Article

The Conservation Laws in Quantum Mechanics

Richard Oldani *

Posted Date: 3 July 2023

doi: 10.20944/preprints202305.0952.v2

Keywords: Non-relativistic quantum mechanics; relativistic quantum mechanics; reality; reductionism; Hamilton’s principle; energy; momentum

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 3 July 2023 doi:10.20944/preprints202305.0952.v2

The conservation laws in quantum mechanics

Richard Oldani

Illinois Institute of Technology: https://orcid.org/0000-0001-6884-5614 email: oldani@juno.com

Abstract: Using arguments introduced by Einstein we show that the reductionist philosophy of quantum mechanics reverses the natural order. In quantum mechanics emission is followed by observation and interpreted classically with the collapse of the wave function, whereas in nature classical absorptions of energy precede emission. Also momentum transfers leading to emissions are asymmetric with respect to spatial orientation and time, but are disregarded in quantum mechanics because molecular impulses cannot be observed. Thus despite irrefutable physical evidence that microscopic processes evolve asymmetrically the wave function is assumed to apply symmetrically in time. The deficiencies are corrected by introducing Hamilton's principle and deriving relativistic equations of motion. The calculus of variations is then applied to the wave function to show that it is an incomplete equation of motion because it yields twice the allowable action minimum thereby allowing the concept of reality to be redefined.

Keywords: Non-relativistic quantum mechanics; relativistic quantum mechanics; reality; reductionism; Hamilton's principle; energy; momentum

1. Introduction

It is often claimed, whether overtly or implied, that the most accurate theory is the best theory. A clock that was accurate to within one second over the age of the universe was later improved upon by a clock accurate to within 100 milliseconds. The most recent clock experiment can differentiate between gravitational potentials of one millimeter [1]. Clock accuracy is continuously being improved upon because it is believed that more accurate time measurement "offers new opportunities for tests of fundamental physics". In another area of research universities are competing to improve upon measurements of the g-2 factor of an electron [2]. Once again the goal is to find a discrepancy with the standard model that will lead to "new physics". Improved instrumentation, more accurate measurements, and better predictions are the procedures that many say will lead to an improved understanding of Nature. We will show in these pages that improved accuracy does not equate with improved understanding, and that it is precisely what cannot be observed that is the key to understanding natural phenomena.

The importance of the observer, and by inference the observation, in scientific inquiry is a sticking point that has a long history in theoretical physics. On one side is Bohr and the majority of physicists. "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." On the other side of the dispute, Einstein believed that, "On principle, it is quite wrong to try founding a theory on observable magnitudes alone." Our purpose is to determine which of these two approaches is the more accurate, the empirical or the intuitive.

2. Non-relativistic quantum mechanics

2.1. Conservation of momentum

Einstein's first attempt at a theory of quantum mechanics was a derivation of Planck's law by statistical methods "in an amazingly simple and general manner" [3]. There he describes the dynamic equilibrium that exists between the thermal energy absorbed by molecules and its subsequent quantum mechanical emission as black body radiation. Heat energy that is absorbed according to classically defined Maxwell Boltzmann statistics is transformed at the molecular level and emitted according to the Planck radiation law. The *absorption* of energy by a molecule leads to momentum +E/c in the direction of propagation, while the *emission* of a photon causes a recoil momentum -E/c that is directed in the opposite direction of propagation. The sudden reversals and random nature of the impulses cause molecular trajectories to be discontinuous, as observed in Brownian motion.

