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Article

Constraining Neutrino Mass in Dynamical Dark Energy Cosmologies with the Logarithm Parametrization and the Oscillating Parametrization

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Abstract: We constrain two dynamical dark energy models that are parametrized by the logarithm form of $w(z) = w_0 + w_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right)$ and the oscillating form of $w(z) = w_0 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1) \right)$. Comparing with the Chevallier-Polarski-Linder (CPL) model, the two parametrizations for dark energy can explore the whole evolution history of the universe properly. Using the current mainstream observational data including the cosmic microwave background data and the baryon acoustic oscillation data as well as the type Ia supernovae data, we perform the χ^2 statistic analysis to global fit these models, finding that the logarithm parametrization and the oscillating parametrization are almost as well as the CPL scenario in fitting these data. We make a comparison for the impacts of the dynamical dark energy on the cosmological constraints on the total mass of active neutrinos. We find that the dark energy properties could significantly change the fitting results of neutrino mass. Looser constraints on $\sum m_\nu$ are obtained in the logarithm and oscillating models than those derived in the CPL model. Consideration of the possible mass ordering of neutrinos reveals that the most stringent constraint on $\sum m_\nu$ appears in the degenerate hierarchy case.

Keywords: dynamical dark energy; neutrino mass; mass hierarchies of neutrinos; the observational constraints

1. Introduction

The fact that neutrinos have masses [1,2] has drawn significant attention from physicists. The squared mass difference between different neutrino species have been measured, i.e., $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$ in solar and reactor experiments, and $|\Delta m_{31}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$ in atmospheric and accelerator beam experiments [2]. The possible mass hierarchies of neutrinos are $m_1 < m_2 \ll m_3$ and $m_3 \ll m_1 < m_2$, which are called the normal hierarchy (NH) and the inverted hierarchy (IH). When the mass splittings between different neutrino species are neglected, we treat the case as the degenerate hierarchy (DH) with $m_1 = m_2 = m_3$.

Some famous particle physics experiments, such as tritium beta decay experiments [3–6] and neutrinoless double beta decay ($0\nu\beta\beta$) experiments [7,8], have been designed to measure the absolute masses of neutrinos. Recently, the Karlsruhe Tritium Neutrino (KATRIN) experiment provided an upper limit of 1.1 eV on the neutrino-mass scale at 2σ confidence level (C.L.) [9]. However, cosmological observations are considered to be a more promising approach to measure the total neutrino mass $\sum m_\nu$. Massive neutrinos can leave rich imprints on the cosmic microwave background (CMB) anisotropies and the large-scale structure (LSS) formation in the evolution of the universe. Thus, the total neutrino mass $\sum m_\nu$ is likely to be measured from these available cosmological observations.

In the standard Λ cold dark matter (Λ CDM) model with the equation-of-state parameter of dark energy $w = -1$, the Planck Collaboration gave $\sum m_\nu < 0.26 \text{ eV}$ (2σ) [10] from the full Planck TT, TE, EE power spectra data, assuming the NH case with the minimal mass $\sum m_\nu = 0.06 \text{ eV}$ (2σ). Adding

the Planck CMB lensing data slightly tightens the constraints to $\sum m_\nu < 0.24$ eV (2σ). When the baryon acoustic oscillations (BAO) data are considered on the basis of the Planck data, the neutrino mass constraint is significantly tightened to $\sum m_\nu < 0.12$ eV (2σ). Further adding the type Ia supernovae (SNe) data marginally lowers the bound to $\sum m_\nu < 0.11$ eV (2σ), which put pressure on the inverted mass hierarchy with $\sum m_\nu \geq 0.10$ eV.

The impacts of dynamical dark energy on the total neutrino mass have been investigated in past studies [11–37]. In the simplest dynamical dark energy model with $w = \text{Constant}$ (abbreviated as w CDM model), the fitting results of $\sum m_\nu$ are $\sum m_{\nu,\text{NH}} < 0.195$ eV (2σ) and $\sum m_{\nu,\text{IH}} < 0.220$ eV (2σ) [33], using the full Planck TT, TE, EE power spectra data and the BAO data as well as the SNe data. From the same data combination, $\sum m_{\nu,\text{NH}} < 0.129$ eV (2σ) and $\sum m_{\nu,\text{IH}} < 0.163$ eV (2σ) [33] in the holographic dark energy (HDE) model [38–45]. The constraint results of $\sum m_\nu$ are different from those in the standard Λ CDM model because of impacts of dark energy properties in these cosmological models.

