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Article

On the Flavour States and the Mass States of Neutrinos

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Abstract: A structure based analysis of the pion's decay path reveals that neutrinos can show up in three eigenstates. It requires a proper understanding of the nature of charged leptons, such as why the loss of binding energy stops the lepton generation at the tauon level. The analysis quantifies this binding energy in terms of the weak interaction strength as embodied by the weak interaction boson and the strength of the energetic background field as embodied by the Higgs boson. The article ends with the conclusion that neutrino oscillation is not a physical phenomenon, but, instead, a measurement interpretation induced from projecting the statistical behaviour of a multi-particle ensemble onto a single particle.

Keywords: neutrino; neutrino oscillation; lepton; flavour state; mass state

1. Introduction

The history of neutrinos dates back from 1927 when Wolfgang Pauli formulated a bold hypothesis on their existence that he sighing posited in a letter to Hans Geiger and Lise Meitner [1]. It was the only way out he could imagine to explain the uniformly distributed energy spectrum of electrons that showed up beta radiation in observations and experiments on radioactivity as already since 1914 been noticed by James Chadwick [2]. It was Enrico Fermi, who took Pauli's hypothesis seriously and who in 1933 developed a theory for beta radiation based on the neutrino existence [3]. In his theory, which presently is regarded as the forerunner of the weak interaction theory, the neutrino is a fermion that eludes observation because of its zero mass and zero charge. Eventually, in 1956, its existence is experimentally confirmed by Reines and Cowan [4].

That experiment marks the start of experimental studies on neutrinos. One of the problems, next to identify suitable physical processes to study the interaction of neutrinos with matter, is the issue how to obtain neutrino fluxes large and strong enough to detect the rare events expected from those processes. Reines and Rowan used a nuclear reactor for the purpose. Their experiment got follow-ups by other ionic neutrino experiments, in particular those based upon knowledge captured in the Standard Solar Model. The idea behind those is, that since the energy of the sun is known and since its major energy production mechanism as well, it is possible to calculate the neutrino flux on earth. This flux is defined as the number of neutrinos that, each second pass through 1 m square surface perpendicular to the direction to the sun. This would enable to develop experimental evidence not only in qualitative terms, but in quantitative terms as well.

And it did. Most remarkably, however, those experiments revealed an unexpected result [5]. The predicted neutrino count showed a deficit of about 50%-70% with respect the actually measured count. The solar neutrino problem was born. What happened with the neutrinos emitted by the sun? Why would those not be capable to produce the predicted neutrino count? The inevitable answer to the problem is the awareness that neutrinos are subject to changes when they move from to the sun to the detectors on earth. The most simple approach to this problem is the assumption that neutrinos come in different flavours. Because they are produced in co-production with charged leptons, they show a specific flavour determined by the co-produced charged lepton. This hypothesis could be affirmatively tested by making the neutrino detectors in the experimental equipment no longer exclusively sensitive to electron neutrinos. Nevertheless, a major problem remained: the neutrino flow

from the sun is produced from nuclear fusion of Hydrogen atoms into Helium nuclei, thereby producing almost exclusively electron neutrinos. How to explain the change of electron neutrinos into a significant amount of other flavours on their route from sun to earth?

Eventually this tantalizing question has resulted into the bold hypothesis, earlier formulated by Pontecorvo in 1957, and later adopted as explanation for the missing neutrinos, that neutrinos are built up by a virtual substructure [6]. Such a virtual substructure would allow neutrino compositions built by three basic eigenstates, different from their flavour states, more or less in the same way as hadrons are composed by quarks. According to this hypothesis, the electron neutrino is in a particular mixture of eigenstates, while a muon neutrino and a tauon neutrino would be in other mixtures. Hypothetically, this would allow oscillations between the flavour states of neutrinos and the loss of coherency would solve the solar neutrino problem.

If substructures are considered as being viable for neutrinos, why would substructures for charged leptons not be viable as well? Why not conceiving the electron, the muon and the tauon as states built by underlying constituents as well? Within the Standard Model, the charged leptons are simply considered as elementary particles, and because in the Standard Model everything comes in a three, even a basic question as “why no charged lepton beyond the tauon” has remained unanswered. This article is aimed to show how these issues of the constrained lepton generation and the mass and origin of neutrinos can be highlighted in the structure based model of particle physics, documented in [7]. In the second paragraph of this article, it is shown how the structural model for charged and uncharged leptons (neutrinos) evolve from the structural model of mesons as developed in previous work. It is the stepping stone for the explanation of some unrecognized phenomena in the Standard Model of particle physics. The first of these is a proof for the non-understood stop of the lepton generation at the tauon level (paragraph 3). It is shown that this can be traced back to the very same reason as why the quark flavour generation stops at the *b* (ottom) level (topquarks are of a different kind [7]). In the fourth paragraph it will be shown why neutrino flavours are composed by eigenstates and how these become manifest as physical mass. The final paragraph contains a discussion on the flavour oscillation phenomenon.

2. Structural models for pion, muon and neutrino

Figure 1 is an illustration of the structural model for a pion as developed by the author over the years [7]. It shows that two quarks are structured by a balance of two nuclear forces and two sets of dipoles. The two quarks are described as Dirac particles with two real dipole moments as the virtue of particular gamma matrices. The vertical one is the equivalent of the magnetic dipole moment of an electron. The (real valued) horizontal dipole moment is the equivalent of the (imaginary valued) electric dipole moment of an electron [8,9].

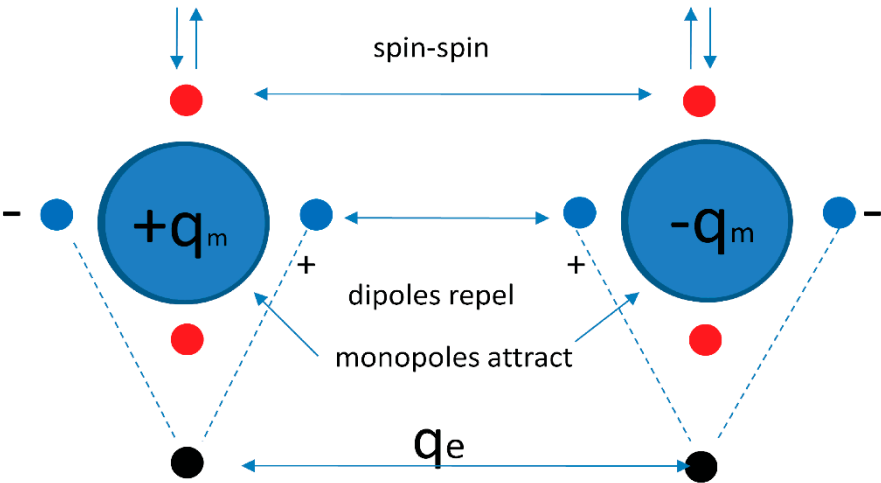


Figure 1. Hypothetical equivalence of the quark’s polarisable linear dipole moment with the magnetic dipole moment of its electric charge attribute.

In a later description, after recognizing that this structure shows properties that match with a Maxwellian description, the quarks have been described as magnetic monopoles in Comay's Regular Charge Monopole Theory (RCMT) [10]. This allows to give an explanation of the quark's electric charge by assuming that the quarks second dipole moments (the horizontal ones) coincide with the magnetic dipole moments of electric kernels q_e . This description allows to conceive the nuclear force as the cradle of baryonic mass (the ground state energy of the created anharmonic oscillator) as well as the cradle of electric charge.