As is typical of Einstein's work he begins his derivation at a fundamental level taking into consideration the conservation laws. "If a radiation beam with a well-defined direction does work on a Planck resonator [quantum oscillator], the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator." He demonstrates here the close association that exists between momentum and energy. Molecular impulses transfer energy to the oscillator which then radiates the energy when an electron decays. If there are no impulses; for example, at absolute zero; the molecule will not radiate. Thus energy absorption is distinguished from energy emission. He continues, "We now turn to the investigation of the motion which our molecules execute under the influence of radiation. In doing this we use a method which is well known from the theory of Brownian movement . . . . . Let a molecule of given kind be in uniform motion with speed **v** along the X-axis of the coordinate system K. We inquire about the momentum transferred on the average from the radiation to the molecule per unit time. To calculate this we must consider the radiation from a coordinate system K′ that is at rest with respect to the given molecule. For we have formulated our hypotheses about emission and absorption only for molecules at rest." The hypotheses he speaks of are, of course, the A and B coefficients. The A coefficient is due to spontaneous emission which occurs randomly so it describes momentum transfers to the molecule that are asymmetric with respect to time. The B coefficients, which are due to stimulated absorption and emission, describe transfers of momentum that occur in random directions. Therefore they are asymmetric with respect to spatial orientation. Despite the presence of well-known physical asymmetries that are known to exist at the microscopic level, the Schrödinger equation is thought to provide "proof" of time reversal symmetry, thereby excluding the need to take momentum into consideration for the equations of motion. Thus greater importance is placed on mathematical expediency than on an accurate description of black body radiation.

In order for the equations of motion of atoms irradiating from a black body to be complete they must include energy exchanges in both K and K', for an atom in complete isolation does not radiate. The emission and absorption of energy is to be described quantum mechanically in the coordinate system K' which is "at rest with respect to the given molecule", while the kinetic energy of molecules will be described in the coordinates of K. To determine the total energy of a gas we sum a classical component due to kinetic energy, as determined by temperature; and a quantum mechanical component due to the energy of excited states. The inclusion of both classical and quantum mechanical energy forms in the same model of radiation is a feature that distinguishes Einstein's methods from all others. It contrasts sharply with the Bohr-Heisenberg method, which derives two independent expressions, one classical and one quantum mechanical, and then links them by using the correspondence principle.

The Schrödinger equation introduces conceptual difficulties by not distinguishing between absorption and emission processes. They are physically independent phenomena, but the wave function combines them into a single continuous process that is symmetrical in time. Distinction between the quantum and classical worlds in K' and K is resolved mathematically by means of wave function collapse. The quantum classical divide was never a problem for Einstein, for he accepted it as a fundamental property of matter. There are classical laws governing energy exchange in K and quantum laws governing energy exchange in K'; two points of view of a single reality, the molecule. His studies were disregarded because they are statistical in nature and cannot be confirmed empirically.

2.2. Conservation of energy

Matrix mechanics evolved from long-standing attempts to describe dispersion phenomena, the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. When complex sets of mathematical rules were discovered that describe the relationship between the observed frequencies and intensities of spectral lines it allowed Heisenberg to formulate a theory of quantum mechanics that reconciles the continuity of radiation fields with the discrete energy states of an atom by expressing electron transitions in the form of a matrix [4].

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{matrix} i\hbar \text{ for } n=m \\ 0 \text{ for } n\neq m \end{matrix} \qquad 1)$$

Although p in the above equation refers to momentum it is not the momentum of a molecule in K used by Einstein, rather it is the momentum due to photon emission in K'. To compare 1) more closely with Einstein's statistically defined measurements of energy fluctuation we refer to the energy matrix [5]. The diagonal elements of the matrix $m=n$ refer to all possible energy states of a quantum system. Because the theory only concerns quantum mechanical phenomena it represents a complete break from classical theory. Its weakness lies in the fact that it does not describe how atoms absorb energy. In other words, equation 1) is formulated exclusively in coordinates relative to K'.