In addition to the w CDM model and the HDE model, the constraints on $\sum m_\nu$ are investigated in the CPL model [46,47] with $w(z) = w_0 + w_1 \frac{z}{1+z}$ (where w_0 and w_1 are two free parameters). Over the years, the CPL parametrization have been widely used and explored extensively. In the model, $\sum m_{\nu,\text{NH}} < 0.290$ eV (2σ) and $\sum m_{\nu,\text{IH}} < 0.305$ eV (2σ) [33] are obtained by using the full Planck TT, TE, EE power spectra data combined the BAO data with the SNe data. The upper limit values of $\sum m_\nu$ are larger than those in the w CDM model and the HDE model, confirming that the constraint results of $\sum m_\nu$ can be changed as the different parametrization forms of w . The CPL model has a drawback that it only explores the past expansion history, but cannot describe the future evolution. Thus the CPL parametrization does not genuinely cover the scalar field models as well as other theoretical models. Such a problem makes the fitting results of $\sum m_\nu$ untenable in the CPL model.

In this paper, we focus on two novel forms of $w(z)$, i.e., the logarithm parametrization $w(z) = w_0 + w_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right)$ and the oscillating parametrization $w(z) = w_0 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1) \right)$ [48], which are correspondingly called the Log model and the Sin model. They can inherit the advantages of the CPL model and explore the whole evolution history of the universe properly. In our present work, the constraints on $\sum m_\nu$ will be investigated in the two models. In fact, there are also some other dark energy parametrizations, such as the Jassal-Bagla-Padmanabhan parametrization [49] and the Barboza-Alcaniz parametrization [50] with the same free parameters w_0 and w_1 . They will be explored with other research motivations in our future work.

On the other hand, in order to better match the current observational result of $w = -1$, we assume the case of $w_0 = -1$ in the CPL parametrization, the logarithm parametrization, and the oscillating parametrization. The forms of $w(z)$ in these models are modified as $w(z) = -1 + w_1 \frac{z}{1+z}$, $w(z) = -1 + w_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right)$, and $w(z) = -1 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1) \right)$ with a free parameter w_1 . We call them the MCPL model, the MLog model, and the MSin model. We also investigate the constraints on them using the same mainstream observational data.

In our work, we first constrain on the Log parametrization and the Sin parametrization by using latest mainstream observational data. Then, we investigate impacts of the dark energy properties on neutrino mass. This paper is organized as follows. In Sect. 2, we provide a brief description of the data and method used in our work. In Sect. 3, we show the constraint results of different dynamical dark energy models and discuss the physical meaning behind these results. At last, we make some important conclusions in Sect. 4.

2. Data and method

Throughout this paper, we only employ the data combination of the CMB data, the BAO data, and the SNe data, which is abbreviated as the CMB+BAO+SNe data. The usage of the data combination facilitates to make a comparison with the results derived from Refs. [10,14,33], in which this typical data combination has also been used to constrain cosmological models. For the CMB data, we use the Planck 2018 temperature and polarization power spectra data at the whole multipole ranges, together

with the CMB latest lensing power spectrum data [10]. For the BAO data, we use the 6dFGS and SDSS-MGS measurements of D_V/r_{drag} [51,52] plus the final DR12 anisotropic BAO measurements [53]. For the SNe data, we use the “Pantheon” sample [54], which contains 1048 supernovae covering the redshift range of $0.01 < z < 2.3$.

For the dynamical dark energy models with the CPL parametrization, logarithm, and oscillating parametrizations, they all have eight free parameters, i.e., the present baryons density $\omega_b \equiv \Omega_b h^2$, the present cold dark matter density $\omega_c \equiv \Omega_c h^2$, an approximation to the angular diameter distance of the sound horizon at the decoupling epoch θ_{MC} , the reionization optical depth τ , the amplitude of the primordial scalar power spectrum A_s at $k = 0.05 \text{ Mpc}^{-1}$, the primordial scalar spectral index n_s , and the model parameters w_0 and w_1 . The priors of these parameters are shown explicitly in the Table 1. When $w_0 = -1$ is fixed, there are seven free parameters in the MCPL, MLog, and MSin models.