The model allows a pretty accurate calculation of the mass spectrum of mesons. It also allows the development of a structural model of baryons including an accurate calculation of the mass spectrum of baryons as well. This calculation relies upon the recognition that the structure can be modelled as a quantum mechanical anharmonic oscillator. Such anharmonic oscillators are subject to excitation, thereby producing heavier hadrons with larger (constituent) masses of their constituent quarks. The increase of baryonic energy under excitation is accompanied with a loss of binding energy between the quarks. This sets a limit to the maximum constituent mass value of the quarks. It is the reason why quarks heavier than the bottom quark cannot exist and why the top quark has to be interpreted different from being the isospin sister of the bottom quark [7].

Because lepton generations beyond the tauon have not been found, they probably don't exist for the same reason. In such a picture the charge lepton structure would result from the decay loss of the magnetic charges of the kernels in the pion structure under simultaneous replacement by their electric charges. This structure is bound together by an equilibrium of the repelling force between the electric charges and the attraction force between the scalar dipole moments. In spite of its resemblance with the pion structure, its properties are fundamentally different. Whereas the pion consists of a particle and an antiparticle making a boson, the charged lepton consists of two kernels making a fermion. Figure 3 shows a naive picture of the decay process. Under decay, the pion will be split up into a muon and a neutrino. In the rest frame of the muon, the muon will obviously contain the electric kernels and some physical mass. The remaining energy will fly away as a neutrino with kinetic energy and some remaining physical mass. Figure 2 shows the model, in which a structureless neutrino is shown next to a muon with a hypothetical substructure.

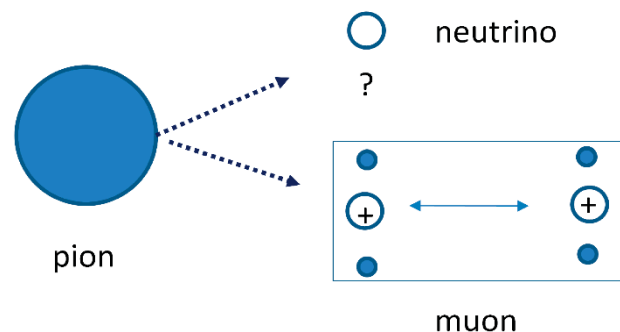


Figure 2. The decay of the pion's atypical dual dipole moment configuration into two typical single dipole moment configurations.

In this picture the muon is considered to be a half spin fermion in spite of the appearance of two identical kernels in the same structure. Assigning the fermion state to the structure seems being in conflict with the convention to distinguish the boson state from the fermion state by a naive spin count. Instead, a true boson state for particles in conjunction should be based upon the state of the temporal part of the composite wave function. In this particular case, the reversal of the particle state into antiparticle of one of the quarks marks a transformation from the bosonic pion state into the fermionic muon state under conservation of the weak interaction bond. Conceptually, the spin-half characteristic of the muon would be the result of the angular dipole moment of two spin-less kernels in a circular orbit. This might seem a conflict with the Pauli doctrine and might give a reason to opponents to reject the concept. Let us take into consideration, though, that those kernels may inherit

their properties from the quarks and that those quarks have two real dipole moments instead of a single one. A generalization of the Pauli doctrine may therefore remove this objection.

Let us proceed from the observation that there is no compelling reason why the weak interaction mechanism between a particle and an antiparticle kernel would not hold for two subparticle kernels. In such model, the structure for the charged lepton is similar to the pion one. It can therefore be described by a similar analytical model. Hence, conceiving the muon as a structure in which a kernel couples to the field of another kernel with the generic quantum mechanical coupling factor g , the muon can be modeled as a one-body equivalent of a two-body oscillator, described by the equation for its wave function ψ ,

$$-\frac{\hbar^2}{2m_m} \frac{d^2\psi}{dx^2} + \{U(d+x) + U(d-x)\}\psi = E\psi; \quad U(x) = g\Phi(x), \quad (1)$$

in which \hbar is Planck's reduced constant, $2d$ the kernel spacing, m_m is the effective mass of the center, $V(x) = U(d+x) + U(d-x)$ its potential energy, and E the generic energy constant, which is subject to quantization. The potential energy $V(x)$ can be derived from a potential $\Phi(x)$. Similarly as in the case of the pion quarks, this potential is a measure for the energetic properties of the kernels. It is characterized by a strength Φ_0 (in units of energy) and a range λ^{-1} (in units of length: the dimension of λ is $[m^{-1}]$).

The potential $\Phi(x)$ of a pion quark has been determined in previous work as,

$$\Phi(x) = \Phi_0 \exp(-\lambda x) \left(\frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x} \right), \quad (2)$$

in which the dimensionless quantity g_m is close to 2. These quantities have more than a symbolic meaning, because in the structural model for particle physics developed so far [7], λ has been quantified by $m'_H = 2\lambda\hbar c$, in which m'_H (≈ 126 GeV) is the energy of the Higgs particle as the carrier of the energetic background field. The quantity Φ_0 has been related with the energy of the weak interaction interaction boson m'_W (≈ 80.4 GeV). The estimate on the magnitude of the gyrometric factor g_m has been found from a derivation of the potential $\Phi(x)$ from the Higgs potential as heuristically adopted in the Standard Model. How this has been done can be read in [13]. Moreover, in paragraph 6 of this article, an historic reflection will be given on this issue.

An equal expression for the potential $\Phi(x)$ would make the muon model to a Chinese copy of the pion model. Instead, we wish to describe the potential $\Phi(x)$ of the muon kernels as,

$$\Phi(x) = \Phi_0 \left(\frac{1}{(\lambda x)^2} - g_m \frac{1}{\lambda x} \right) \quad (3)$$

The rationale for this modification is rather trivial. In the structural model for the pion, the exponential decay is due to the shielding effect of an energetic background field. If the muon is a true electromagnetic particle, there is no reason why its potential field would be shielded. This may explain why an additional energetic particle is required to compensate for the difference between the shielded and the unshielded potential. This may explain the origin of the neutrino. Considering that the potential is a measure of energy and that the break-up of a pion into a muon and a neutrino takes place under conservation of energy, it is fair to conclude that the neutrino can be described in terms of a potential function as well, such that

$$\Phi_{muon}(x) = \Phi_{pion}(x) + \Phi_{neutrino}(x) \quad (4)$$

We may even go a step further by supposing that, similarly as the muon, the neutrino can be modelled by a composition of two kernels. If so, each of these neutrino kernels have a potential function $\Phi_{\nu}(x)$, such that

$$\Phi_{\nu}(x) = \Phi_0 \{1 - \exp(-\lambda x)\} \left\{ \frac{1}{(\lambda x)^2} - \frac{g_m}{\lambda x} \right\} \quad (5)$$

It is instructive to emphasize that the potential function of a particle, be it a quark, a charged lepton or a neutrino, does not contain any information about its mass. In that respect it is not different from the potential function of a charged particle like an electron. Furthermore it is of interest to emphasize that, like mentioned before, the quantities Φ_0 and λ have a physical meaning in quantitative terms.

The muon is not a stable particle. It loses its weak interaction bond under decay into electrons. Figure 3 shows an interpretation of the process. It shows how the weak interaction boson that binds the pion quarks desintegrates into the muon and the muon neutrino and how the two may recoil into a weak interaction boson that decays into an electron and electron antineutrino. This picture and the description just given evoke two basic questions.

The first one is this. If it is true indeed that the behaviour of the muon can be modelled as an anharmonic oscillator, why would the muon not be subject to a similar excitation mechanism as shown by the pion? The answer is that the muon is subject to excitation indeed, thereby producing the tauon state. Actually, this has been documented in previous work [11]. But, unlike as in the case of pions, it is a single stop. This excitation mechanism will be summarized in the next paragraph. It will be shown that this analysis will give a firm support to the model captured by the equations (2-7).

