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Abstract: In order to describe the time evolution of energy states we choose to abandon the Hamiltonian method of quantum mechanics, which has been the standard for over a century, in favor of the more fundamental Lagrangian method. Integral equations of motion for the absorption and emission of radiation are derived to replace the differentially motivated Schrödinger equation. This new interpretation for the time evolution of energy states is used to interpret several well-known experiments, among them Planck's law, the electron cyclotron, and chaos theory; thereby demonstrating an improved physical model of quantum mechanical and classical phenomena.

Keywords: non-relativistic; relativistic quantum mechanics; Hamilton's principle; symmetry

1. Introduction

There are two distinct approaches for obtaining the equations of motion of a material system. The more common method is to choose a particular equation and use it to make predictions. We evaluate the correctness of the equation by how well it matches our predictions. There is an alternate method we can use if we are not sure which equation is the correct one. This method allows one to collect together all the equations and express them as the stationary property of a certain action function, the time-integral of the Lagrangian [1]. The Lagrangian method has the advantage that it can easily be expressed relativistically, on account of the action function being a relativistic invariant. It calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. This second method, commonly referred to as "Hamilton's principle", allows the differential equations of motion for any physical system to be re-formulated as an equivalent integral equation.

2. Theory

In quantum mechanics the Hamiltonian operator is used to generate the time evolution of quantum states. The validity of equations in non-relativistic theory is evaluated by calculating a theoretical probability distribution of measurements and then comparing it to the distribution obtained from actual measurements. An alternate method using the Lagrangian seeks to express the meaning of the entire set of differential equations describing the paths, paths of both high and low probability. Comparison of the two methods shows that the Hamiltonian method violates the conservation laws by failing to include absorption energy in the equations of motion [2]. Energy must be absorbed before it is emitted, for the simple reason that the unilateral creation of energy is a violation of the conservation laws. The wave function, a product of the Hamiltonian method, combines absorption and emission into a single mathematical expression describing excitation and decay, so it includes two paths of an electron. By using the Lagrangian method we easily recognize that the wave function cannot represent the true path for it includes twice the action minimum, one for each path of the electron. To satisfy the conservation laws we shall define radiation processes as transformations of energy from continuous forms to discrete.

To describe the time evolution of a quantum state as an integral equation we interpret the wave function as a two-body system; that is, the superposition of two field geometries, the electromagnetic field of a photon $\phi_{i,\mu}$ and the electric field ϕ_i if an electron [2]. Using symmetry arguments we define the electron shells as surfaces equidistant from the nucleus specified by the generalized coordinates R_1 and R_2. Because quantization consists of two distinct electron paths to complete a cycle, two

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equations of motion are required. The first, describing the electron's path during excitation is given in generalized coordinates by the action minimum of a Lagrangian.

$$S[r(t)] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} (T - V)dt = \hbar \tag{1}$$

The action, S[r(t)], describes the absorption process four dimensionally as a continuous path with circular orbits for the ground and excited states. It has as its argument an infinite number of functions, the possible electron trajectories r(t). The action minimum is not equal to zero as in classical mechanics, but ħ.

Emission immediately follows absorption and consists of a Lagrangian density of the fields and their first derivatives £(ϕ_i, $\phi_{i,\mu}$) in the region of space-time between the excited and ground states R_2 and R_1 [2]. This allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the electromagnetic field strength.

$$S[\phi_i(t)] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} £(\phi_i \phi_{i,\mu}) d^3x dt = \hbar \tag{2}$$

The action S[$\phi_i(t)$] is a functional, a function of the values of generalized coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The photon is described therefore as a four-dimensional localization of fields contained within the volume d^3x and the time interval t_2-t_1 . Thus discrete and continuous properties of the energy transformation process are described in a single equation.