Table 1. Priors on the free parameters for the two-parametrization dark energy models.

Parameter	Prior
$\Omega_b h^2$	[0.005, 0.100]
$\Omega_c h^2$	[0.001, 0.990]
$100\theta_{\text{MC}}$	[0.5, 10.0]
τ	[0.01, 0.80]
$\ln(10^{10} A_s)$	[2, 4]
n_s	[0.8, 1.2]
w_0	[-3.0, -0.01]
w_1	[-4, 9]

We consider the case that $\sum m_\nu$ serves as a free parameter with different hierarchies of neutrino mass. For the NH, IH, and DH cases, the priors of $\sum m_\nu$ are [0.06, 3.00] eV, [0.10, 3.00] eV, and [0.00, 3.00] eV. The neutrino mass spectrum is described as

$$(m_1, m_2, m_3) = (m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + |\Delta m_{31}^2|})$$

with a free parameter m_1 for the NH case,

$$(m_1, m_2, m_3) = (\sqrt{m_3^2 + |\Delta m_{31}^2|}, \sqrt{m_3^2 + |\Delta m_{31}^2| + \Delta m_{21}^2}, m_3)$$

with a free parameter m_3 for the IH case, and

$$m_1 = m_2 = m_3 = m$$

with a free parameter m for the DH case.

In order to check the consistency between dynamical dark energy models and the CMB+BAO+SNe data, we employ the χ^2 statistic [55–57] to do the cosmological fits. A model with a lower value of χ^2 is more favored by the CMB+BAO+SNe data combination. Our constraint results are derived by modifying the August 2017 version of the camb Boltzmann code [58] and the July 2018 version of CosmoMC [59].

3. Results and discussions

We constrain the sum of the neutrino mass $\sum m_\nu$ in these dynamical dark energy models by using the CMB+BAO+SNe data. In the following discussion, we will present the fitting results with the $\pm 1\sigma$ errors of cosmological parameters. But for the constraints on $\sum m_\nu$, we only provide the 2σ upper limit. Meanwhile, we also list the values of χ_{min}^2 for different dark energy models.

3.1. Comparison of dynamical dark energy models

We constrain the models parameterized by $w(z) = w_0 + w_1 \frac{z}{1+z}$, $w(z) = w_0 + w_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right)$ and $w(z) = w_0 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1) \right)$. The fitting results are listed in Table 2. We find that the current CMB+BAO+SNe data favor the constraint results of $w_0 = -1$ and $w_1 = 0$ in the three models. For the CPL model, we obtain $\Omega_m = 0.3059 \pm 0.0077$ and $H_0 = 68.37 \pm 0.83$ km/s/Mpc, with $\chi^2_{\min} = 3821.214$. For the Log model, we have $\Omega_m = 0.3060 \pm 0.0075$ and $H_0 = 68.37 \pm 0.81$ km/s/Mpc, with $\chi^2_{\min} = 3821.150$. For the Sin model, we have $\Omega_m = 0.3056 \pm 0.0077$ and $H_0 = 68.41 \pm 0.83$ km/s/Mpc, with $\chi^2_{\min} = 3821.164$. The fit values of Ω_m and H_0 are similar for the three models. According to the $\chi^2_{\min}$ values, the models provide a similar fit to the CMB+BAO+SNe data.

Table 2. The fitting values for the six dynamical dark energy models.

Parameter	CPL	Log	Sin	MCPL	MLog	MSin
w_0	-0.968 ± 0.079	$-0.968^{+0.065}_{-0.072}$	$-0.973^{+0.059}_{-0.058}$	-1	-1	-1
w_1	$-0.24^{+0.33}_{-0.27}$	$0.93^{+0.79}_{-1.11}$	$0.36^{+0.28}_{-0.40}$	$-0.12^{+0.13}_{-0.11}$	$0.52^{+0.39}_{-0.48}$	$0.22^{+0.16}_{-0.21}$
Ω_m	0.3059 ± 0.0077	0.3060 ± 0.0075	0.3056 ± 0.0077	$0.3048^{+0.0070}_{-0.0071}$	$0.3045^{+0.0069}_{-0.0068}$	0.3044 ± 0.0068
H_0 [km/s/Mpc]	68.37 ± 0.83	68.37 ± 0.81	68.41 ± 0.83	68.47 ± 0.76	68.53 ± 0.73	68.55 ± 0.73
σ_8	0.822 ± 0.011	0.822 ± 0.011	0.823 ± 0.011	0.822 ± 0.011	0.823 ± 0.011	0.824 ± 0.011
$\chi^2_{\min}$	3821.214	3821.150	3821.164	3821.310	3821.288	3821.290