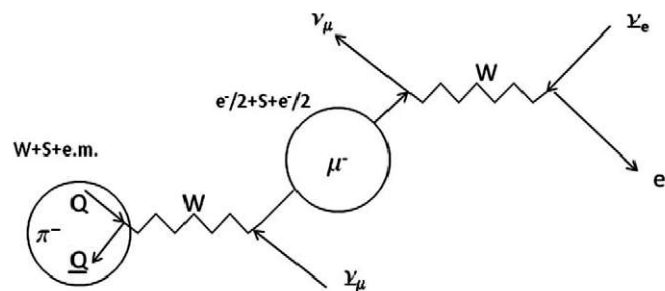


Figure 3. A charged pion decays into a charged lepton (muon) and its associated neutrino because of emission of the vector weakinteraction boson. Subsequently, the muon and the neutrino recoils into a weak interaction boson that subsequently decays into a (Pauli) shower of electrons and antineutrinos.

The second issue is the question how the muon decays into an electron and a neutrino. This has been documented in previous work as well [12]. In this work it has been shown that this can adequately be described in a way as originally proposed by Fermi. Its predictive power, however, has not been properly understood before.

This paragraph is now concluded with the statement that the leptons show up in three generations of charged-uncharged twins. The muon twin is the result of the pion's loss of its bond with energetic background field. The tauon twin is an excitation and the electron twin is the elementary one.

It might be worthwhile to remark that there is no reason why the picture shown in Figure 3 would not be reversible, like shown by all quantum mechanical processes. It means that the neutrinos may change flavour, although not on their own. An electron neutrino with high energy may recoil with an electron to produce a weak interaction boson that subsequently disintegrates into a muon and an muon antineutrino. This observation, however, does not solve the solar neutrino problem mentioned before, because that problem points to neutrino interactions on their own.

Let us proceed by considering the structure of the charged leptons.

3. The charged leptons

The formulation of the anharmonic oscillators shown by (2-4) does not contain any particular quantum mechanical property. A binomial expansion of the potential energy $V(x)$ allows to rewrite the wave equation as,

$$-\frac{\hbar^2}{2m_m} \frac{d^2\psi}{dx^2} + g\Phi_0 \{k_0 + k_2\lambda^2 x^2 + \dots\} \psi = E\psi, \quad (6)$$

In which k_0 and k_2 are dimensionless coefficients that depend on the spacing $2d$ between the quarks and in which g is a generic quantum mechanical coupling factor. The oscillator vibrates with a certain frequency ω , subject to the classical relationship,

$$\frac{1}{2} m_m \omega^2 = g\Phi_0 k_2 \lambda^2 \rightarrow \frac{m'_m (\hbar\omega)^2}{(\hbar c)^2} = 2g\Phi_0 k_2 \lambda^2. \quad (7)$$

Conventionally, m_m represents is the central mass of the oscillator. However, in this non-relativistic center-of-mass model, it does not represent the individual masses of the two bodies, but, like stated before, it is an equivalent mass that captures the energy of the field. Similarly as in the pion case, it will make sense to normalize the wave function equation to,

$$-\alpha_0 \frac{d^2\psi}{dx'^2} + V'(x') = E'\psi, \quad (8)$$

in which $\alpha_0 = \frac{\lambda^2 \hbar^2}{2m_m g\Phi_0}$, $x' = x\lambda$, $d' = d\lambda$, $E' = \frac{E}{g\Phi_0}$, $U'(x') = \frac{U(\lambda x)}{g\Phi_0}$ and

$$V'(x') = U'(d' + x') + U'(d' - x') = k_0(d') + k_2(d')x'^2 + \dots; U(x) = g\Phi(x).$$

The oscillator settles itself into a minimum state of energy at $d' = d'_{\min}$.

As can be proven by analysis, under adoption of the potential function (3), we have the simple conditions,

$$d'_{\min} = \frac{g_m}{2}; \quad V'(d'_{\min}) = -\frac{g_m^2}{2}$$

$$\frac{k_a^2}{k_b} = \frac{k_0^2(d'_{\min})}{k_2(d'_{\min})} = 2; \quad \left| \frac{k_b}{k_a} \right| = \frac{g_m^2}{4} \quad (9)$$

Curiously, the condition of minimum energy is independent of g_m . Consistency with earlier work on the meson model imposes to consider the boson $\hbar\omega$ as the weak interaction boson $\hbar\omega = \hbar\omega_w$ in virtual state and to relate its energy with the binding energy because of the massless nature of the kernels. Hence,

$$\hbar\omega = \hbar\omega_w = 2 |k_a| g\Phi_0. \quad (10)$$

Substitution of (10) into (7) reveals

$$m'_m = \frac{k_b}{|k_a|} \frac{(\lambda\hbar c)^2}{\hbar\omega}. \quad (11)$$

The factor $\lambda\hbar c$ can be related with the kernel spacing $2d'_{\min}$. This spacing will be kept constant by the weak interaction boson. Similarly as in the case of mesons, the spacing is about the half wave length $L_w / 2$ of this meson. Hence

$$2d'_{\min} = \alpha \frac{L_w}{2} = \frac{\alpha}{2} c \frac{2\pi}{\hbar\omega_w} = \alpha\pi \frac{\hbar c}{m'_w} \rightarrow \lambda\hbar c = \frac{2d'_{\min}}{\alpha\pi} m'_w; d'_{\min} = d\lambda, \quad (12)$$

in which α is of the order 1. While in the meson case the weak interaction boson $\hbar\omega_w \approx 80.4 \text{ GeV}$ in the center-of-mass frame transforms into the rest mass $m'_\pi \approx 140 \text{ MeV}$ of the pion, in the muon case $\hbar\omega_w$ can be considered as the weak interaction boson in virtual state with a mass value equal to the rest mass of the pion. Hence, from (12), (14) and (9),

$$m'_m = \frac{k_b}{|k_a|} \left(\frac{\lambda\hbar c}{m'_w} \right)^2 m'_\pi = \frac{k_b}{|k_a|} \left(\frac{2d'_{\min}}{\alpha\pi} \right)^2 m'_\pi = \frac{g_m^2}{4} \left(\frac{g_m}{\alpha\pi} \right)^2 m'_\pi \quad (13)$$

Subsequent substitution of this result into the α_0 expression in (8) reveals

$$\alpha_0 = \frac{\lambda^2 \hbar^2}{2m_m g\Phi_0} = \frac{k_a^2}{k_b} = 2 \quad (14)$$

Let us proceed by observing Figure 4. It shows the potential (energy) $V'(x)$ of the muon's center of mass, defined by (1), (8) and (14) as a function of its deviation from the spatial center. Each curve in the figure is characterized by a particular parameter value for the (normalized) spacing d' between the poles. There is a clear minimum for $d' = d'_{\min}$, an increase of the curvature for $d' < d'_{\min}$ and a decrease of the curvature for $d' > d'_{\min}$. If the two poles are spaced in the state of minimum potential ($d' = d'_{\min}$), the vibration energy of the muon is in the ground state. As long as the curves show a minimum with a negative value, the configuration shows an amount of stability-preserving binding energy. It will be clear from figure 4 that the binding energy is lost for narrow spacing. In Figure 5 it is illustrated that the energy constant level of first excitation in ground state may correspond with the ground state energy constant of the configuration at a smaller spacing.

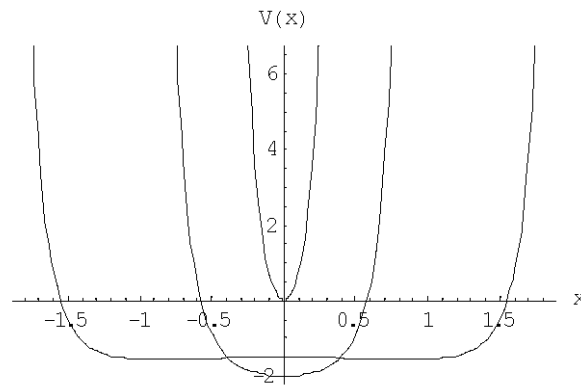


Figure 4. The potential energy of the muon's center of mass as a function of spacing between the poles. The stable ground state of the muon occurs at maximum binding energy ($V(x) = -2$; $d' = 1$). The binding energy is lost for spacing $d' = 0.5$.