Off-diagonal elements of the energy matrix $m\neq n$ refer to continuous resonances that occur between radiation and an atom's valence electrons without the emission of discrete energy quanta. Elements of the array E_{mn} that are above the diagonal have a plus sign because they represent energy absorption and elements of the array that are below the diagonal E_{nm} have a minus sign representing energy emission. Thus a single matrix describes two physical events and noncommutation is the result of differences in their physical order. If absorption precedes emission the angular momentum of an atomic oscillator is ħ, and if emission precedes absorption the angular momentum is equal to zero [6]. The off-diagonal elements are assigned a value of zero because the energy of an absorption offsets the energy of an emission except for a difference in phase. However the impulses between molecules can exchange energy without changes of state. Spectral lines also broaden due to exchanges of momentum caused by thermal energy that cannot be avoided. Although the collisions cannot be observed individually they are evident statistically in measurements of gas temperature. Heisenberg did not take these microscopic interactions into account because he believed that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable" [7]. This caused him to focus exclusively upon the intensity and frequency of the spectral lines, which are the emission properties of a radiating atom or molecule.

Describing energy by means of emission processes alone is a violation of energy conservation. The meaning of energy is not embodied in emission alone, nor in absorption alone for due to conservation laws energy can neither be created nor destroyed. It is correctly described as a transformation from one form to another. Before it can be emitted it must be absorbed. Of the different formulations of quantum mechanics only Einstein's treats energy as a transformation [3], "In *theoretical* investigations these small effects [molecular impulses] are definitely as important as the more prominently appearing *energy* transfers by radiation [photon emission], because energy and momenta are always intimately linked together." What he neglected to say was that *energy and momentum are linked together by the conservation laws.* Thermal energy is absorbed by oscillators during black body radiation by means of momentum exchange and subsequently emitted quantum mechanically as radiation. If the classically defined absorption energy in K is discounted because it is unobservable or too small to measure one-half of all radiation processes are arbitrarily eliminated.

2.3. Feynman's theory

In the path integral approach to quantum mechanics we can see violations of the conservation laws even more clearly. In the following passage Feynman considers absorption and emission by first using the field approach of classical theory and then of quantum field theory, but he then rejects both in favor of a particle model of *only* emission [8].“If one solves the problem of an atom being perturbed by a potential varying sinusoidally with time, which would be the situation if matter were quantum mechanical and light classical, one finds indeed that it will in all probability eject an electron whose energy shows an increase of hν, where ν is the frequency of variation of the potential. When, however, we come to spontaneous emission and the mechanism of the production of light, we come much nearer to the real reason for the apparent necessity of photons. The fact that an atom emits spontaneously at all is impossible to explain by the simple picture given above. In empty space an atom emits light and yet there is no potential to perturb the systems and so for it to make a transition. The explanation of modern quantum mechanical electrodynamics is that the atom is perturbed by the zero-point fluctuations of the quantized radiation field. It is here that the theory of action at a distance gives us a different viewpoint. It says that an atom alone in empty space would, in fact, *not* radiate. Radiation is a consequence of the interaction with other atoms (namely, those in the matter which absorbs the radiation). We are then led to the possibility that the spontaneous radiation of an atom in quantum mechanics also, may not be spontaneous at all, but induced by the interaction with other atoms, and that all of the apparent quantum properties of light and the existence of photons may be nothing more than the result of matter interacting with matter directly and according to quantum mechanical laws.”

Feynman’s discussion begins in a way that is consistent with the conservation laws; that is, atoms absorb energy before radiating and atoms in empty space do not radiate at all. The realization that energy conservation governs the whole radiation process, and not emission alone, did not stimulate thoughts about how to incorporate absorption into a *complete* theory of radiation, rather it caused him to disregard fields and formulate emission as a particle theory [9]. The paths in his path integral formulation contribute to probability amplitudes which are added in order to obtain a probability amplitude *for the event*. The square of the final amplitude gives the probability of finding a particle. Although we treat the paths as if they are *possible* trajectories, they are not like anything we have ever experienced for they are unrestricted by the conservation laws. They may form loops, extend to infinity, go backwards in time, or exceed the speed of light. In the next section we shall introduce equations of motion that are consistent with respect to relativity theory.