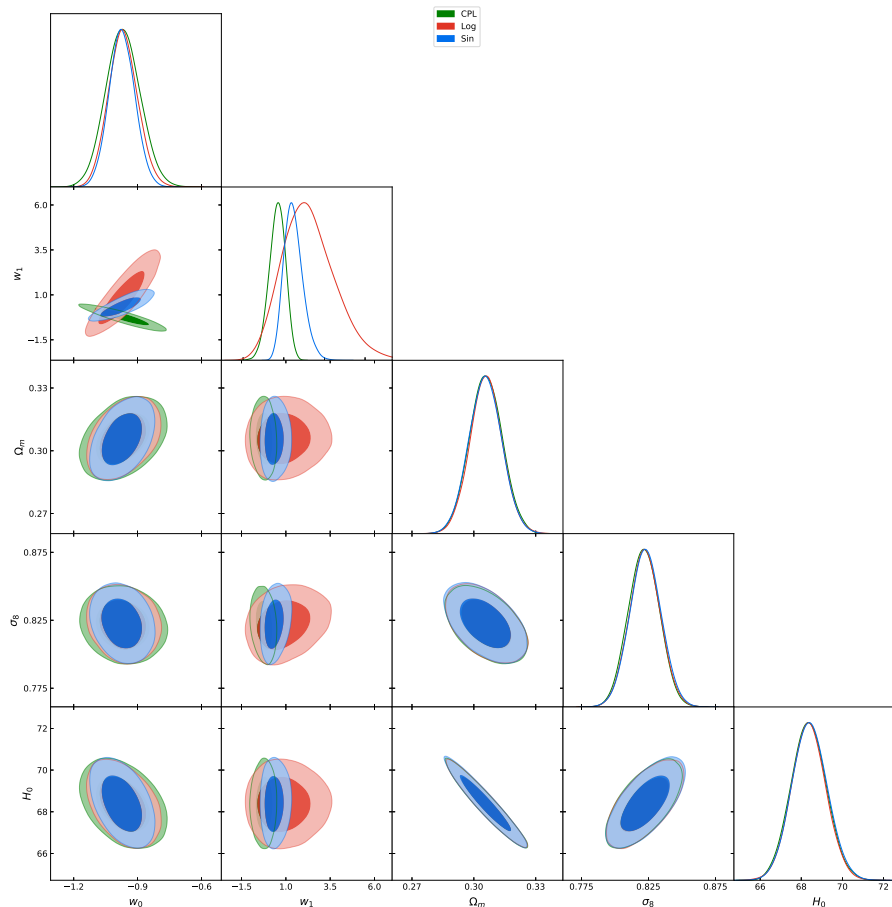


Figure 1. One-dimensional marginalized distributions and two-dimensional contours at 1σ and 2σ level for the CPL, Log, and Sin models.