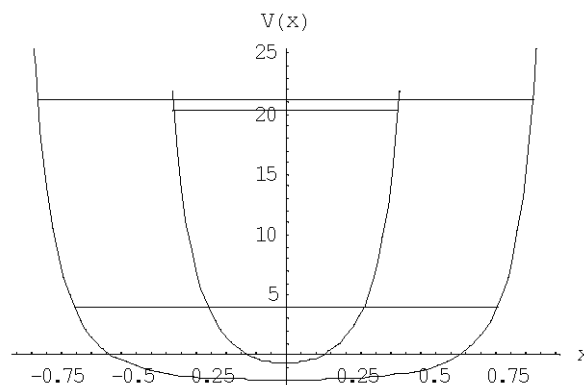


Figure 5. The jump from the muon state to the tauon state is a jump from the first excitation level of the muon state to the ground state of the heavier tauon. It happens at a spacing $d' = 0.54$, where the energy constants (not to be confused with the massive energies) match, under preservation of a slight amount of binding energy.

This is the reason why, under excitation conditions, the configuration may jump from the muon state to the tauon state. It will be clear that the jump to the level of second excitation cannot be made under preservation of negative binding energy. Hence, charged leptons beyond the tauon particle are non-existing.

The normalized wave equation allows to calculate by simple computer code the ground state energy as a function of the spacing d' . The computation of the ground state curve results into a behavior as shown in the upper part of Figure 6. It also allows to calculate the excitation levels. The dashed line in that part shows the level of the first excitation. Note that in accordance with (8), the reference value has been defined as $E_{ref} = g\Phi_0$. It may seem as if the energy level of the first excitation is beyond reasonable physical expectation, because an ideal harmonic oscillator would show at first excitation only three times the ground state value. In that respect the picture is somewhat misleading. One should, however, take into consideration that both these values must be established from the bottom level, determined by the binding energy. In this picture the binding energy level in relative amounts is -2, the ground state energy is 4.02 and the first excitation level is 21.3. This makes the excitation to ground state ratio about 3.85. The difference with the factor 3 of the ideal harmonic oscillator shows the effectiveness of the computer code, in which the anharmonic nature of the oscillator is taken into account. The eigen state quantization feature, though, is similar.

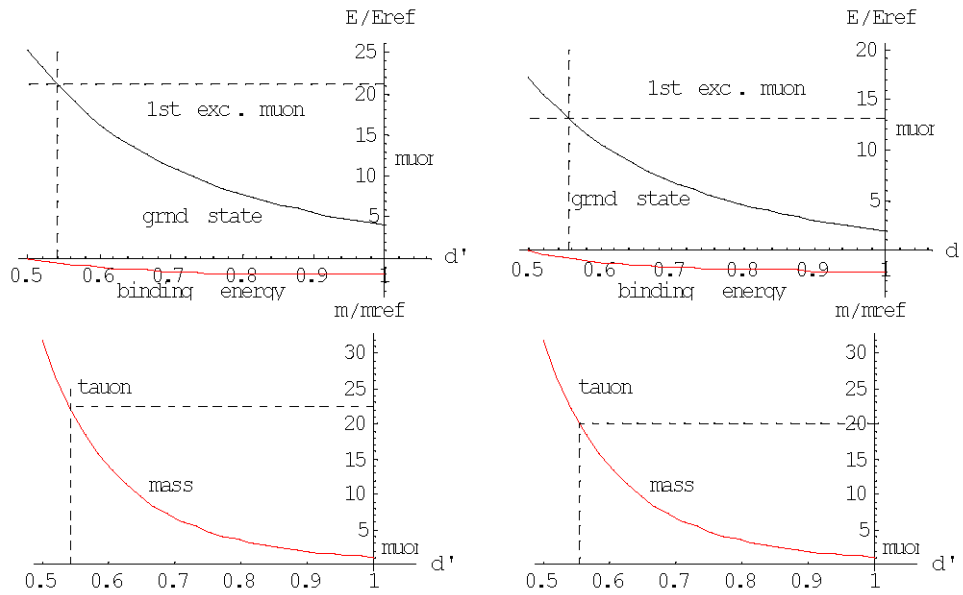


Figure 6. The lower curves show the dependence of the lepton's physical mass on the pole spacing. The upper graphs show that the pole spacing of the tau particle is determined by the equality in vibration energy of the muon's first excitation level and the ground state vibration energy of the heavier tau particle. Note that the binding energy of the tau particle is just slightly negative. The right-hand figures are a first-order correction on the left-hand part ones.

Let us consider relationship (10) in more detail. It requires a careful inspection because of the dependency on d' for k_2 as well as for k_0 . The dependency of k_0 , though, is irrelevant for the curves shown in Figure 6, because the energy E in the upper curve and the mass m in the lower curve both are normalized upon $E_{ref} = g\Phi_0 = |k_a| \hbar\omega$. Hence,

$$m'_m = k_2(d') \left(\frac{g_m}{\alpha\pi} \right)^2 E_{ref} \quad (14)$$

The expression allows to relate the rest mass of the tauon with the one of the muon without elaborating on the mass m_m itself. The lower part of Figure 3 shows the mass curve as derived from $k_2(d')$ as a function of d' , in which $k_2(d'_{min})$ is taken as the reference. The lower curve shows that, at spacing $d' = 0.54$, the relative mass value of the structure amounts to 22.6. Although it compares fair with the mass ratio of the tauon over the muon, which amounts to about 18, the discrepancy needs an explanation. A re-inspection of (10) allows to do so. In the meson case, which relies upon the very same model, the pion decays into a muon under mediation of the weak interaction boson $\hbar\omega_w$. This has given the reason to regard the weak interaction boson as the relativistic value of the pion's rest mass. But where comes its energy from? Eq. (10) tells that it is built up in equal shares of the binding energy $|k_a| g\Phi_0$ and the ground state energy referenced to zero. The actual computation, though, shows an asymmetry. This asymmetry can be accounted for by defining

$$\hbar\omega_w = |k_a| g\Phi_0 \left(1 + \frac{E_{binding}}{E_{ground}} \right) = |k_a| g\Phi_0 (1 + \beta); \beta = \frac{E_{binding}}{E_{ground}}. \quad (15)$$

In retrospect, it might well be that the asymmetry is due to the assumption $\hbar\omega = \hbar\omega_w$ in (10), while there is no particular reason why this should be true. Let us drop this condition and let us redefine (10) as,

$$\hbar\omega = 2\gamma |k_a| g\Phi_0. \quad (16)$$

The modification of (10) into (16) has an impact on the calculation of α_0 , such that now

$$\alpha_0 = 2\gamma^2. \quad (17)$$

The computation is rather sensitive for parameter α_0 . By playing with γ , it appears being possible to restore the symmetry between E_{binding} and E_{ground} . Computation shows that this happens for $\gamma = 0.78$. Doing so, consistency with previous work is maintained, because we have restored

$$\hbar\omega_w = 2 |k_a| g\Phi_0. \quad (18)$$

Note: the true reason for this relationship can be found from the general relativistic view, documented in [13,19], once more highlighted in paragraph 5 of this article.