3. Relativistic quantum mechanics

3.1. The action principle

To determine particle paths that agree with the conservation laws we apply Hamilton’s principle. It assumes that a particle, in this case the electron, begins at a particular position P_1 and point in time t_1 and travels at a certain velocity to arrive at a position P_2 after a given amount of time [10]. The tentative paths are not restricted in any way. Not only can we choose any path between the points but the particle’s motion in time may change freely as well. All that we require is that motions begin at the observed time t_1 and end at the observed time t_2. The time integral of the energy extended over the entire motion between P_1 and P_2 is called “the action”. The quantity that we use as a measure of the action S is the time integral of the difference between the kinetic and potential energies T-V.

$$S = \int_{P_1}^{P_2} \int_{t_1}^{t_2} (T - V)\, dt \qquad 2)$$

Each of the paths between the same two end-points P_1 and P_2 will have a different action. The principle of least action asserts that the particle path actually taken is the one for which the action is a minimum.

3.2. The absorption of energy by an atomic system

Even the simplest atomic system, the hydrogen atom, includes an infinite number of paths that connect the initial and final points. Hamilton's principle expresses the meaning of the entire set of differential equations describing the paths. It calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. The principle of least action is independent of any particular system of coordinates and is invariant for all coordinate transformations.

Hamilton's principle is valid for systems upon which work is performed; that is, non-conservative systems. Radiating atoms that absorb and emit energy are non-conservative systems. Work is performed on them causing energy to be taken from the environment and transferred to the system. Neither Heisenberg's nor Feynman's methods include energy absorption so they treat the atom as a conservative system. Another reason that Hamilton's principle is preferred for describing atomic systems can be found in experiments with atomic clocks. Measurements of single clock periods t_1 to t_2 have been performed with extremely high precision [1]. Thus the initial and final points of an electron's path can be confirmed experimentally with nearly perfect accuracy relative to time.

To describe the energy absorption of an atom we utilize the symmetry of an atomic system to apply Hamilton's principle 2) in generalized coordinates. The transition of an electron from the ground state to an excited state is characterized in generalized coordinates by a 6-dimensional space, three to describe its position on the electron shells R_1 and R_2, and three to describe its trajectory. We can think of the Schrödinger equation as a collection of all possible paths of an electron between electron shells, each of which describes a path with a different action. The actual path of the electron, the one chosen by nature, is the one whose action is minimum.

Energy is absorbed by the atom from a radiation field due to the superposition of transverse electromagnetic fields. The path taken by the electron initiates its motion at a fixed point on the equipotential surface R_1 of the ground state at time t_1, proceeds along a path r, and upon arriving at the excited state R_2 adopts the circular path $2\pi r$ of an orbital thereby assuming orbital angular momentum. The action minimum S[r(t)] for a path between stationary points on R_1 and R_2 yields not zero as in classical dynamics, but the reduced Planck's constant ħ due to angular momentum.

$$S[r(t)] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} (T - V)\,dt = \hbar \qquad 3)$$

The action, S[r(t)], is a functional that describes the absorption process in four dimensions. It has as its argument an infinite number of functions, the possible electron trajectories r(t). The path of the electron is the one actually followed, with action minimum not equal to zero as in classical mechanics, but ħ.

3.3. Energy emission

At the relativistic or high end of the energy spectrum in quantum field theory, particles are treated as excited states of the more fundamental underlying quantum fields. As Nobel laureate Frank Wilczek noted [11], "In quantum field theory, the primary elements of reality are not individual particles, but underlying fields." On the other hand, when we examine lower energy interactions in nonrelativistic theory we find that interpretations are almost exclusively about particles. It seems that theoreticians use fields when it is convenient to use fields, and particles when it is convenient to use particles. However, the physical principles that underlie mathematical models demand a level of consistency that goes beyond simple expediency.

