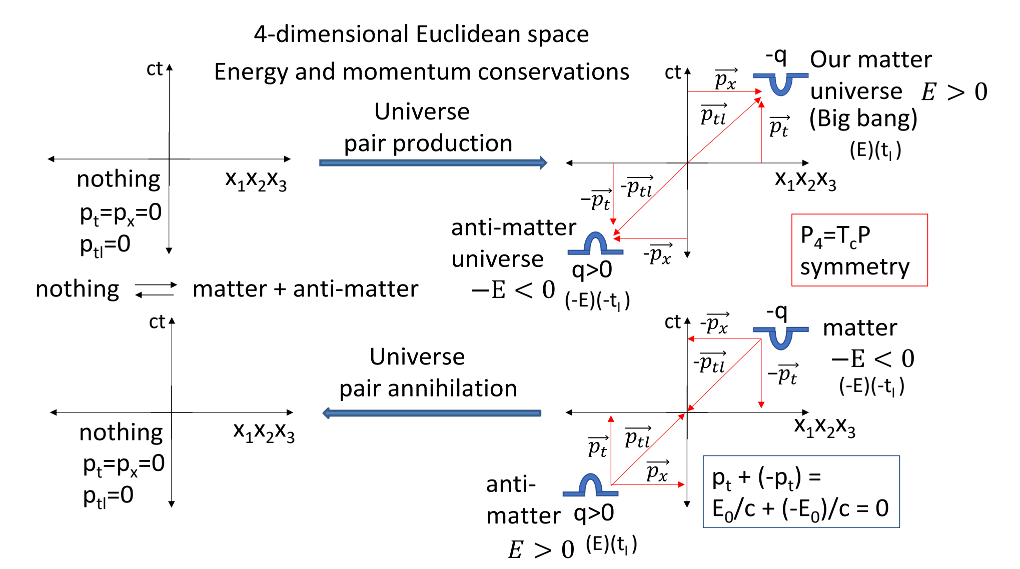

Fig. 16. The big bang event is explained by the P4 symmetry.

In Fig. 13, the Schrodinger equation includes the rest mass energy. $E_s = E - E_0$ is defined for the simplicity. The plane wave function is $\Psi(t_l, x) = Ae^{-i(E_s t_l - p_x x)/\hbar}$. And the Schrodinger equation of the free particle is $E_s\Psi(t_l, x) = \frac{p_x^2}{2m_0}\Psi(t_l, x)$. The Schrodinger equation of the particle with the potential barrier of V is $H_s\Psi(t_l, \mathrm{x}) = E_s\Psi(t_l, \mathrm{x}) = (E - E_0)\Psi(t_l, \mathrm{x}) = (\frac{p_x^2}{2m_0} + V(x))\Psi(t_l, \mathrm{x})$ with V(x). The solutions of the Schrodinger equations and Klein-Godon equation are explained in the next section. In Fig. 14, the particle probability of $\rho = \bar{\Psi}\gamma^0\Psi = |\Psi_1|^2 + |\Psi_2|^2 + |\Psi_3|^2 + |\Psi_4|^2 > 0$ is shown from the Dirac equation.

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The Klein-Godon and Dirac equations have the negative solutions in Fig. 14. The negative energy solutions are explained as the antiparticle in the negative energy universe if the positive energy solutions are for the particle in the positive energy universe in terms of the 3-D quantized space model (TQSM) in Fig. 1. The antiparticle with the negative energy and particle with the positive energy are related by the 4-D P_4 = TcP symmetry in Fig. 1 and Fig. 14. Note that the negative energy solutions are used to define the antiparticle with the positive energy by the Feynman and Stuckelberg in terms of the standard model (SM) in Fig. 14.