We will use the time evolution of quantum states given by 1) and 2) to introduce new perspectives for deriving the laws of nature. Rather than formulating classical laws separately from quantum mechanical laws we seek equations of motion that include classical and quantum concepts together. Thus the time evolution of a quantum state consists of a continuous energy increase, a discrete energy transformation, and a localized emission of energy, or photon; where energy states are separated by discrete field boundaries. We will show in the following that the physical model described by 1) and 2) has heuristic value and leads to further insights into the behavior of natural phenomena.

3. Applications

3.1. Time asymmetry

Using arguments originating with Einstein, energy and momentum at the molecular level are shown to be causally related [2]. The causal relationship leads inevitably to his derivation of the A and B coefficients, precursors to the experimentally confirmed theory of lasers [3]. The coefficient B_{12} (excitation) refers to energy transfer caused by momentum in the direction of propagation and the coefficients B_{21} and A_{21} (decay) refer to the recoil momentum of induced and spontaneous emission in a direction opposed to the direction of propagation. Because different probability laws govern the momentum exchange of absorption and emission processes it was common knowledge that [4], "Einstein believes that irreversibility is exclusively due to reasons of probability". Ordinarily arguments based on the conservation of momentum would provide sufficient cause to question a theory. Similar arguments resulted in Pauli's proposal for the existence of an unknown particle to explain beta decay, and in Fermi's theory of the neutrino which eventually confirmed it. Even though Einstein's theory of the A and B coefficients has proven fundamental to our understanding of microscopic processes, the reversibility of the Schrödinger equation has not been questioned.

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3.2. The time parameter

We use the same time variable to describe time as a parameter and the time evolution of energy states despite a fundamental difference. Time the parameter is measurable whereas the time evolution of states need not be. To illustrate the difference consider Planck's radiation law $B_\nu(\nu, T)$ which describes the relationship between the temperature T and the frequency ν of the emitted radiation.

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}, \tag{3}$$

Time appears as a measurable with specific value in 3). It does not show continuous energy absorption as described by 1) or discrete localizations of energy due to photon emission described by 2). The purpose of 3) is to provide experimental confirmation of Planck's law describing the spectral radiance of a black body. It includes non-relativistic physical variables measured at a particular point in time. Temperature, for example, cannot be transformed relativistically [5].

Einstein introduced the idea that quantized packets of energy account for the form of Planck's radiation law in his paper on the photoelectric effect [6]. It explains the spectral radiance of black body radiation explicitly in terms of ν and T measurements, but it does not describe how the evolution of a quantum system occurs relative to the background time. He was not satisfied with Planck's derivation and continued to study the question of radiation for the next four years. He was then able to obtain a detailed balance equation to his satisfaction relating the statistically defined fluctuation of energy due to photon numbers on one hand, and interference on the other [7]. The equation he found describes both absorption and emission so two terms appear in the equation.

$$\langle \epsilon(\nu, T) \rangle = \left(h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2 \right) \nu d\nu \tag{4}$$

"According to the current theory, the expression would be reduced to the second term (fluctuation due to interference). If the first term alone were present, the fluctuations of the radiation pressure could be completely explained by the assumption that the radiation consists of independently moving, not too extended complexes of energy *hν*. In this case, too, the formula says that in accordance with Planck's formula the effects of the two causes of fluctuation mentioned act like fluctuations (errors) arising from mutually independent causes."

There are very noticeable differences in the equations 3) and 4), both of which describe Planck's radiation law, for the simple reason that each one is designed for a particular purpose. Equation 3) includes the physical variables temperature and frequency that determine spectral radiance at a particular point in time. On the other hand, Einstein's equation 4) is based on his desire to understand how black body radiation evolves with respect to background time as a statistical balance between the energy resulting from the incoming wave properties of electromagnetic radiation and the outgoing particle properties due to photon emission.