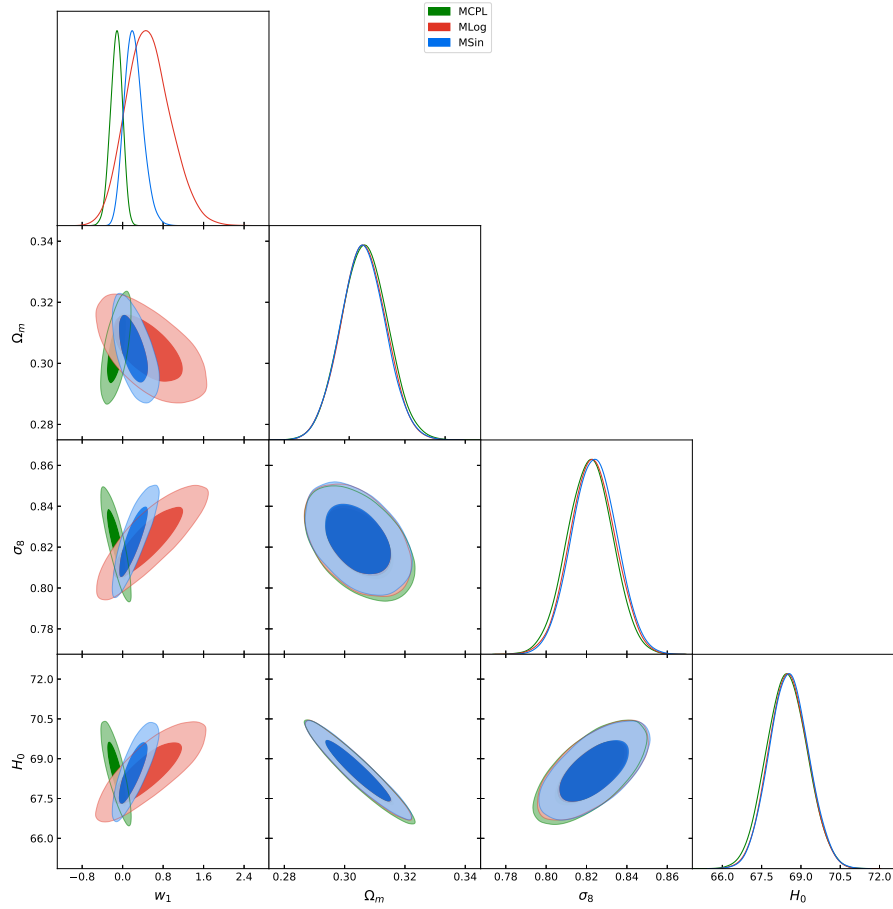


Figure 2. One-dimensional marginalized distributions and two-dimensional contours at 1σ and 2σ level for the MCPL, MLog, and MSin models.

As described in Sect. 1, when $w_0 = -1$ is fixed in the above models, the form of $w(z)$ is modified with a free parameter w_1 . The fitting results are also given in the last three columns of Table 2. In the MCPL model, $w(z) = -1 + w_1 \frac{z}{1+z}$. In the MLog model, $w(z) = -1 + w_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right)$. In the MSin model, $w(z) = -1 + w_1 \left(\frac{\sin(1+z)}{1+z} - \sin(1) \right)$. We obtain $w_1 = -0.12^{+0.13}_{-0.11}$, $w_1 = 0.52^{+0.39}_{-0.48}$, and $w_1 = 0.22^{+0.16}_{-0.21}$, showing a slight deviation to $w_1 = 0$ in the MLog model and the MSin model. This is because w_1 is intrinsically correlated with w_0 , as shown in Figure 1 (w_1 is anticorrelated with w_0 in the CPL model, but the correlation between them is opposite in the Log model and the Sin model). When the value of w_0 is fixed to -1 , the fitting value of w_1 will be changed to a certain extent.

Furthermore, we focus on the $\chi^2_{\min}$ values for the three models. We obtain $\chi^2_{\min} = 3821.310$ in the MCPL model, $\chi^2_{\min} = 3821.288$ in the MLog model, and $\chi^2_{\min} = 3821.290$ in the MSin model. Similarly, almost identical $\chi^2_{\min}$ values are presented in the three models. In Figures 1 and 2, we also provide the one-dimensional marginalized distributions and two-dimensional contours at 1σ and 2σ level for these dynamical dark energy models. The fitting results of the parameter Ω_m , H_0 , and σ_8 hardly change in these models despite of $w(z)$ parametrized by different forms.

3.2. Constraints on neutrino masses

We investigate the constraints on total neutrino mass in these models. For the neutrino mass measurement, we consider the NH case, the IH case, and the DH case. The fitting results are listed in Tables 3–5. In the CPL+ $\sum m_\nu$ model, we obtain $\sum m_\nu < 0.285$ eV for the NH case, $\sum m_\nu < 0.304$ eV for the IH case, and $\sum m_\nu < 0.254$ eV for the DH case (see Table 3). In the Log+ $\sum m_\nu$ model, we have $\sum m_\nu < 0.302$ eV for the NH case, $\sum m_\nu < 0.317$ eV for the IH case, and $\sum m_\nu < 0.282$ eV

for the DH case (see Table 4), showing that much looser constraints are obtained than those in the CPL+ $\sum m_\nu$ model. In the Sin+ $\sum m_\nu$ model, the constraint results become $\sum m_\nu < 0.327$ eV for the NH case, $\sum m_\nu < 0.336$ eV for the IH case, and $\sum m_\nu < 0.311$ eV for the DH case (see Table 5), which are looser than those in the Log+ $\sum m_\nu$ model. All the above fitting upper limits on $\sum m_\nu$ are larger than those obtained in the standard Λ CDM model (in the Λ CDM model, the constraint results are $\sum m_\nu < 0.156$ eV for the NH case, $\sum m_\nu < 0.184$ eV for the IH case, and $\sum m_\nu < 0.121$ eV for the DH case [33,60]), indicating that the dynamical dark energy with the logarithm form and the oscillating form can affect significantly the fitting value of $\sum m_\nu$.