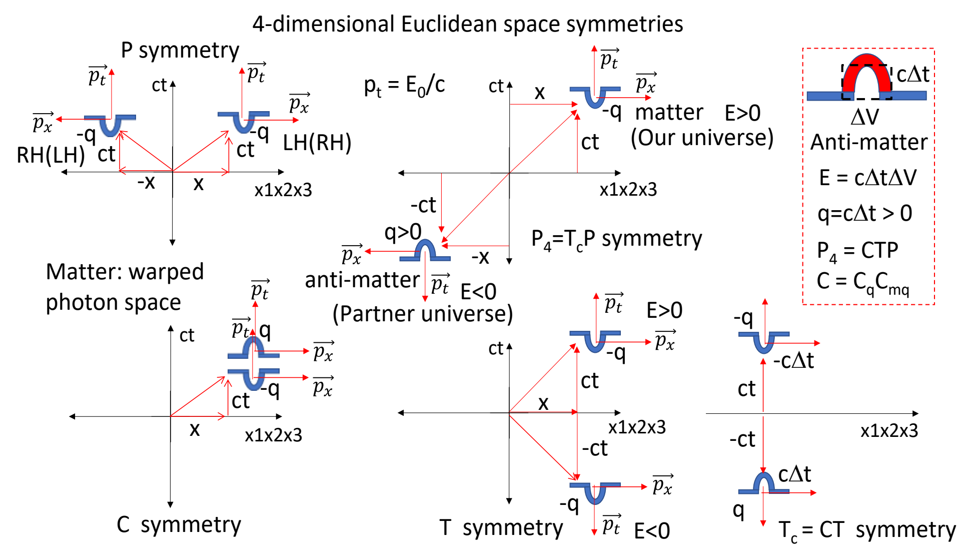

Fig. 17. The P, C, T, Tc and TCP symmetry are explained.

4. Negative energy solutions and $P_4 = T_cP$ symmetry

In Fig 15, the negative energy solutions of the Klein-Godon and Dirac equations are described on the 4-D Euclidean space. Negative energy solutions support the negative energy universe with the antimatters from the P_4 symmetry in Figs. 1 and 15. $E^2 = E_0^2 + P_x^2c^2$ for the positive energy particles and $(-E)^2 = (-E_0)^2 + (-P_x)^2c^2$ for the negative energy particles. If the negative energy solutions are applied to the big bang event as shown in Fig. 16, the matter universe with the positive energy and the antimatter universe with the negative energy were created from the big bang. This explains why the matters are dominating within our universe. In Fig. 17, the P, C, T and TC = T_c symmetries are compared. In Figs. 18 and 19, more examples of the $P_4 = T_cP$ symmetry are shown. In Fig. 18, three kinds of space fluctuations are shown. The pair production of the neutrino and anti-neutrino is explained as the P_4 symmetry. And the pair production of two photons is explained as the P_4 symmetry. The dark energy to cause the accelerated space expansion is thought to be originated from these two processes. In Figs. 19 and 1, the negative energy states and positive energy states with the P_4 symmetry are compared. The 4-D momenta are defined based on the 4-D Euclidean space. The momentum operators of the negative energy states are compared with those of the positive energy states. The corresponding plane wave functions are shown with the proper sign changes. For the negative energy particles, the space momentum directions are opposite to the particle velocity directions because the energy is negative in Figs. 1 and 19. For the positive energy particles, the space momentum directions are same to the particle velocity directions because the energy is positive.

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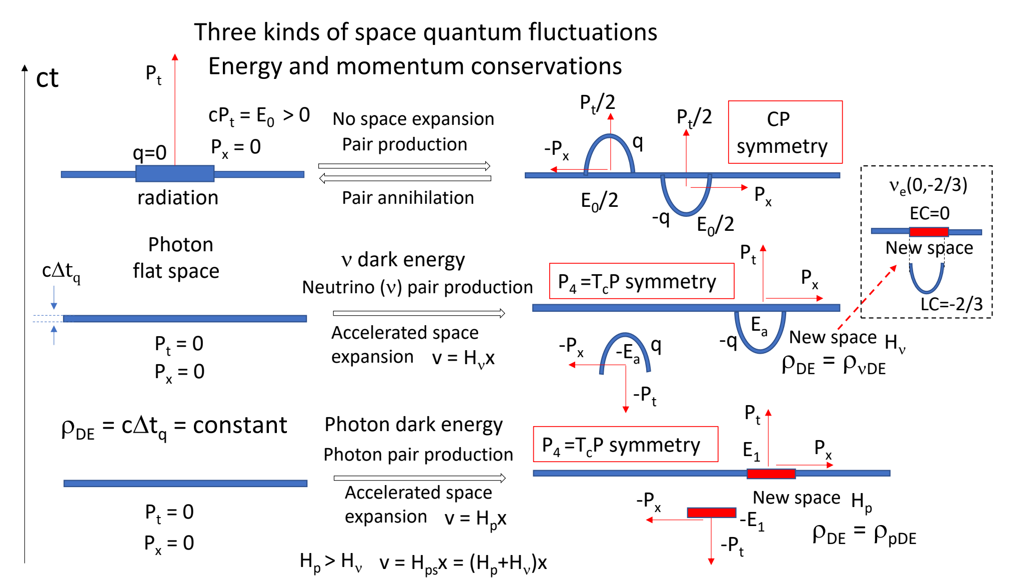

Fig. 18. Two examples of the P_4 symmetry are shown.

