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Time Evolution of Energy States

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Abstract: In order to describe the time evolution of energy states we choose to abandon the non-relativistic Hamiltonian method, which has been the standard for nearly a century, in favor of a more fundamental, relativistically correct Lagrangian method based on theories originally proposed by Dirac and Einstein. Integral equations of motion for the absorption and emission of radiation are derived that underlie and anticipate the differentially motivated Schrödinger equation. This new interpretation applies to a large volume of experimental evidence of both classical and quantum mechanical origin. Among the examples discussed in support are Planck's law describing black body radiation, the function of the simplest quantum mechanical system an electron cyclotron, atomic clocks, matrix mechanics, chaos theory, and evolutionary biology.

Keywords: non-relativistic; relativistic; quantum mechanics; Hamilton's principle; Lagrangian; symmetry; atomic clocks

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1. Introduction

There are two distinct approaches for obtaining the equations of motion of a material system. The more common method is to derive equations that coincide with predictions. We evaluate the correctness of the equation by how well it matches our predictions. This is how the Hamilton formulation of quantum mechanics originated and was manifested formally in matrix mechanics, wave mechanics, and the path integral formulation. There is an alternate, more fundamental method we can use if we are not sure which equation is the correct one. This method allows one to collect together all the equations and express them as the stationary property of a certain action function, the time-integral of the Lagrangian [1,2]. The Lagrangian method has the advantage that it can easily be expressed relativistically, on account of the action function being a relativistic invariant. It calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. This second method, also referred to as "Hamilton's principle", allows the differential equations of motion for any physical system to be re-formulated as an equivalent integral equation.

2. Theory

The Schrödinger equation is a linear partial differential equation based on Hamiltonian mechanics that determines how the wave function evolves over time. A probability distribution of all possible particle paths is obtained by taking the square of the wave function. Only one of the paths is realized, the path that results in a particle measurement. However, there is an alternate method that describes the quantum system by taking the time integral of a Lagrangian. It expresses the meaning of the entire set of differential equations describing the paths, irrespective of whether a path is confirmed by measurement. The path realized is the path for which the action is minimized.

The Hamiltonian formulation ignores paths that are not observed. Thus the wave function describes the time evolution of quantum states, excitation and decay, as a single event. The Lagrangian method differs fundamentally for it holds that observability and measurement are not the only determinations of path, rather that all paths, also unobservable paths, must be accounted for due to the conservation of energy. Thus the time evolution of a quantum system encompasses two paths of the electron, the unobserved path of excitation and the observed path of decay.

To describe the time evolution of a quantum state using the Lagrangian method we use symmetry arguments and define the electron shells as surfaces equidistant from the nucleus as specified by the generalized coordinates R_1 and R_2 [3]. Because quantization consists of two distinct electron paths to complete one cycle, two equations of motion are required. The first, describing the electron's path during excitation is given in generalized coordinates by the action minimum of a Lagrangian.

$$S[r(t)] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} (T - V)dt = \hbar \qquad 1)$$

The action functional, S[r(t)], describes the absorption process four-dimensionally as a continuous path that proceeds from the ground state to that of the excited state. It has as its argument an infinite number of functions, the possible electron trajectories r(t). The action minimum is not equal to zero as in classical mechanics, but ħ.

Emission immediately follows absorption and consists of a Lagrangian density of the fields and their first derivatives $£(\phi_i, \phi_{i,\mu})$ in the region of space-time between the excited and ground states R_2 and R_1. This allows for a complete accounting of the energy interactions, where ϕ_i is the current density due to a transitioning electron and $\phi_{i,\mu}$ is the localized electromagnetic field strength that results in the creation of a photon.

$$S[\phi_i(t)] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} £(\phi_i \phi_{i,\mu}) d^3x dt = \hbar \qquad 2)$$

The action $S[\phi_i(t)]$ is a functional, a function of the values of generalized coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The photon is described therefore as a four-dimensional localization of fields contained within the volume d^3x and the time interval t_2-t_1 . Thus discrete and continuous properties of the energy transformation process are described in a single equation.

We will use the time evolution of quantum states given by 1) and 2) to introduce new perspectives for deriving the laws of nature. Rather than formulating classical laws separately from quantum mechanical laws we seek equations of motion that include classical and quantum concepts together. Thus the time evolution of a quantum state consists of a continuous energy increase of classical origin, a discrete quantum mechanical transformation, and the four-dimensional localization of energy during emission. We will show in the following that this physical model, as described by 1) and 2), has heuristic value and leads to further insights into the behavior of natural phenomena.