A correction on (13) because of the introduction of γ results into,

$$m'_m = \frac{k_b}{\gamma |k_a|} \left(\frac{2d'_{\min}}{\alpha\pi} \right)^2 m'_\pi = \frac{g_m^2}{4\gamma} \left(\frac{g_m}{\alpha\pi} \right)^2 m'_\pi \quad (19)$$

Sticking to the condition that the two kernels are spaced apart by the weak interaction boson, α has to be equated as $\alpha = 1$. Hence, from (19), m'_m is related with g_m by

$$m'_m = \frac{k_b}{4\gamma |k_a|} \left(\frac{2d'_{\min}}{\alpha\pi} \right)^2 m'_\pi = \frac{g_m^2}{2.12} \left(\frac{g_m}{\pi} \right)^2 m'_\pi \quad (20)$$

If $g_m = 2$, we would have $m'_m = 107$ MeV, which is rather close to the 105 MeV value reported by PDG.

The condition for equal shares for binding energy and ground state energy gives the result as shown in the right-hand part of Figure 6. The excitation ratio now amounts to 20, which compares somewhat better to the value 18 reported by the P(article)D(ata)G(roup). The condition that the two kernels are kept apart by the half wavelength of the weak interaction boson fixes the parameter λ . Another choice for this fix would imply another value for α . It is instructive to keep this in mind, because in the meson study λ has been fixed by the condition $\lambda\hbar c = m'_H / 2$, in which m'_H is the weak interaction boson. This strong interaction boson does not play a role in the case of leptons. Later in this article it will be shown that the condition $g_m = 2$ is imposed by the consistency of this theory with gravity, shown before in the work on mesons. In Table 1 the results of the analysis have been summarized.

Table 1. the mass formula.

	muon	tauon
\mathcal{G}_m	2	
$ k_0 $	2	
k_2	2	
d'_{\min}	1.00	0.54
γ	0.78	
α	1	
ref	140 MeV	
mass	105 MeV	20 x

The calculated result for the muon corresponds with a kinematic analysis. Figure 7 illustrates how the pion’s rest mass (139.57 MeV/c²) is splitted up into the rest mass of the muon (105.66 MeV/c²) and the kinetic energy of the neutrino (33.91 MeV). Some algebraic evaluation [14] shows that, in the rest frame of the muon, the pion gets a slight amount of kinetic energy of 4.12 MeV/c². Stated otherwise: the pion explodes and breaks up into two pieces, a heavy part flying away at low speed and a light part flying away at high speed, such as illustrated in Figure 7.



Figure 7. The break up of a pion into a (low speed) muon (left) and a (high speed) neutrino (right).
The kinetic energy of the neutrino can be related with its rest mass because of the relativistic relationship

$$E_{\nu} = \frac{m'_{\nu}}{\sqrt{1 - v^2 / c^2}} \tag{21}$$

If we would know the neutrino’s velocity, we would be able to calculate the neutrino mass from its kinetic energy. Unfortunately, the only thing known is an estimate of the neutrino mass from direct measurements with, like reported by PDG [17], doubtful reliability. These direct measurements show an upper limit for the muon neutrino mass of $m_{\nu\mu} < 190 \text{ keV}/c^2$.

Note: In the muon theory as has been developed in this paragraph, the muon no longer is a pointlike elementary particle. In fact, an estimate of its physical size can be estimated from

$$d = \frac{d'_{\min}}{\lambda} = 2d'_{\min} \frac{\hbar c}{m'_H} \approx 1.82(197/125) \times 10^{-18} \text{ [m]}. \tag{22}$$

It is my guess that the difference from the pointlike size is the reason why the magnetic dipole moment of the muon is slightly different from the magnetic dipole moment of the electron. The accurate fit (up to the 27th decimal) between the calculated dipole moment of the electron and its measurement is the greatest triumph of Quantum Electro Dynamics (QED), both for scientists in theoretical physics and, not in the last place, for scientists in experimental physics. The fit in the muon case deviates in the 11th decimal [16].

As long as the difference in the potential functions (2) and (3) is taken into account, the model is generic for the muon and for the pion. The awareness that the spin-out energy of the pion under decay should be consistent with the requirement as expressed by (15) gives reason to give a minor revision of the documented work on mesons. The difference between the muon case as just discussed and the meson case is the difference between the potential functions (2) and (3).

To guarantee that the spin-out energy of the meson under decay equals the energy of the weak interaction boson, condition (18) has to be met. Similarly as in the case of muons, the only way to guarantee an equality in share between the contributions from the binding energy and the ground state energy, is the adoption of the conclusion that $\hbar\omega \neq \hbar\omega_W$, such that $\hbar\omega = \gamma\hbar\omega_W$. The computation shows that it happens for $\gamma = 1.1$. Similarly as in the muon case, the values k_a, k_b and d'_{\min} are not affected by γ . The eigen state issue of the mesons, though, is fundamentally different from the eigen state issue of the leptons. Similarly as discussed so far, pion scales to a kaon if the ground state energy of the kaon equals the energy of the pion in first excitation. But this scaling takes place under invariance of the ratio Φ_0 / λ . This implies that, unlike as in the case of charged leptons, the kernels are subject to changes in Φ_0 , making different quarks, while in the lepton case Φ_0 is invariant. Whereas in the lepton case the mass relationship between muon and tauon is the consequence from a spacing condition, the spacing relationship between the pion and the kaon is the consequence from a mass condition. The eigen state mechanism of the pion is illustrated in Figure 8. It has to be emphasized, though, that the eigen state issue of the mesons is much more complicated than in the case of leptons, because of the influence of the spin-spin interaction between the quarks [11]. It gives rise to a much larger number of eigen states than shown in the figure.

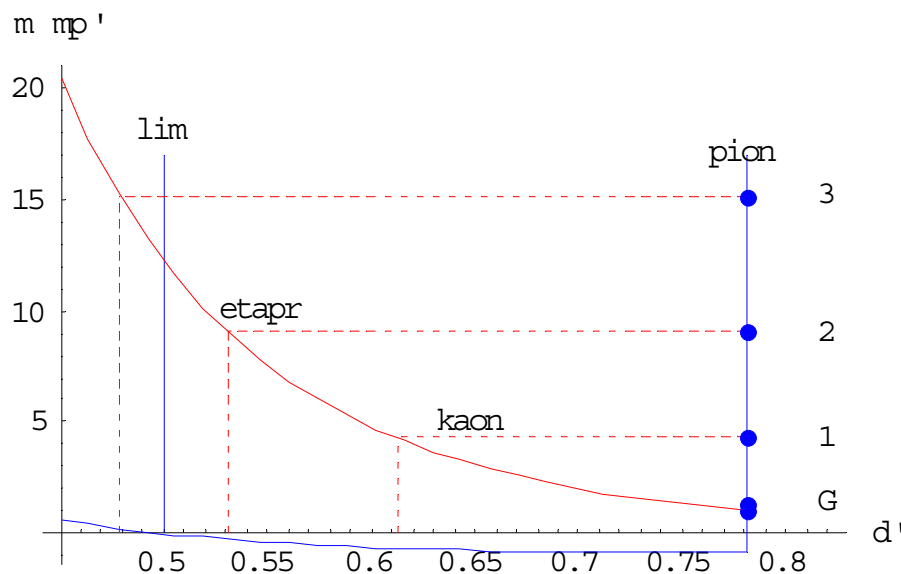


Figure 8. The scaling mechanism in mesons. The kaon and pion are two eigenstates. Unlike as in the case of leptons, the kaon is generated from the pion as soon as the mass curve of the pion under stress (the k_2 curve) crosses the mass of the pion in first level excitation.