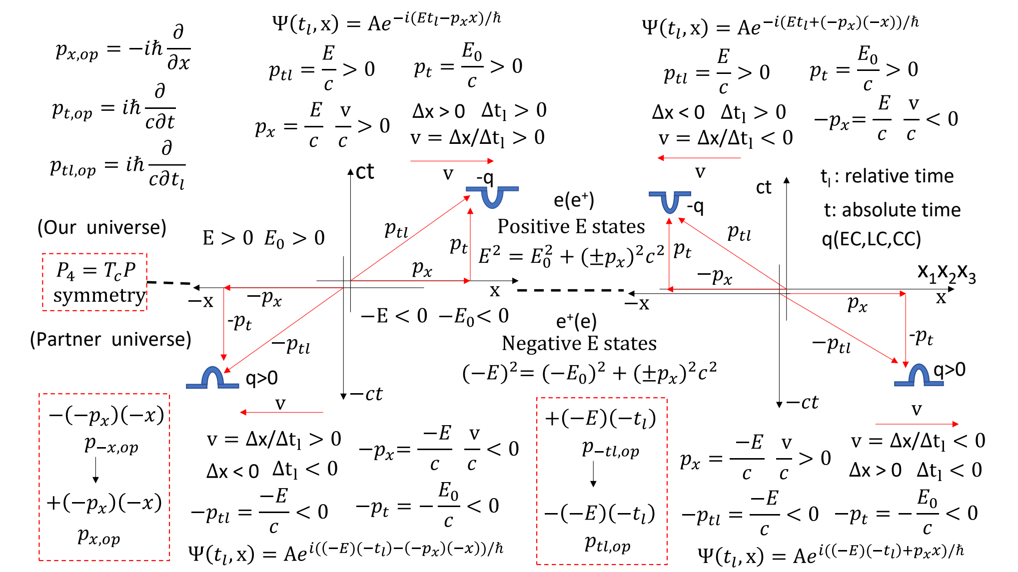

Fig. 19. The negative energy solutions are explained by the P_4 symmetry. The corresponding plane wave functions are shown. The 4-D momentum conservation (energy and momentum conservations) is applied.

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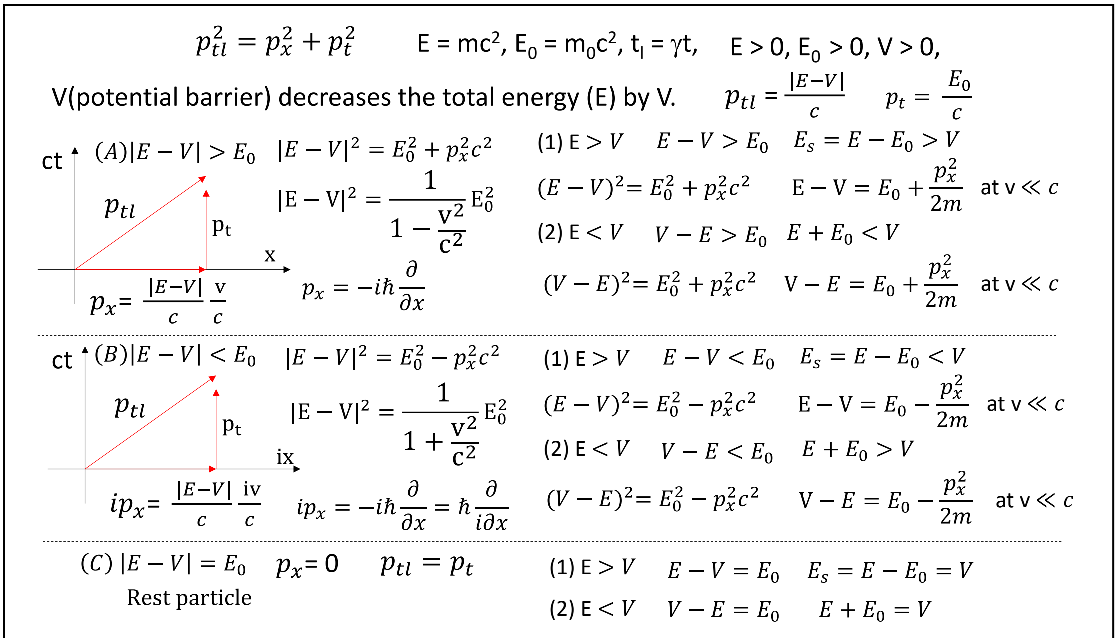

Fig. 20. The Klein-Gordon equation with the potential barrier is solved.