In quantum field theory particles are described by an action functional $S[\psi(x_i)]$ where the fields $\psi(x_i)$ of particles are defined throughout space. Each of the elementary particles has a field and the action depends on all of the fields. The equations of motion are determined by minimizing the Lagrangian in a region of space-time. We will follow these same practices in order to extend the field interpretation to nonrelativistic theory. The region of space-time that is of interest for lower energy interactions lies between the two states of an atomic oscillator. Within the space-time region between the excited and ground states we define a Lagrangian density of the fields and their first derivatives $£(\phi_i, \phi_{i,\mu})$ which allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the electromagnetic field strength. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated in a way that is consistent with special relativity theory, where emission initiates from the excited state $R_2 = (x_2,y_2,z_2)$ at time t_2 and it finalizes at the ground state $R_1 = (x_1,y_1,z_1)$ at time t_1. Applying Hamilton's principle we require the integral of the Lagrangian density over the region of space-time between the excited and ground states to be a minimum for all small variations of the coordinates inside the region, where the action minimum for an arbitrary quantum system is defined in angular measure to be the reduced Planck's constant ħ.

$$S\left[\phi_i(t)\right] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} \pounds(\phi_i \phi_{i,\mu}) d^\beta x dt = \hbar \qquad 4)$$

The energy states |2> and |1> coincide with the electron shells and determine invariant field boundaries where fields vanish. The change in action yields a relativistic formulation of emission that is invariant, the same for all observers. The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The field boundaries are uniquely fixed in four dimensions by the volume d^3x and the time interval t_2-t_1 causing photon emission to be described as a four-dimensional localization of fields.

3.4. Comparison of the relativistic and non-relativistic models

The overriding problem in quantum mechanics historically has been how to describe two physical processes, absorption and emission, with a single equation. According to the Schrödinger wave equation they occur as a single process that evolves symmetrically in time. A spin ½ particle, or fermion must perform two complete rotations, or a total of 720 degrees, to return to its original state. If the electron is represented mathematically by a vector in Hilbert space |ψ> then one rotation of 2π results in a negative value -|ψ> and a second rotation of 2π brings the electron back to its original state |ψ>. The rotation occurs in abstract space so no physical interpretation is possible.

Quantization described by an action functional occurs in real space and real time as a four-dimensional absorption of energy by the electron 3), followed by a four-dimensional localization of field 4) and release of a photon. We interpret the first rotation of the wave function not as a rotation in *abstract space*, but as the change in phase of electromagnetic fields from 0 to 2π in *real space* during absorption. Thus one "rotation" of the wave function is interpreted as one full cycle of an electromagnetic wave and an increase in the electron's energy from the ground state to an excited state. The second rotation occurs as the electron returns to its original state, the ground state, and is interpreted as a localization of electromagnetic field energy and emission of a photon. The dual wave-particle nature of the photon is thereby realized in a physical transformation.

4. Discussion

To determine the true evolution of a quantum system we use Hamilton's principle and take advantage of the symmetries of the system, by using generalized coordinates to describe the electron shells R_2 and R_1. Then the actual path the electron follows between the initial and final stationary points of a complete cycle between t_1 and t_2 is the one which minimizes the action. However, the true path is not necessarily the one that is the most precisely determinable. Thus the Schrödinger equation is the most accurate differential equation of motion known. We have no better way to describe the path of an electron other than to calculate the probability of where it will arrive at a detector. However, probable paths do not minimize the action. Only the actual path minimizes the action. Electron excitation is one path and decay is another. The fact that two events cause one result, a photon emission, is to a large extent what makes quantum theory seem so mysterious. The electron experiences two transitions resulting in the emission of a single photon.

The purpose of the wave function is to describe that entire radiation process with a single expression. To do that it must describe both absorption and emission with a single equation. As a result it includes an incoming energy quantum and an outgoing energy quantum and twice the minimum allowable action for a complete electron cycle. Therefore it does not represent the true path of an electron transition.