Table 3. The fitting values for the CPL+ $\sum m_\nu$ and MCPL+ $\sum m_\nu$ models considered mass hierarchy cases of NH, IH, and DH.

Parameter	CPL			MCPL		
	NH	IH	DH	NH	IH	DH
w_0	$-0.940^{+0.085}_{-0.095}$	$-0.929^{+0.083}_{-0.097}$	$-0.950^{+0.082}_{-0.092}$	-1	-1	-1
w_1	$-0.49^{+0.46}_{-0.33}$	$-0.59^{+0.48}_{-0.32}$	$-0.39^{+0.47}_{-0.30}$	$-0.24^{+0.18}_{-0.13}$	$-0.30^{+0.18}_{-0.14}$	$-0.17^{+0.19}_{-0.13}$
$\sum m_\nu$ [eV]	< 0.285	< 0.304	< 0.254	< 0.250	< 0.276	< 0.228
Ω_m	$0.3094^{+0.0081}_{-0.0087}$	$0.3103^{+0.0081}_{-0.0082}$	$0.3077^{+0.0083}_{-0.0090}$	0.3069 ± 0.0073	0.3078 ± 0.0072	$0.3058^{+0.0072}_{-0.0079}$
H_0 [km/s/Mpc]	68.27 ± 0.82	$68.27^{+0.83}_{-0.81}$	68.32 ± 0.84	68.47 ± 0.76	68.49 ± 0.75	$68.45^{+0.77}_{-0.76}$
S_8	0.825 ± 0.012	0.823 ± 0.012	0.827 ± 0.012	0.824 ± 0.011	0.822 ± 0.011	0.826 ± 0.012
$\chi^2_{\min}$	3822.102	3822.516	3821.168	3822.144	3823.046	3821.112

Table 4. The fitting values for the Log+ $\sum m_\nu$ and MLog+ $\sum m_\nu$ models considered mass hierarchy cases of NH, IH, and DH.

Parameter	Log			MLog		
	NH	IH	DH	NH	IH	DH
w_0	$-0.946^{+0.071}_{-0.080}$	$-0.938^{+0.073}_{-0.081}$	$-0.955^{+0.069}_{-0.079}$	-1	-1	-1
w_1	$1.90^{+1.00}_{-1.70}$	$2.20^{+1.10}_{-1.70}$	$1.52^{+0.95}_{-1.63}$	$1.02^{+0.50}_{-0.78}$	$1.21^{+0.51}_{-0.76}$	$0.77^{+0.47}_{-0.80}$
$\sum m_\nu$ [eV]	< 0.302	< 0.317	< 0.282	< 0.268	< 0.288	< 0.250
Ω_m	0.3094 ± 0.0082	$0.3106^{+0.0083}_{-0.0082}$	$0.3080^{+0.0081}_{-0.0089}$	0.3066 ± 0.0072	0.3078 ± 0.0072	0.3056 ± 0.0074
H_0 [km/s/Mpc]	68.31 ± 0.82	$68.27^{+0.83}_{-0.82}$	$68.33^{+0.82}_{-0.81}$	68.54 ± 0.74	68.52 ± 0.75	$68.53^{+0.75}_{-0.74}$
S_8	0.825 ± 0.012	0.823 ± 0.012	0.827 ± 0.012	0.824 ± 0.012	0.822 ± 0.011	$0.826^{+0.013}_{-0.012}$
$\chi^2_{\min}$	3822.100	3822.180	3821.048	3822.458	3823.538	3821.284

Table 5. The fitting values for the Sin+ $\sum m_\nu$ and MSin+ $\sum m_\nu$ models considered mass hierarchy cases of NH, IH, and DH.