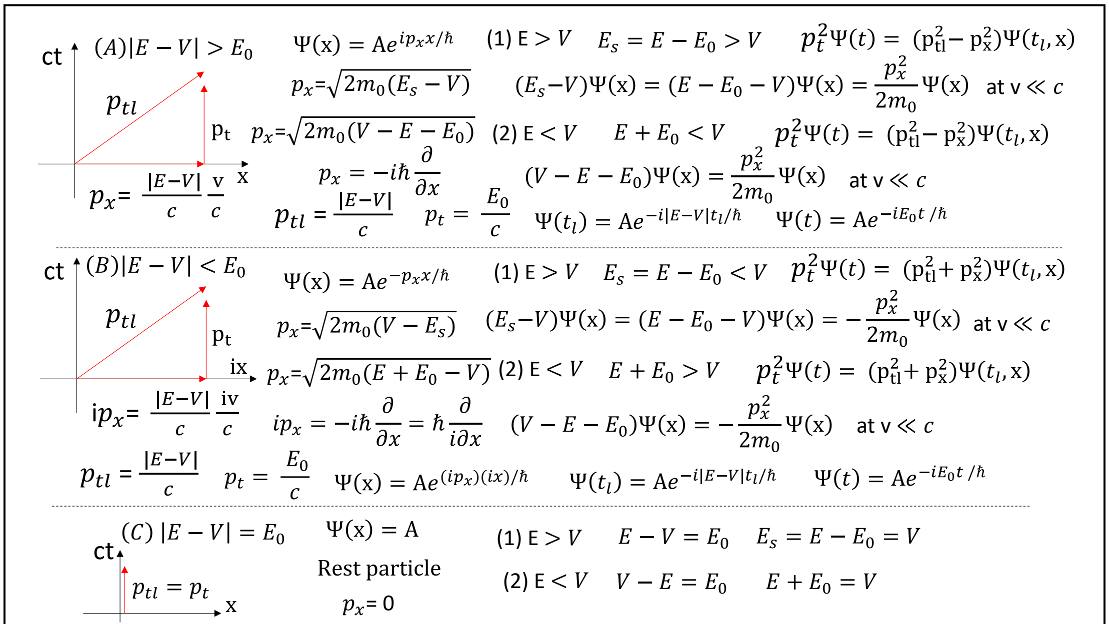

Fig. 21. The Klein-Gordon equation with the potential barrier is solved. The plane wave functions are shown based on the 4-D Euclidean space.

5. Klein-Gordon and Schrodinger equations with the potential barriers and Klein paradox

The continuity equations of the Klein-Gordon, Dirac and Schrodinger equations are derived in section 2. In this section, the Klein-Gordon and Schrodinger equations with the potential barriers

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of V are solved. For the simplicity, only the positive energy states (E>0 and E_0>0) are considered in solving those equations in Figs. 20 - 24. The solutions for the positive energy states (E>0 and E_0>0) can be easily extended to the negative energy states (-E<0 and -E_0<0). In Fig. 24, the Klein paradox is explained based on the 4-D Euclidean space.

The potential barrier energy decreases the total energy by V > 0. Therefore, E is replaced with |E-V|. There are 5 cases of (A1) E - V > E_0, (A2) V - E > E_0, (B1) E - V < E_0, (B2) V - E < E_0 and (C) V - E = E_0 in Figs. 20 and 21. A1 and A2 cases have the energy that is increased from the rest mass energy by the equation of $|\mathrm{E}-\mathrm{V}|^2 = \frac{1}{1-\frac{v^2}{c^2}}\mathrm{E}_0^2$. This means that the length of the particle is expanded by the equation of $\Delta x^2 = \frac{1}{1-\frac{v^2}{c^2}}\Delta \mathrm{x}_0^2$. It is because the energies are the 4-D volumes of |E-V| = cΔtΔV and E_0 = cΔtΔV_0. Here ΔV is the space volume of the particle. The A1 and A2 cases are given by the real Δx and v numbers along the real x axis because the rest mass energy is conserved as the constant minimum energy of the particle and meets the equation of $(E-V)^2 =$