3.0. Applications

3.1. Time asymmetry

Using arguments originating with Einstein, energy and momentum at the molecular level are shown to be causally related [3,4]. The causal relationship leads inevitably to his derivation of the A and B coefficients, precursors to the experimentally confirmed theory of lasers. The coefficient B_{12} (excitation) refers to energy transfer caused by momentum exchange due to molecular impulses in the direction of propagation. The coefficients B_{21} and A_{21} (decay) refer to recoil momentum due to the induced and spontaneous emission of photons in a direction opposed to the direction of propagation. These arguments enable him to describe black body radiation at the molecular level using statistical arguments, and he concludes that excitation is governed by Maxwell-Boltzmann statistics and the emission is governed by Planck statistics. Because different probability laws are used to describe the momentum exchange of absorption and emission processes it was well known that [5], "Einstein believes that irreversibility is exclusively due to reasons of probability".

Einstein's theory of the causal relationship between energy and momentum has been proven experimentally by laser technology; however, his views on irreversibility are not accepted

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universally. The pure quantum state is believed to be reversible and that irreversibility is determined by "collapse" of the wave function when a measurement is performed, a non-relativistic effect; and not by probability, a relativistic effect dependent upon gravitational potential. Ordinarily arguments based on the conservation of momentum would provide sufficient cause to question a theory. Similar arguments based on momentum exchange resulted in Pauli's proposal for the existence of an unknown particle to explain beta decay, and in Fermi's theory of the neutrino which eventually confirmed it. In the next section we provide experimental proof that elementary processes are irreversible.

3.2. *The geonium atom*

We have discussed the time evolution of energy states in abstract terms by using the theoretically derived Equations (1) and (2) . We wish now to compare theory and practice with a fascinating series of experiments that could not possibly have been imagined by the founders of quantum mechanics in 1926. The experiments are superior in precision and simplicity to the single-electron hydrogen atom used to derive non-relativistic quantum mechanics. In fact the "geonium atom" is the simplest possible quantum system since it replaces the nucleus with an external apparatus bound to the earth [6]. It is possible to trap single electrons in a "bottle" made of intersecting electric and magnetic fields and cool them to near absolute zero. The fields may then be used to manipulate the motion of an electron as it transitions between energy levels at rates many magnitudes slower than the electrons in an atom. The trapped electron constitutes an artificial atom or "quantum cyclotron". The measurements are so sensitive that the influence of the earth's gravitational field must be taken into account in order to calculate the equations of motion.

Applying a relatively large constant magnetic field to the trap causes the electron to execute two different types of motion simultaneously; circular orbits perpendicular to the field and axial drifts parallel to it. The experiments are used to precisely observe the absorption and emission of energy by a quantum oscillator. As they describe it, "There is a small alternating magnetic field in the particle's rest frame, which is perpendicular to the large constant magnetic field. This alternating magnetic field has a frequency component and so a spin-flipping resonance occurs when the drive frequency equals the anomaly frequency." In other words, the electron is stimulated by a small classically defined magnetic field with varying frequency. The drive frequency gradually shifts upwards with increasing energy until it causes a discrete "spin-flip" to occur.

According to the standard model electromagnetic field energy is quantized before it is absorbed. Experiments performed with geonium, the simplest possible quantum system, clearly indicate otherwise. Energy is absorbed continuously due to classical magnetic resonance and emitted discretely in the form of spin-flips. The belief that the time reversal symmetry of pure quantum states is denied here by experimental proof. Spin flips cannot be used to reproduce the continuously variable potential of magnetic fields. Thus experimental techniques have outstripped theoretical principles derived a century ago when relatively simple spectroscopic measurements were all that was available. Experiments with geonium confirm the validity of Equations (1) and (2).

3.3. *Non-inertial frames*

In the early years of quantum mechanics it was assumed that the influence of the gravitational field on an atom could be neglected due to its extremely small effect compared to that of the electric field. At the time there were very few objections and no hard evidence to the contrary. Heisenberg's uncertainty principle proved that the paths of electrons are indeterminate so it seemed reasonable to formulate quantum mechanics in inertial frames; that is, in the absence of gravitational fields.

Recent experiments require that we rethink those ideas. Researchers have designed an atomic clock using a single crystal of 100,000 strontium atoms together with ultraviolet light to differentiate between the gravitational potential of the crystal's upper and lower surfaces, a distance of one millimeter [7]. The fractional frequency instability given for that experiment, 7.6×10^{-21}, makes it possible to determine the fractional shift in wavelength $\Delta\lambda$ corresponding to the thickness of the electron shell.

$$\Delta\lambda = \frac{\Delta f}{f}\lambda = (7.6\times 10^{-21})\times(6.98\times 10^{-7}\,m)$$

$$= 5.3 \mathrm{X} 10^{-27}\,\mathrm{m}$$

Therefore each cycle of an atomic clock's "pendulum", an oscillating electron, is carried out between surfaces of thickness with uncertainty $\Delta x \leq 5.3 \mathrm{X} 10^{-27}$ m. The ticks of the clock are referred to as "non-demolition measurements" because the uncertainty of the ticks does not increase from their measured value as the system evolves [8]. This means that the collapse of the wave function does not occur when time measurements are performed. Continued clock improvements are only limited by experimental factors so the accuracy of time measurement is believed to be theoretically unbounded. If a different clock rate exists at each point in space, as this experiment suggests, then formulations of quantum mechanics in inertial frames; that is, in the absence of gravitational fields, are incomplete. We require instead a *relativistic* formulation of quantum mechanics in non-inertial frames as has been carried out in more detail in a previous communication [9].