3.3. Matrix mechanics

Despite its unusual and highly complex mathematics, matrix mechanics is related to the topic of black body radiation as well. To understand how we shall inspect its formulation of the energy matrix [8].

$$\sum_k \left(p_{nk} q_{km} - q_{nk} p_{km} \right) = \begin{matrix} i\hbar \text{ for } n=m \\ 0 \text{ for } n \neq m \end{matrix} \tag{5}$$

The diagonal elements n=m represent the observable properties of energy, the transition probabilities and frequencies, which are emissions formulated in coordinates of the atom. Non-diagonal matrix elements n≠m refer to the resonances of radiation with an atom's valence electrons which result in net positive exchanges of momentum, but time-averaged zero exchanges of energy. Exchanges of momentum lead to a net increase in the kinetic energy of molecules; however, because

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they cannot be observed individually Heisenberg did not take them into account believing that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable".

Although he eliminated exchanges of momentum, from consideration due to their unobservability Heisenberg soon realized that something was missing, lamenting to Pauli in a letter [9], 'But the worst thing is that I am quite unable to clarify the transition [of matrix mechanics] to the classical theory.' If he had taken Einstein's 1917 paper seriously perhaps he would have realized that he had skipped over classical theory and gone directly to quantum theory. Without classical absorptions there can be no quantum emissions so matrix mechanics is an incomplete formulation of quantum mechanics.

3.4. Non-inertial frames

In the early years of quantum mechanics it was assumed that the effect of the gravitational field in an atom could be neglected due to its extremely small effect compared to that of the electric field. At the time there were very few objections and no hard evidence to the contrary. Heisenberg's uncertainty principle proved that the paths of electrons are indeterminate so it seemed reasonable to formulate quantum mechanics in inertial frames; that is, in the absence of gravitational fields.

Recent experiments require that we rethink those ideas. Researchers have designed an atomic clock using a single crystal of 100,000 strontium atoms together with ultraviolet light to differentiate between the gravitational potential of the crystal's upper and lower surfaces, a distance of one millimeter [10]. The fractional frequency instability given for that experiment, $7.6X10^{-21}$, makes it possible to determine the fractional shift in wavelength $\Delta\lambda$ corresponding to the thickness of the electron shell.

$$\Delta\lambda = \frac{\Delta f}{f}\lambda = (7.6\times 10^{-21})\times(6.98\times 10^{-7}\,m) = 5.3 \text{ X } 10^{-27}\,\text{m}$$

Therefore each cycle of an atomic clock's "pendulum", an oscillating electron, is carried out between surfaces of thickness with uncertainty $\Delta x \leq 5.3 \text{ X } 10^{-27}$ m. The ticks of the clock are referred to as "non-demolition measurements" because the uncertainty of the ticks does not increase from their measured value as the system evolves [11]. This means that the collapse of the wave function does not occur when time measurements are performed. Continued clock improvements are only limited by experimental factors so the accuracy of time measurement is believed to be theoretically unbounded. If a different clock rate exists at each point in space, as this experiment suggests, then formulations of quantum mechanics in inertial frames; that is, in the absence of gravitational fields, are incomplete. We require instead a *relativistic* formulation of quantum mechanics in non-inertial frames as has been proposed previously [12].

3.5. The "geonium atom"

We have discussed the time evolution of energy states in abstract terms by using the theoretically derived equations 1) and 2) . We wish now to compare theory and practice with a fascinating series of experiments that could not possibly have been imagined by the founders of quantum mechanics in 1926.

It is possible to trap single electrons in a "bottle" made of intersecting electric and magnetic fields and cool them to near absolute zero. The fields may then be used to manipulate the motion of an electron as it transitions between energy levels at rates many magnitudes slower than the electrons in an atom [13]. The trapped electron constitutes an artificial atom or "quantum cyclotron", the simplest quantum mechanical system possible. The measurements are so sensitive that the influence of the earth's gravitational field is taken into account.