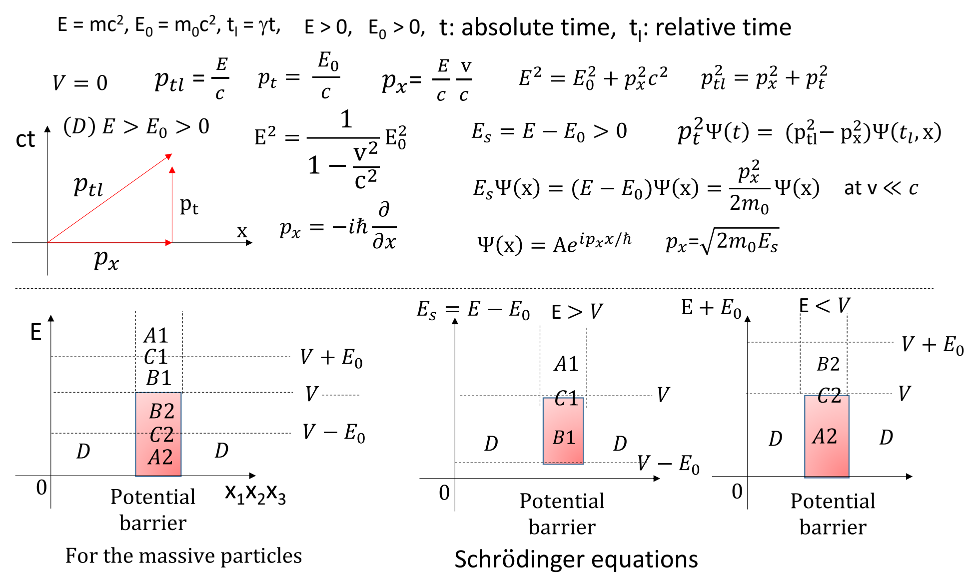

Fig. 22. The Klein-Gordon equation with the potential barrier is solved.

$E_0^2 + p_x^2c^2$. Then the space momentum is the real space momentum of $p_x = \frac{|E-V|}{c}\frac{v}{c}$. But the B1 and B2 cases have the energy that is decreased from the rest mass energy by the equation of $|\mathrm{E}-\mathrm{V}|^2 = \frac{1}{1+\frac{v^2}{c^2}}\mathrm{E}_0^2$. This means that the length of the particle is contracted by the equation of $\Delta x^2 = \frac{1}{1+\frac{v^2}{c^2}}\Delta \mathrm{x}_0^2$. It is because the energies are the 4-D volumes of |E-V| = cΔtΔV and E_0 = cΔtΔV_0. Here ΔV is the space volume of the particle. The B1 and B2 cases are given by the imaginary iΔx and iv numbers along the imaginary ix axis because the rest mass energy is conserved as the constant minimum energy of the particle and meets the equation of $(E-V)^2 = E_0^2 - p_x^2c^2$. Then the space momentum is given as the imaginary space momentum of $ip_x = \frac{|E-V|}{c}\frac{iv}{c}$.

The plane space wave functions of the A1 and A2 cases are $\Psi(x) = Ae^{ip_x x/\hbar}$. The plane space wave functions of the B1 and B2 cases are $\Psi(x) = Ae^{(ip_x)(ix)/\hbar} = Ae^{-p_x x/\hbar}$. The C case is for the rest particle with $p_x = 0$. The D case in Fig. 22 is for the particle without the potential barrier.