Whereas the mass formula is in the pion case eminently suited to calculate the mass spectrum of mesons with the pion mass taken as a reference, the formula falls short to calculate the pion mass itself. This is due to the high impact of the exponential decay of the pion's potential function (2), which is missing in the potential function (3) of the muon. Whereas the pion's potential is affected by the omni-present energetic background field, showing up as the far field part of the quark's potential, of the the muon's potential is not. In other words: the pion's potential field is influenced by strong interaction (the far field of the quark) as well as by weak interaction (the near field of the quark), whereas the muon's potential is only influenced by weak interaction. This gives a substantial

difference in the k_b / k_a ratio in the mass expression (14). Whereas this mass expression allows to calculate the rest mass of the muon, it is not suited to calculate the rest mass of the pion, while (14) nevertheless enables to calculate the mass spectrum of the mesons. More on mesons is beyond the scope of this article. Details can be found in [11].

Similarly as the tauon shows up as an eigen state next to the muon, the kaon shows up as an eigen state of the pion. The results shown in Table 1 and Table 2 cannot be obtained from the Standard Model. In the Standard Model these particles are considered as independent elementary ones.

Table 2. kaon as eigen state of the pion.

	pion	kaon
g_m	2	
$ k_0 $	0.84	
k_2	3.97	
d'_{\min}	0.78	0.63
γ	1.1	
mass	140 MeV	505 MeV

4. The eigenstates of neutrinos

Considering that the tauon is the result of the excitation of the muon modelled as an anharmonic oscillator built up by two kernels, it is worthwhile to consider the possibility that the muon neutrino is subject to excitation as well. We have concluded before that the muon neutrino is built up by two kernels, each having a potential function given by (5). Similarly as in the case of the pion and the muon, the two kernels may compose an anharmonic oscillator that can be described by the wave equation in the general format (1). Similarly as in the muon case, this equation can be written as,

$$-\frac{\hbar^2}{2m_m} \frac{d^2\psi}{dx^2} + g\Phi_0 \{k_0 + k_2\lambda^2 x^2 + \dots\} \psi = E\psi \quad (23)$$

The only difference so far is that the expansion of the potential function (4) will result in other values for the dimensionless constants k_0 and k_2 . Rewriting (23) in a normalized format gives,

$$-\frac{(\hbar c\lambda)^2}{2m'_m} \frac{1}{g\Phi_0} \frac{d^2\psi}{dx'^2} + (k_0 + k_2 x'^2) \psi = E'\psi \quad (24)$$

Invoking the relationships (8) and $\lambda\hbar c = m'_H / 2$, we have, as before,

$$-\alpha_0 \frac{d^2\psi}{dx'^2} + (k_0 + k_2 x'^2) \psi = E\psi; \quad \alpha_0 = \frac{k_0^2}{k_2}. \quad (24)$$

The wave equation can be solved numerically with the same computation model as for mesons and the charged leptons. This model with its computer code has been documented in previous work [13].

Figure 9 is the equivalent of Figure 6 shown for the charged lepton. It illustrates that the ground state energy of a neutrino with a smaller spacing between the poles equals the energy levels of the first excitation and the second excitation of the neutrino in a minimum state of energy. Whereas the level of the first excitation is at $d' = 0.72$ and well before the cross-over from negative binding energy to positive binding energy at $d' = 0.5$, the level of the second excitation at $d' = 0.48$ is just beyond.

The implication of this effect is that neutrinos may show up, apart from association with an electron, in association with a muon and with a tauon. Hence, in three different eigenstate modes. Whereas the potential function of the charged lepton, similarly as the meson, shows a polynomial behavior, the potential function of the neutrino almost has a “brick wall” characteristic. This shapes the resulting wave functions to almost harmonic standing waves, like shown in Figure 10.

It has to be emphasized that this result is obtained by analysis and a numerical solution of the wave equation. How to interpret this analytical result in physical terms? It seems as if the pion acts under decay as an emitter for energetic standing wave packets, similarly as antennas do in electromagnetic communication. These packets seem to possess a similar kind of coherency as lasers have in radiation of their photons. Because the potential field of the neutrino poles have no other physical parameters apart from Φ_0 as fixed by the weak interaction boson, and λ as fixed by the Higgs field, it is fair to conclude that neutrinos will exclusively show up in three modalities, because loss of binding energy will prevent other ones.

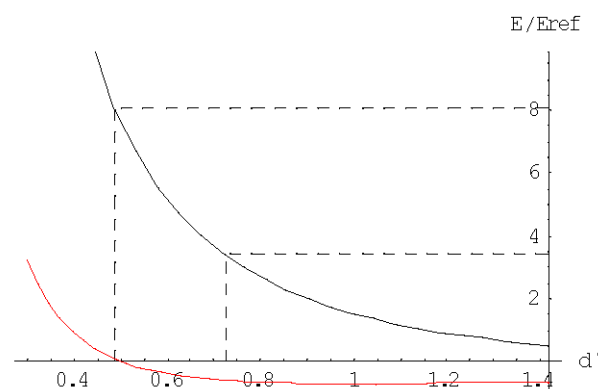


Figure 9. The pole spacing of the neutrino is determined by the equity in vibration energy of the neutrino's first excitation level, respectively the second excitation energy from ideal ground state with its the ground state vibration energy at reduced pole spacing. Note that the binding energy (shown in a different scale as the lower curve) just allows three modes of eigenstates.

This eigenstate model for neutrinos shares quite some characteristics with the present adopted model of neutrinos as in 1957 heuristically proposed Pontecorvo. Quite some questions, though, still remain. It is for instance not clear why neutrinos should interact on their own, making different flavour states, without being influenced by their charged partners. One of the issues to be taken into consideration, though, is the statistical character of particle physics. Taking as an example the breakdown of the weak interaction of muons into electrons and neutrinos, one might expect that the produced neutrinos are not only subject to a statistical distribution of their kinetic energy, but that their distribution over the three possible eigenstates is subject to a statistical behaviour as well. While in this view any of the neutrinos will physically be in different defined eigenstates out of three, one may adopt a mathematical description captured in a single matrix, common for all neutrinos in the decay of a specific neutrino flavour, such as done in the now commonly accepted theory of neutrinos [17]. In fact, however, this is no more than a projection of the statistical behaviour of a multi-particle system on a single virtual particle.

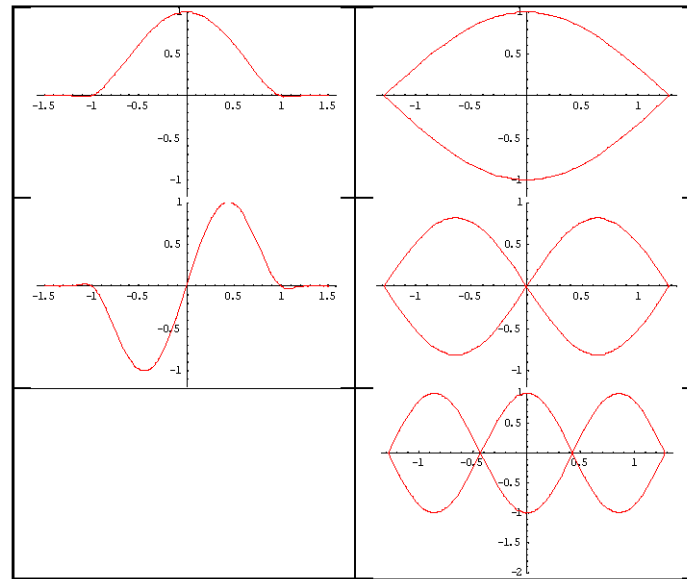


Figure 10. Comparison of the wave function formats between the charged lepton and the neutrinos. Whereas the wave function of the charged lepton is close to the one of an harmonic quantum mechanical oscillator, the wave function of a neutrino is a non-electromagnetic standing wave packet. .

Taking this in mind, let us consider the decay process of the pion, once more. There is no reason why the picture shown in Figure 4 would not be reversible, like shown by all quantum mechanical processes. It means that the neutrinos may change flavour, but not on their own. An electron neutrino with high energy may recoil with an electron to produce a weak interaction boson that subsequently disintegrates into a muon and an muon antineutrino. This observation, however, does not solve the solar neutrino problem mentioned before. And that problem is usually tackled by assuming that neutrinos may change for whatever reason flavour on their own under influence their eigenstate behaviour. But why should they do so?