Applying a relatively large constant magnetic field to the trap causes the electron to execute two different types of motion simultaneously; circular orbits perpendicular to the field and axial drifts parallel to it. The experiments are used to precisely observe the absorption and emission of energy by a quantum oscillator. As they describe it, "There is a small alternating magnetic field in the

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particle's rest frame, which is perpendicular to the large constant magnetic field. This alternating magnetic field has a frequency component and so a spin-flipping resonance occurs when the drive frequency equals the anomaly frequency." In other words, the electron is stimulated by a small classically defined magnetic field with varying frequency. The drive frequency gradually shifts upwards with increasing energy until it causes a discrete "spin-flip" to occur.

According to the standard model electromagnetic field energy is quantized before it is absorbed. Experiments performed with geonium, the simplest possible quantum system, clearly indicate otherwise. Energy is absorbed continuously due to classical magnetic resonance and emitted discretely in the form of spin-flips. The belief that the Schrödinger equation possesses time reversal symmetry as suggested by mathematical arguments is denied here by physical evidence. For a second time (see 2.4) experimental techniques have outstripped theoretical principles derived a century ago when relatively simple spectroscopic measurements were all that was available. Experiments with geonium confirm the validity of equations 1) and 2).

3.6. Liquid helium in a box

Quantum mechanics is believed to be restricted to applications at the microscopic level. There are, however, many classical experiments which look suspiciously like the quantum jump. Consider the flow of heat through a quantity of liquid helium confined in a box [14]. When a very small temperature difference of a mere 0.001°C is applied to the upper and lower sides of the box, heat energy flows continuously causing cylindrical rolls of fluid to be created by the thermal currents. The rolls conduct a continuous flow of energy from high temperature to low. We can associate the paths of helium molecules with action minima as described by equation 1).

A slight increase of heat causes a wobble to appear and then the period doubles, or "bifurcates" as a second oscillation is added. The bifurcation, which increases energy flow by multiplying the number of dissipation centers, may be described as a four-dimensional localization of Lagrangian density (See eqn 2). The flows do not interfere with each other so the superposition is continuous. However, the appearance of the new dissipation center occurs discontinuously. Continuous increases of energy are followed by discrete transitions to new center of flow and a heightened flow level overall. Our hypothesis concerning the universality of the time evolution of discrete energy states stated in 2.0 is supported. For the first time quantum mechanical principles describing the evolution of microscopic states are successfully applied to a macroscopic phenomenon.

3.7. Pendulum connected to an external drive

We can also observe bifurcations graphically by tracing a pendulum's motion in terms of equally spaced points in time [15]. In the Figures 1 and 2) we see the time-asymptotic phase-space orbit of a pendulum as it transitions from period one motion to period two motion due to an external drive. In period one motion the pendulum repeats the same motion over and over. In period two motion the cycles *are distinguishable from each other and* are repeated on alternate periods of the external drive. The transition from period one to period two occurs unpredictably at a single moment in time.

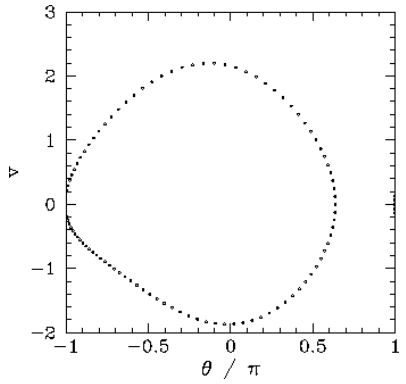

Figure 1. Period one motion.

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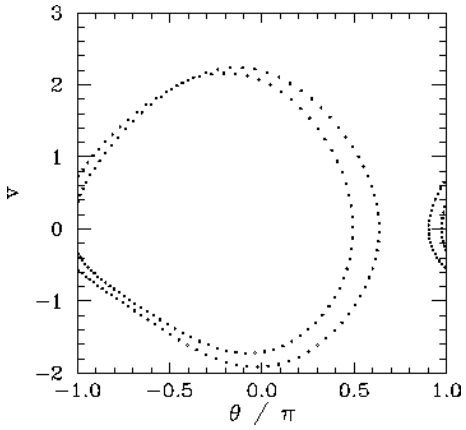

Figure 2. Period two motion.