5. Conclusion

When posed the question, What is reality?" Nobel laureate Anton Zeilinger replied recently [12], "In physics, we have always made great progress without answering the question of what this is. We only answer the question of what can be measured and how can we observe something. We can observe reality, we can make measurements, but I don't think we can say anything about the essence of reality."

The preferred method for determining the nature of reality is through the process of reductionism. It seems perfectly reasonable to start with the very small, presumably simpler world, and add to it to explain more complex phenomena. Although reductionist methods are successful in the classical world for understanding complex structures, they fail in the quantum world. The reason quantum mechanics cannot say anything about reality is because it reverses the natural order. Measurement begins with emission whereas Nature begins with absorption. Thus reductionism reverses the natural order. Natural processes do not begin with the simple and proceed to the complex as in reductionism, Nature proceeds from the more complex classical world to the simpler quantum world.

Energy absorption from the classical world precedes and determines what can or cannot be emitted and observed. An object that is isolated from the classical world at absolute zero is neither measurable nor observable. To gain access to the quantum world we must pass through the classical world. In black body radiation, for example, energy is added to matter by heating it until it begins to radiate. The temperature determines what frequencies are emitted and the distribution of frequencies is described by Planck's law. Einstein used statistics to show that energy is more than a quantum, it is a transformation of classical energy to quantum mechanical energy. No one else followed his lead because they continued to believe in the reductionist philosophy which proceeds from the simple to the complex.

More and more precise measurements are an integral part of reductionist programs. They can measure what is real, however they do not assist in determining the nature of reality because they are performed at single points in time. Nothing of interest in science; particle trajectories, the emission of a photon, a pendulum, or a life form; is static. To describe a particle continuously in time we use action, energy times time, and integrate over a period of time. Instead of describing particle motion as a probability distribution of all possible paths measured at a particular point in time, we single out the one true path whose action is minimum over a specific, well-defined period of time; in other words, the time integral of a Lagrangian. The trajectory of a particle is not contained in a distribution of paths expressed by a probability law, it is the one true path in a collection of paths whose action is minimum. We obtain proof by subjecting the Schrödinger equation to the calculus of variations. One rotation of the wave function is required to excite an electron and another to return it to its original state and emit a photon. Each rotation of 2π represents a *transfer of energy* with action minimum $\hbar$.

References

[1] T. Bothwell *et al*. "Resolving the gravitational red shift within a millimeter atomic sample" *Nature* **602**, Issue 7897, 420 (2022) arXiv:2109.12238

[2] Guellati-Khelifa, S. "Searching for new physics with the electron's magnetic moment" *Physics* **16**, 22 (2023).

[3] A. Einstein, "Quantum theory of radiation" *Phys Z* **18**, 121 (1917), Einstein (1917) Quantum theory of radiation

[4] Heisenberg W. "Quantum mechanical re-interpretation of kinematic and mechanical relations", *Z Phys* **33**, 879 (1925) in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics*, (Dover, 1968).

[5] Oldani, R. "Einstein on measurement theory: A heuristic point of view" *Physics Essays* **3** (1), (2022) doi:https://doi.org/10.4006/0836-1398-35.1.51

[6] Oldani, R. “Application of Einstein’s methods in a quantum theory of radiation” (Intechopen, London, 2021).

[7] W. Heisenberg *Z Phys* **33** (1) (1925), in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics* (Amsterdam, 1967) .

[8] Brown, L.M., *Feynman’s thesis: A new approach to quantum theory* (World Scientific Publishing Co., 2005), p. 4.

[9] Feynman, R. “A space-time approach to non-relativistic quantum mechanics” *Rev Mod Phys* **20**, 367-387 (1948).

[10] Lanczos, C. *The Variational Principles of Mechanics* 4th edn. (Dover, 1970)

[11] Wilczek, F. "Mass Without Mass I: Most of Matter," *Physics Today* **52** (11), 11-13 (1999).

[12] Ansede, M., Interview with Anton Zeilinger, Nobel laureate, *El Pais newsletter*, June 14, 2023