It is in the author's view quite probable that the answer to this question has to do with an interpretation of experimental evidence. A neutrino can only be detected if it produces its charged lepton partner. Such a production can be understood from the reverse process just described. As compared to the forward mode, the reverse mode is not impossible, but unlikely. This implies that the instrumentation for neutrino detection is based on the counting of rare events over considerable time. A beautiful example is the method used in the Super-Kamiokande experiment, which is based upon the detection of Cherenkov radiation [18]. This radiation is produced by electrons and muons in water that may propagate in water at a faster speed than the light in water does. The radiation profile from electrons and muons produced by the matter interaction between neutrinos and (heavy) water molecules is slightly different. This difference enables to distinguish between electron-neutrinos and muon-neutrinos. The experimental evidence that the sum of the electron-neutrino counts and the muon-neutrino counts is equal to the corresponding calculated amount of solar electron-neutrinos is presently taken as proof that neutrinos change flavours on their own, thereby solving the mystery of the missing solar neutrinos.

Another explanation could be that the production of neutrinos in the reverse decay process is somewhat different from the production of neutrinos in the forward process. Whereas in the forward process the neutrinos are emitted in a certain flavour dependent distribution over the three eigenstates, the production of charged leptons in the reverse process might be selective on eigenstates. This would mean, that flavour changes between neutrinos on their own are non existing. It also implies that the oscillation phenomenon as observed in instrumentation based upon the detection of beat frequencies of propagating neutrino wave functions is not due to physical interaction between the neutrino flavours, but that this phenomenon is a result from non interacting physical eigenstates

propagating at slightly different speeds. This means that the observed phenomenon is incorrectly interpreted as an oscillation between flavour states.

5. Gravity as a proof for the consistency of the theory

One of the issues still to be discussed is the consistency with previous work. The new element in this article is the unification of mesons, charged leptons and neutrinos into the same framework. In the canonical theory these particles are not connected, but are regarded as independent from each other with quite a number of independent elementary particles. Apart from the argumentation presented in this article by theoretical analysis, such a unification, in which we have only a single quark, a single electron and a single neutrino as elementary particles (next to their antiparticles), makes sense from a logic point of view. Even more convincing would be if further proof could be given for the bridge between gravity and the quantum mechanics of particle physics, such as earlier shown in previous work on mesons [13,19]. To the author's regret, so far, this bridge is ignored in present literature, possibly because its argumentation has been regarded as highly speculative, in spite of a verifiable numerical proof.

For a proper understanding of the consistency issue, it is instructive to summarize the considerations that have led to the meson model shown in Figure 1, which is generalized to a model for leptons as well. This configuration is built from a model for the spatial potential of the archetype quark. The spatial potential has the format of the internucleon potential that in 1928 was dubbed by George Gamov [20] as the liquid drop model. This model inspired me to show that the very same format allows to give an interpretation for the origin of the Higgs potential different from the Spontaneous Symmetry Breaking (SSB) adopted in the Standard Model [21]. The way to do so is a straightforward application of the Euler-Lagrange equation on the functional format of the Higgs potential as heuristically formulated in the Standard Model. In principle, this approach should produce a classical field solution in spatial format. This road appears not being free from problems. First of all, the high non-linearity of the Higgs potential prevents an analytical solution. However, a simple computational procedure allows to obtain a numerical solution that nicely fits to a liquid drop shape. The result is a classical spatial function, so far still consistent with the Higgs potential, hence with the SSB hypothesis. A classical physical interpretation, though, gives some problems. How to explain the exponential decay and how to explain the near field characteristics with its decay by r^{-2} ? Over the years, these problems have been gradually removed. The renormalization problem associated with the r^{-2} decay of the near field part, has been avoided by hypothesizing that the energetic pointlike source of the potential, i.e. the quark, shows a scalar dipole field adjacent to the far field monopole field. Such a field shows a x^{-2} decay along the axis of the dipole. The exponential decay problem disappears under the hypothesis that the quark's field is shielded akin to the shielding of the scalar field of a charged electric charge in an ionic plasma, known as the Debye effect. This plasma is an energetic background energy that can be modeled as uniformly distributed space charge consisting of tiny elementary dipoles that are subject to polarization. This vacuum polarization is present in cosmology as well, although the cosmological shielding is enhancing instead of suppressing. Einstein's Lambda in his Field Equation for general relativity has been invoked for its explanation. It means that the hypothesis of the existence of an energetic background field makes sense from a logic point of view.

The toughest problem is finding an explanation for the existence of a scalar dipole field next to the monopole field of a quark. Commonly, the quark is considered as a Dirac particle. A Dirac particle of the canonical type, though, does not show a scalar dipole field. Dirac's theory shows two dipole moments. For an electron, the first one has been identified as an anomalous magnetic dipole moment. It has a real value. The value of the second one, though, is imaginary. Hence, an anomalous electric dipole moment of an electron is non-existing. This awareness has triggered me to study Dirac's theory closely. Playing with Dirac's gamma matrices revealed the possible existence of a real valued second dipole moment. It is a recent result, published in 2020 [8] and updated somewhat later [9]. It met some opposition because it violates the Lorentz invariance. This problem disappears under

confinement of the quark in hadron compositions [22]. An example of such a hadron composition is the meson structure shown in Figure 1.

Like shown in the preceding text, the analytical description of this configuration shows an unknown gyrometric parameter g_m , which balances the monopole far field of the quark against the dipole near field. It may seem as if its value is heuristically chosen. In fact, however, its value can be traced back to, maybe, the most fundamental issue of the theory developed in this and previous work. The awareness that two quarks may compose a stable structure under the balance of the scalar part of their monopole fields and the fields from their scalar dipole moments allows to describe the energy in their geometric center in terms of Einstein's stress-energy tensor. This allows to derive a geodesic equation for its center of equivalent baryonic mass. Because this equation is an equation of motion, it can be transformed into a wave equation. This can be done under application of the basic quantum mechanical theorem in which momenta are transformed into operators on a wave function. This approach has been described in earlier work [13,19] and has resulted into the derivation of a numerically verifiable expression for the gravitation constant in quantum mechanical parameters, thereby showing a bridge between gravity and quantum mechanics. Unfortunately, so far this work has not gained the momentum that it deserves. The work can be summarized by the obtained expression for the gravitational constant. It reads as,

$$G = 2 \frac{g^2 c^4}{m_W'^2} \left(\frac{k_a}{k_b} \right)^2 \left(\frac{\alpha \pi}{2d'_{\min}} \right)^4 \Delta_\pi \quad (25)$$

in which Δ_π is a term for relativistic correction, which is determined by,

$$\sqrt{\Delta_\pi} = \frac{(\hbar c) \ln(2)}{m'_{0\pi} c t_{0\pi}} \left(\frac{\alpha \pi}{2d'_{\min}} \right) \quad (26)$$

These expressions contain, apart from the lifetime $t_{0\pi}$ of the pion, a dimensionless factor α . This factor relates the quark spacing $2d'_{\min}$ with the half wave length $L_W / 2$ of the weak interaction boson. Hence,

$$2d_{\min} = \alpha \frac{L_W}{2} = \frac{\alpha}{2} c \frac{2\pi}{\omega_W} = \alpha \pi \frac{\hbar c}{m'_W} \rightarrow \frac{2d'_{\min}}{\alpha \pi} = \frac{m'_H / 2}{m'_W} \quad (27)$$

This relationship, not recognized by me before, removes from the result α as an unknown parameter. Hence,

$$G = 2 \frac{g^2 c^4}{m_W'^2} \left(\frac{k_a}{k_b} \right)^2 \left(\frac{\alpha \pi}{2d'_{\min}} \right)^4 \Delta_\pi \rightarrow G = 2 \frac{g^2 c^4}{m_W'^2} \left(\frac{k_a}{k_b} \right)^2 \left(\frac{m'_W}{m'_H / 2} \right)^4 \Delta_\pi \quad (28)$$

$$\sqrt{\Delta_\pi} = \frac{(\hbar c) \ln(2)}{m'_{0\pi} c t_{0\pi}} \left(\frac{\alpha \pi}{2d'_{\min}} \right) \rightarrow \frac{(\hbar c) \ln(2)}{m'_{0\pi} c t_{0\pi}} \left(\frac{m'_W}{m'_H / 2} \right)$$

The only way to influence this result is the choice for the gyrometric ratio g_m in the potential function of the quark, as defined by (2). In Table 3 the result is summarized,

Table 3.