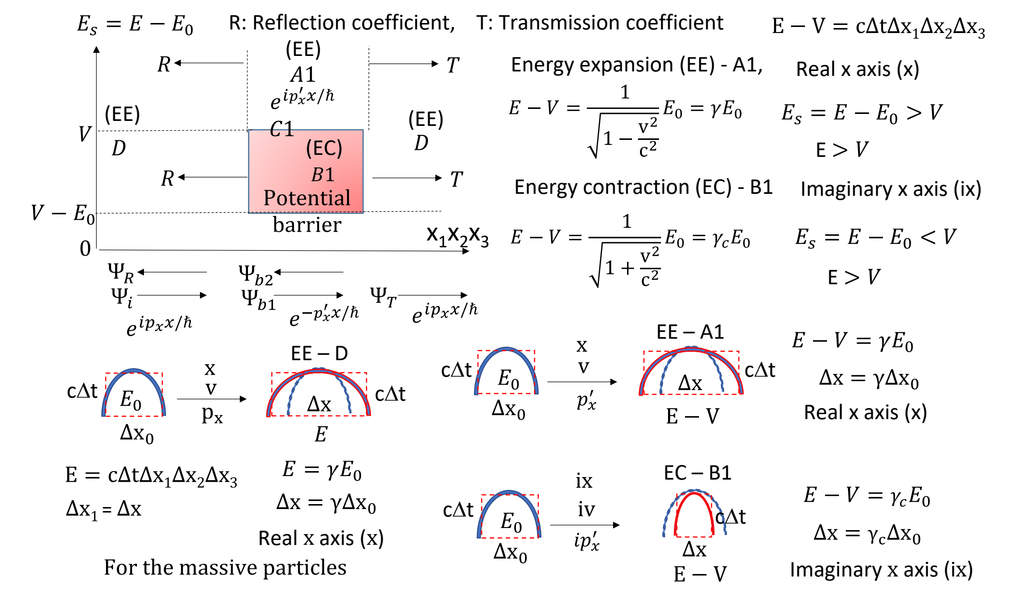

Fig. 23. The Klein-Gordon equation with the potential barrier is solved.

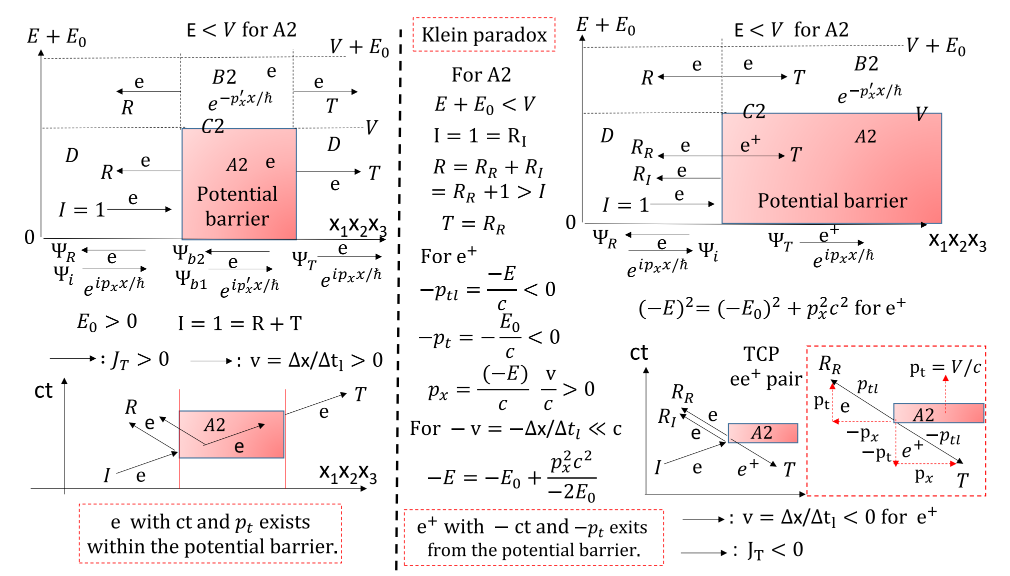

Fig. 24. The Klein-Gordon equation with the potential barrier is solved. The Klein paradox is explained by using the P_4 symmetry. The 4-D momenta (energy and space momenta) defined in Fig. 2 are conserved for the pair production of the e and e^+ pair under $P_4 = T_cP = TCP$ symmetry.