When combined the two diagrams represent the time evolution of a classical energy state. To compare them with the time evolution of a quantum state we apply Hamilton's principle which expresses the meaning of the entire set of differential equations describing the paths. The symmetry of the experiment calls for generalized coordinates in the plane of the orbits for the same reasons that generalized coordinates on electron shells are called for by atomic transitions in equations 1) and 2). The energy is now determined by positions and velocities rather than by a Lagrangian density of the fields. We obtain the path actually taken in the same way that the path of an electron in an atom is obtained; that is, by minimizing the action, where the action is given by the time integral of a Lagrangian.

$$S[q(t)]=\int_{t_1}^{t_2} L(\dot{q}(t),q(t),\mathrm{t})dt \tag{6}$$

The transition between period one and period two motion is referred to as a "bifurcation" and is caused by a gradual increase in the external drive. We describe it *relativistically* as a continuous increase of energy followed by a discrete transition. A *non-relativistic* description of the same phenomenon would seek to predict the energy difference between period 1 motion and period 2 motion. This is because the Schrödinger equation describes transition energies with time-independent values. Transition energies are non-relativistic because they occur at single points in time, are measurable, and occur independently of the background time. As shown in paragraph 3.2 it is the way we describe the evolution of states relative to the background time that determines whether a theory is relativistic or non-relativistic.

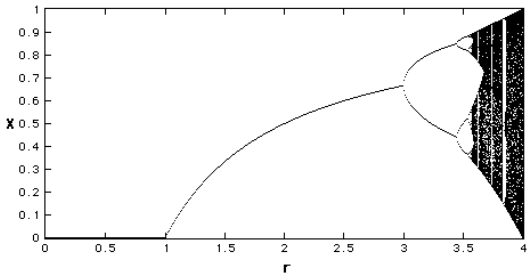

Figure 3. Period doubling on the way to chaos.

As the energy of a dissipative system increases bifurcations are followed by additional bifurcations in an endless series referred to as "period doubling", where each bifurcation is the result of a transition energy. The above logistic map shows the discrete points in time of transitions due to period doubling as they occur relative to the continuously evolving background time. It is a graphical representation of the action functionals described earlier which describe natural phenomena

mathematically by using continuous and discrete forms of time. Although we are using the logistic maps here as a visual aid to illustrate the behavior of microscopic and macroscopic processes they originated in the biological sciences to model population dynamics [16].

4. Conclusion

The Hamiltonian formulation of quantum mechanics proves adequate for predicting experimental findings at the microscopic level as confirmed by measurements performed at particular points in time. However, it does not give a complete description of events when considered over more extended time periods for it neglects what cannot be observed or measured precisely, the absorption energy. This does not mean it is wrong. Quantum mechanics still gives us the best description of microscopic phenomena possible. However, it does not give us a true picture of Nature, one that reflects the way events actually occur. The reason as stated earlier is that the wave function represents two events, absorption and emission, and the first (absorption) must be carried out to completion before the second one (emission) begins. Each one is governed by the uncertainty principle and each one follows a path determined by the action minimum ħ.

An improved description of the way natural phenomena occur is obtained with the Lagrangian formulation by including the background time in descriptions of the time evolution of states. The differences between the non-relativistic Hamiltonian formulation and the more fundamental Lagrangian formulaton become especially apparent when macroscopic phenomena are considered. Whereas in quantum mechanics continuous absorption processes cannot be measured so they are ignored, in macroscopic experiments such as period doubling absorptions are prominent, are easily measured, and it is impossible to ignore them. Thus changes of state of any type, quantum or classical, may be described with Lagrangian methods by isolating them from environmental influence and applying equations 1) and 2) to show a gradual increase of energy followed by a discrete transformation and immediately thereafter emission in the form of localized field geometries. This new physical model for describing the time evolution of natural phenomena has heuristic value in the search for a more complete understanding of Nature.

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