Physics	This theory	Calculated
$\hbar c = 197 \text{ MeV fm}$	$g_m = 2$	$G = 6.89 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
$m'_H = 125 \text{ GeV}$	$k_a = -0.84$	
$m'_W = 80.4 \text{ GeV}$	$k_b = 3.97$	Measured
$g^2 = 1/137$		$G = 6.67 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
$m'_{0\pi} = 139.6 \text{ MeV}$		
$t_{0\pi} = 2.60 \times 10^{-8} \text{ s}$		

Taking the gravitational constant as the true reference , the gyrometric factor g_m , originally derived from the heuristically defined Higgs potential, cannot be different from $g_m = 2$. In the Appendix A yet another relationship is shown for further support of this conclusion.

6. Conclusion

Whereas the Standard Model of particle physics shows a weakness in its ability to include neutrinos in the framework under explanation of present experimental evidence of their features, the structural model for particle physics [7,22], developed and documented over the years by the author of this article, shows a strength. It provides a theoretical basis for the oscillatory behaviour between the neutrino flavour states as an alternative for the heuristic PMNS matrix (Pontecorvo, Maki, Nakagawa, Sakata) model of neutrinos [6]. As discussed in paragraph 4, in a statistical interpretation it can even been seen as a confirmation.

Moreover, the work shows that neutrinos, charged leptons and mesons can be unified in a single framework, in which any of the three families is consisting of particles that are related to each other as eigenstates. Apart from the basic constituents quark, electron and electron neutrino, the eigenstates are not truly pointlike. The amount of eigenstates is limited by an upper bound due to the loss of binding energy in their substructure: a reason why leptons heavier than a tauon and the tauon neutrino cannot exist.

The work presented in this article is consistent with previous work in which particle physics theory has been connected with gravity. The theory is built upon the awareness that the quark is a Dirac particle with different gamma matrices than those of the electron. As a consequence it has two real anomalous dipole moments, instead of a single real one next to an imaginary one as shown by an electron. Moreover, the quark has Maxwellian properties akin to those as shown in Comay’s RCMT concept. The second awareness is that the vacuum is not empty, but filled with an ubiquitous background energy that becomes manifest as the Higgs boson. It is one of the two bosons responsible for the strong interaction and the weak interaction in particle physics theory. The other one is the weak interaction boson that shows up in free state as well as in virtual state. It governs the eigenstate mechanism of mesons and leptons by keeping the constituents of their structure apart in a spacing determined by half of its de Broglie wavelength.

Appendix A. The Gyrometric Factor

One of the intriguing issues that comes forward from the view as exposed in this article related with previous work is the wonder that one may have on the appearance of a gyrometric that shows

up as a constant of just 2. Although the gravity proof is quite decisive on it, it is instructive to consider it from a different perspective. This appendix is meant to do so.

In the theory developed before for mesons, now in this article applied to leptons as well, the quark is an energetic source with Maxwellian properties. Moreover, it is a Dirac particle that gains a constituent mass by interaction with fields from other sources. Although Maxwellian, the particle may have two real dipole moments, instead of a single real one next to an imaginary one such as shown by Dirac particles of the electron-type [8,22]. In this article it has been shown that the archetype meson, the pion, flips the spin of one of its constituents thereby producing a structure with two kernels, while preserving a configuration in which the repulsive force between the kernels is balanced with the attractive force of the quark's real second dipole moment. The behaviour of this balance is governed by weak interaction in the sense that the spacing between the kernels is kept constant with a value determined by the half wave length of the weak interaction boson in virtual state. In the Maxwellian view, as proposed by Comay in his R(egualer)C(harge)M(onopole) T(heory) concept [10], the quark can be viewed as a magnetic monopole, charged as q_m . In the view as exposed in this article and before in related work, the quark is a Dirac particle of a particular type, in which the particle's dipole moments both are real valued. In this view, the each quark in the pion as well as each kernel in the muon can be characterized by some potential function. Denoting the constituent mass as m , the magnetic monopole charge as q_m and the associated real valued magnetic dipole moment as μ_m , the potential function along the dipole axis is,

$$\Phi(x) = \frac{q_m}{x} + \frac{\mu_m}{x^2}; \mu_m = \frac{q_m}{2m} \frac{\hbar}{c}. \quad (\text{A1})$$

Hence,

$$\Phi(x) = \frac{\lambda q_m}{\lambda x} + \frac{\lambda^2 \mu_m}{\lambda^2 x^2} = \lambda q_m \left(\frac{1}{\lambda x} + \frac{\lambda}{\lambda^2 x^2} \frac{\hbar c}{2mc^2} \right) = \lambda q_m \left(\frac{1}{\lambda x} + \frac{\lambda \hbar c}{2m' \lambda^2 x^2} \right) \quad (\text{A2})$$

$$\Phi(x) = \lambda q_m \frac{2m'}{\lambda \hbar c} \left(\frac{1}{(\lambda x)^2} + \frac{2m'}{\lambda \hbar c} \frac{1}{\lambda x} \right) = \Phi_0 \left(\frac{1}{(\lambda x)^2} + g_m \frac{1}{\lambda x} \right); g_m = \frac{2m'}{\lambda \hbar c}. \quad (\text{A3})$$

Whereas in the pion case the center-of-mass frame is different from the lab frame, both frames coincide in the muon case. That makes the muon more comprehensible than the pion. From the analysis of the muon model, we have from (11) and (12),

$$\frac{m'_\mu}{\lambda \hbar c} = \frac{k_b}{|k_a|} \frac{\lambda \hbar c}{\hbar \omega}; \quad \lambda \hbar c = \frac{2d'_{\min}}{\pi} \hbar \omega_w \rightarrow \frac{\lambda \hbar c}{\hbar \omega_w} = \frac{2d'_{\min}}{\pi} \quad (\text{A4})$$

$$\frac{m'_\mu}{\lambda \hbar c} = \frac{k_b}{|k_a|} \frac{2d'_{\min}}{\pi} \left(\frac{\hbar \omega}{\hbar \omega_w} \right) = \frac{k_b}{|k_a|} \gamma \frac{2d'_{\min}}{\pi} \quad (\text{A5})$$

In the muon frame we have $m_\mu = 2m$. Hence, from (A3),

$$g_m = 2 \frac{k_b}{|k_a|} \gamma \frac{4d'_{\min}}{\pi} \quad (\text{A6})$$

Because for the muon, $|k_a| = k_b$, $d'_{\min} = 1$ and $\gamma = 0.78$, the result is just $\gamma = 2$.

It supports the hypothesis that the quark is an RCMT monopole carried by a Dirac particle with two real dipole moments. This hypothesis is not essential for the view as explained in this article. It just gives an intriguing physical interpretation.

Data Availability Statement: The numerical data quoted in this article have been generated from rather simple computer programs written in Wolfram's Mathematica [13]. The computer code can be made available on request.

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