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In Figs. 22 and 23, the total six cases of A1, A2, B1, B2, C and D are graphically compared. In Fig. 23, the A1 and B1 cases with the potential barriers are solved. Note that the rest mass energy of E_0 is always included. The energy expansion (increase) of $E - V = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} E_0 = \gamma E_0$ and energy contraction of $E - V = \frac{1}{\sqrt{1+\frac{v^2}{c^2}}} E_0 = \gamma_c E_0$ are observed for the A1 and B1 cases, respectively. To explain the energy contraction for the B1 case, the imaginary concept is introduced for the x, v and p_x values. Therefore, the ix, iv and ip_x values are used for the B1 case. In Fig. 24, the A2 and B2 cases with the potential barriers are solved. In Fig. 24, the electron current is considered for the easy explanation. The electrons with the input coefficient (I = 1) have the reflection coefficient (R) and transmission coefficient (T). Then 1 = I = R + T for the weak potential barrier. The electrons can exist within the potential barrier of V and go through the potential barrier of V as shown in Fig. 24. In this case, I = 1, R + T = 1, R < 1 and T < 1 as expected. There are no problems with these solutions. For the strong potential barrier of V, R = 1 and T = 0 are expected. But the R > 1 and $T \neq 0$ are the calculated results. Even the current density (J_T) of the transmission (T) has the negative x direction. This is called as the Klein paradox [15-19]. The Klein paradox has been explained by using the pair production of the electron and positron at the front border of the potential barrier of V [15-19]. In this explanation, Klein paradox has supported the Feynman - Stueckelberg interpretation that the positron with the positive energy (E>0) and forward time direction (t>0) is the same as the electron with the negative energy (-E) and backward time direction (-t) because of the (-E)(-t) = (E)(t) relation. But in the present TQSM model, the $P_4 = T_cP$ symmetry is applied for the explanation of the Klein paradox in Fig. 24. In other words, the pair production of the electron with the positive energy and positron with the negative energy takes place at the front border of the potential barrier of V under the 4-D momentum conservation (energy and momentum conservations) with the 4-D $P_4=T_cP=TCP$ symmetry. In Fig. 24, the positrons make the transmission coefficient of T. Tthe positrons with the negative energy are moving along the negative time (-t < 0) direction and positive space (x > 0) direction in Fig. 24. The positrons with the negative energy move on the 4-D Euclidean space with x > 0 and -t < 0. Therefore, the $\Delta x > 0$ and $\Delta t_l < 0$. The particle velocity of v is $v = \Delta x / \Delta t_l < 0$. However, the space momentum is $p_x > 0$ and the positron current density is $J_T < 0$ as shown in Figs. 1, 19 and 24. The negative positron current density means the positive space momentum in the negative energy coordinates as shown in Figs. 1, 19 and 24. This is what the calculations of the Klein paradox show in Fig. 24. The Klein paradox can be explained by using the 4-D $P_4 = TCP = T_cP$ symmetry.

The potential barrier at the rest state has the positive potential energy of V in Fig. 24. The potential energy is the rest mass energy of the potential barrier. Therefore, the potential barrier has the positive time momentum of $p_t = V/c$ along the ct axis. The reflected electrons have the positive time momentum of $p_t = E_0/c$ along the ct axis in Fig 24. The transmitted positrons have the negative time momentum of $-p_t=-E_0/c$ along the -ct axis. Therefore, the potential barrier and reflected electrons with the positive time momenta and the transmitted positrons with the negative time momentum should be separated toward the opposite time direction as shown in Fig. 24.

6. Summary

In the standard model (SM) the Klein-Gordon, Dirac and Schrodinger equations have been used based on the 4-D Minkowski space with the relative time. The negative energy solutions of these

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equations are used to make the definition of the antiparticle with the positive energy and the forward time direction. Then it is proposed from the Feynman - Stueckelberg interpretation that the positron with the positive energy (E>0) and forward time direction (t>0) is the same as the electron with the negative energy(-E) and backward time direction (-t) because of the (-E)(-t) = (E)(t) relation. Often this concept of the antiparticle like the positron has been applied in the Feynman diagram. As one of many general applications, the relation of the nonlinear Klein-Gordon equation and the symmetron or chameleon was recently published [29].