Parameter	Sin			MSin		
	NH	IH	DH	NH	IH	DH
w_0	$-0.956^{+0.063}_{-0.070}$	$-0.952^{+0.065}_{-0.066}$	-0.962 ± 0.063	-1	-1	-1
w_1	$0.80^{+0.37}_{-0.70}$	$0.91^{+0.41}_{-0.69}$	$0.66^{+0.34}_{-0.69}$	$0.49^{+0.21}_{-0.38}$	$0.57^{+0.22}_{-0.38}$	$0.37^{+0.19}_{-0.39}$
$\sum m_\nu$ [eV]	< 0.327	< 0.336	< 0.311	< 0.298	< 0.318	< 0.277
Ω_m	$0.3097^{+0.0083}_{-0.0090}$	$0.3106^{+0.0082}_{-0.0083}$	0.3081 ± 0.0084	0.3069 ± 0.0072	0.3079 ± 0.0072	0.3058 ± 0.0073
H_0 [km/s/Mpc]	$68.33^{+0.83}_{-0.84}$	$68.32^{+0.84}_{-0.83}$	68.37 ± 0.82	$68.57^{+0.72}_{-0.73}$	68.55 ± 0.73	$68.56^{+0.74}_{-0.73}$
S_8	0.825 ± 0.012	0.823 ± 0.012	0.826 ± 0.012	0.824 ± 0.012	0.822 ± 0.012	0.826 ± 0.012
$\chi^2_{\min}$	3822.408	3823.456	3821.080	3822.876	3823.574	3821.224

Considering the same neutrino mass ordering, the fitting value of $\sum m_\nu$ is smallest in the CPL model and largest in the Sin model, confirming that the fitting values of $\sum m_\nu$ can be changed by modifying the $w(z)$ forms. In Figure 3, we provide two-dimensional marginalized contours (68.3% and 95.4% confidence level) in the $\sum m_\nu$ - w_0 plane of the CPL, Log, and Sin models, considered mass hierarchy cases of NH, IH, and DH. In the three two-parametrization models, $\sum m_\nu$ is positively correlated w_0 , which ensures the same observed acoustic peak scale in the cosmological fit using the Planck data. When we compare the constraint results of $\sum m_\nu$ for the three different cases of neutrino mass orderings, we find that the smallest value of $\sum m_\nu$ is obtained in the DH case, and the largest value of $\sum m_\nu$ corresponds to the IH case, which mean that considering the mass hierarchy can also affect the fitting values of $\sum m_\nu$.

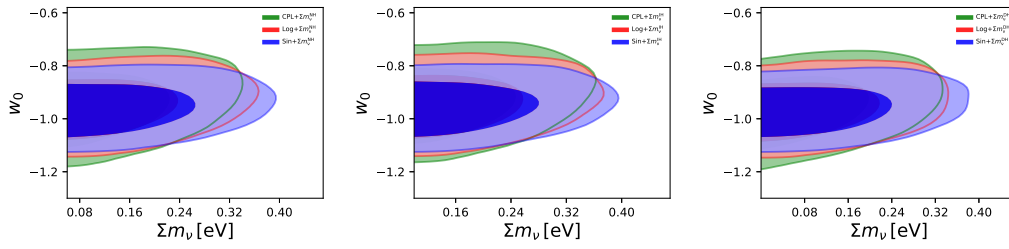


Figure 3. Two-dimensional marginalized contours (68.3% and 95.4% confidence level) in the $\Sigma m_\nu - w_0$ plane of the CPL, Log, and Sin models considered mass hierarchy cases of NH, IH, and DH.

In the $\text{CPL} + \Sigma m_\nu$ model, we obtain $\chi^2_{\min} = 3822.102$ for the NH case, $\chi^2_{\min} = 3822.516$ for the IH case, and $\chi^2_{\min} = 3821.168$ for the DH case (see Table 3). In the $\text{Log} + \Sigma m_\nu$ model, we have $\chi^2_{\min} = 3822.100$ for the NH case, $\chi^2_{\min} = 3822.180$ eV for the IH case, and $\chi^2_{\min} = 3821.048$ for the DH case (see Table 4). In the $\text{Sin} + \Sigma m_\nu$ model, the constraint results become $\chi^2_{\min} = 3822.408$ for the NH case, $\chi^2_{\min} = 3823.456$ for the IH case, and $\chi^2_{\min} = 3821.080$ for the DH case (see Table 5). Obviously, the small difference of the $\chi^2_{\min}$ values among the three mass hierarchies only stems from the different prior ranges of the parameter Σm_ν , which does not help to distinguish the neutrino mass orderings.

We also discuss the constraints of Σm_ν in the MCPL model, the MLog model, and the MSin model, in which $w(z)$ is parameterized with a single free parameter w_1 . In the $\text{MCPL} + \Sigma m_\nu$ model, we obtain $\Sigma m_\nu < 0.250$ eV for the NH case, $\Sigma m_\nu < 0.276$ eV for the IH case, and $\Sigma m_\nu < 0.228$ eV for the DH case (see Table 3). In the $\text{MLog} + \Sigma m_\nu$ model, we have $\Sigma m_\nu < 0.268$ eV for the NH case, $\Sigma m_\nu < 0.288$ eV for the IH case, and $\Sigma m_\nu < 0.250$ eV for the DH case (see Table 4). In the $\text{MSin} + \Sigma m_\nu$ model, the constraint results become $\Sigma m_\nu < 0.298$ eV for the NH case, $\Sigma m_\nu < 0.318$ eV for the IH case, and $\Sigma m_\nu < 0.277$ eV for the DH case (see Table 5). Not surprisingly, the constraint results of Σm_ν are largest in the MSin model and smallest in the MCPL model.

Furthermore, comparing constraint results of Σm_ν with those derived from the two-parametrization models, we find that the values of Σm_ν are smaller in these one-parametrization models, indicating that a model with less parameters tends to provide a smaller fitting value of Σm_ν . The two-dimensional marginalized contours in the $\Sigma m_\nu - w_1$ plane are shown in Figure 4. We see that Σm_ν is positively correlated with w_1 in the MCPL and MLog models, but is anti-correlated with w_1 in the MSin model. The different degeneracies between them ensure that the ratio of the sound horizon and angular diameter distance remains nearly constant.

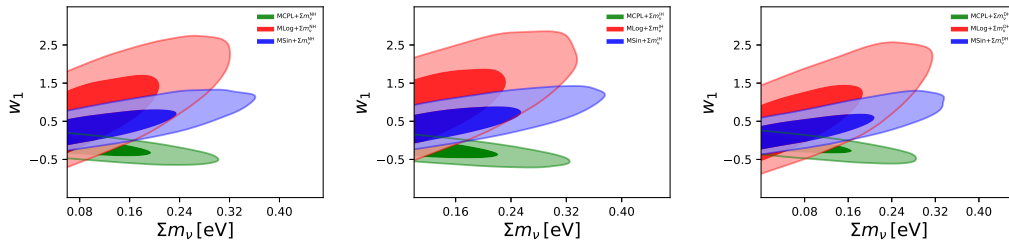


Figure 4. Two-dimensional marginalized contours (68.3% and 95.4% confidence level) in the $\Sigma m_\nu - w_1$ plane of the MCPL, MLog, and MSin models considered mass hierarchy cases of NH, IH, and DH.

4. Conclusions

In this paper, we constrain three dynamical dark energy models parameterized by two free parameters, w_0 and w_1 . They correspond to the CPL parametrization, the logarithm parametrization, and the oscillating parametrization. The difference from the CPL model is that the logarithm parametrization and the oscillating parametrization can overcome the future divergency problem, and successfully probe the dynamics of dark energy in all the evolution stages of the universe. We constrain these dynamical dark energy models by using current cosmological observations including

the CMB data, the BAO data, and the SNe data. We find that the Log model and the Sin model behave as the same as the CPL model in the fit to the CMB+BAO+SNe data.

We investigate the constraints on the total neutrino mass $\sum m_\nu$ in these dynamical dark energy. Simultaneously, we consider the NH case, the IH case, and the DH case of three-generation neutrino mass. We confirm the fact that the different neutrino mass hierarchies can affect the constraint results of $\sum m_\nu$ significantly. The smallest fitting value of $\sum m_\nu$ is obtained in the DH case, and the largest value of $\sum m_\nu$ corresponds to the IH case. We reconfirm that the dark energy properties could indeed significantly change the fitting results of $\sum m_\nu$. The values of $\sum m_\nu$ in the Log and Sin models are larger than those derived from the CPL model. In addition, our results does not provide more evidence for determining the neutrino mass orderings because of the similar values of $\chi^2_{\min}$ obtained for different neutrino mass hierarchies.

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