In the present 3-D quantized space model (TQSM) [28], the Klein-Gordon, Dirac and Schrodinger equations are developed on the 4-D Euclidean space with two times of the absolute time (t) and relative time (t_l). The negative energy solutions of these equations support the existence of the antiparticles with the negative energies (-E < 0) and negative time direction (-t < 0) in Figs. 1 and 19. The positive energy solutions and negative energy solutions of the Klein-Gordon and Dirac equations show the 4-D P_4 = TCP = T_cP symmetry. Based on these concepts, the plane wave functions are derived, and several examples are explained. Those examples include the big bang event, dark energies and Klein paradox. The plane field wave functions for the photons are derived from $\frac{\partial^2 \mathcal{E}}{c^2 \partial^2 t_l} = \frac{\partial^2 \mathcal{E}}{\partial^2 x}$ by assigning the effective photon energy and effective photon velocity. $\mathcal{E}(t_l, \mathrm{x}) = \mathrm{A}(t_l, \mathrm{x}) e^{-i(Et_l - p_x x)/\hbar}$ for the electric or magnetic fields. The photon momentum along the relative time axis is $p_{tl} = \frac{\mathrm{E_{tl}}}{\mathrm{c_{eff}}} = \frac{\sqrt{2}\mathrm{E}}{\sqrt{2}\mathrm{c}} = \frac{\mathrm{E}}{\mathrm{c}} = \mathrm{p_x} = \mathrm{p_t}$ in Fig. 7. The effective velocity ($\mathrm{c_{eff}} = \sqrt{2}$ c) and effective energy ($E_{tl} = \sqrt{2}$E) along the relative time axis (ct_l) give the relative time momentum of $p_{tl} = \frac{\mathrm{E_{tl}}}{\mathrm{c_{eff}}} = \frac{\sqrt{2}\mathrm{E}}{\sqrt{2}\mathrm{c}} = \frac{\mathrm{E}}{\mathrm{c}} = \mathrm{p_t} = \mathrm{p_x}$. This indicates that the photons with the energy of E have the constant photon velocity of c and the constant photon momentum of $p_{tl} = \frac{\mathrm{E_{tl}}}{\mathrm{c_{eff}}} = \frac{\sqrt{2}\mathrm{E}}{\sqrt{2}\mathrm{c}} = \frac{\mathrm{E}}{\mathrm{c}} = \mathrm{p_t} = \mathrm{p_x}$ on the 4-D Euclidean space. For the massive particles, $Et_l = \overrightarrow{p_{tl}} \cdot \overrightarrow{ct_l} = (\overrightarrow{p_t} + \overrightarrow{p_x}) \cdot (c\vec{t} + \vec{x}) = E_0 t + p_x x$. And the obtained plane wave function is $\Psi(t) = \mathrm{A}(\mathrm{t}) e^{-iE_0 t/\hbar} = \Psi(t_l, \mathrm{x}) = \mathrm{A}(t_l, \mathrm{x}) e^{-i(Et_l - p_x x)/\hbar}$. For the negative energy particles, it is thought that the space momentum directions are opposite to the particle velocity directions because the energy is negative in Figs. 1 and 19.

The continuity equations of $\frac{\partial}{\partial t}\rho = -\nabla \cdot \vec{J}$ are derived for the Klein-Gordon, Dirac and Schrodinger equations based on the 4-D Euclidean space. The Klein-Gordon and Schrodinger equations with the potential barriers are solved by using the 4-D P_4 = TCP = T_cP symmetry and 4-D Euclidean space. Within the potential barriers, the particles experience the particle energy expansion (increase) or particle energy contraction (decrease). The particle energy expansion is connected to the real space axis (x) and real space momentum (p_x). And the particle energy contraction is connected to the imaginary space axis (ix) and imaginary space momentum (ip_x). The Klein paradox is explained by using the 4-D P_4 = TCP = T_cP symmetry. The Klein paradox is explained by the pair production of the electron with the positive energy (E>0) and positive time direction (t_l>0) and the positron with the negative energy (-E<0) and negative time direction (-t_l<0). The 4-

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D momenta (energy and space momenta) defined in Fig. 2 are conserved for the pair production of the e and e^+ pair under $P_4=T_cP=TCP$ symmetry in Fig. 24. The potential barrier and reflected electrons with the positive time momenta and the transmitted positrons with the negative time momentum should be separated toward the opposite time direction as shown in Fig. 24.

And the matter collapse to the photons at the very high particle velocity is discussed from the length expansion. The reason why the gamma ray burst (GRB) takes place has not been well understood. The origin of the gamma ray burst is the challenging research topic in the theoretical point of view. In the present work, for the first time it is proposed that the gamma ray burst is originated from the matter collapse to the photons near the black hole in Fig. 5. The matters are falling and swirling toward the black hole by the strong gravitational force as shown in Fig. 5. Then the velocity of the matters is very rapidly increasing. Then, the velocity of the matters is close to the photon velocity near the black hole. Then at the very high velocity, the matters are collapsing to the photons without the gravitational force with the black hole as shown in Fig. 5. Then the strong gamma rays are emitted to make the gamma ray burst. The gamma ray burst takes place for the very short period because the whole matters are rapidly collapsed to the photons for the very short time. In other words, this successful explanation of the gamma ray burst supports the matter collapse to the photons. Also, the wave function collapse is explained from the length expansion. It is concluded that the wave function collapse takes place when the measurement makes the particle velocity to be zero.

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