3.4. The time parameter

We use the same time variable to describe time as a parameter and the time evolution of energy states despite a fundamental difference. Time the parameter is measurable at single points in time whereas the time evolution of energy states occurs continuously over a period of time. To illustrate the difference consider Planck's radiation law $B_\nu(\nu, T)$ which describes the relationship between the temperature T and the frequency ν of the emitted radiation.

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}, \qquad 3)$$

Time appears as a measurable with specific value in 3). It does not show continuous energy absorption as described by 1) or discrete localizations of energy due to photon emission described by 2). The purpose of 3) is to provide experimental confirmation of Planck's law describing the spectral radiance of a black body. It includes physical variables measured at a particular point in time which are manifestly non-relativistic. Temperature, for example, cannot be transformed relativistically [10].

Einstein introduced the idea that quantized packets of energy are required in Planck's radiation law in his paper on the photoelectric effect [11]. It explains the spectral radiance of black body radiation explicitly in terms of ν and T measurements, but it does not describe how a quantum system evolves relative to the background time. He was not satisfied with Planck's non-relativistic derivation so he continued studying the question. Four years later he was finally able to obtain a detailed balance equation in real time describing the statistically defined fluctuation of energy due to photon numbers in relation to that determined by the superposition of electromagnetic fields [12]. The equation he found describes the fluctuation of energy due to both sources simultaneously.

$$\langle \epsilon(\nu, T)\rangle = \left(h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2\right)\nu d\nu \qquad 4)$$

As Einstein describes it, "According to the current theory, the expression would be reduced to the second term (fluctuation due to interference). If the first term alone were present, the fluctuations of the radiation pressure could be completely explained by the assumption that the radiation consists of independently moving, not too extended complexes of energy *hν*. In this case, too, the formula says that in accordance with Planck's formula the effects of the two causes of fluctuation mentioned act like fluctuations (errors) arising from mutually independent causes."

There are very noticeable differences in the Equations (3) and (4), both of which describe Planck's radiation law, for the simple reason that each one is designed for a particular purpose. Equation (3) includes the physical variables temperature and frequency that determine spectral radiance at a particular point in time. It is a non-relativistic equation because it is only valid in inertial systems relative to absolute time; that is, in the absence of the influence of gravitational fields. On the other hand, Einstein's Equation (4) is formulated in continuous time as a statistical balance between the energy resulting from the incoming wave properties of electromagnetic radiation and the outgoing particle properties due to photon emission. As a balance equation formulated relative to the continuous background time it holds irrespective of the intensity of gravitational fields. Because it corrects naturally for time dilation we say it is formulated relativistically.

3.5. Matrix mechanics

Despite its unusual and highly complex mathematics, matrix mechanics is related to the topic of black body radiation and the emission and absorption of radiation. To understand how we shall inspect its formulation of the energy matrix [13].

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{array}{l} i\hbar \text{ for } n=m \\ 0 \text{ for } n \neq m \end{array} \qquad 5)$$

The diagonal elements n=m represent the observable properties of energy, the transition probabilities and frequencies, which are emissions formulated in coordinates of the atom. Non-diagonal matrix elements n≠m refer to the resonances of radiation with an atom's valence electrons which result in net positive exchanges of momentum, but time-averaged zero exchanges of energy. Exchanges of momentum lead to a net increase in the kinetic energy of molecules; however, because they cannot be observed individually Heisenberg did not take them into account believing that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable".

Although he eliminated exchanges of momentum, from consideration due to their unobservability Heisenberg soon realized that something was missing, lamenting to Pauli in a letter [14], 'But the worst thing is that I am quite unable to clarify the transition [of matrix mechanics] to the classical theory.' If he had taken into consideration Einstein's findings that momentum and energy are causally related (see 3.1) he would have realized that he had skipped over classical theory in violation of the conservation of energy and gone directly to quantum theory. Because quantum emissions cannot occur without the classical absorption of heat energy matrix mechanics is an incomplete formulation of quantum mechanics.

3.6. Liquid helium in a box

Quantum mechanics is believed to be restricted to applications at the microscopic level. There are, however, many classical experiments which look suspiciously like the quantum jump. Consider the flow of heat through a quantity of liquid helium confined in a box [15]. When a very small temperature difference of a mere 0.001°C is applied to the upper and lower sides of the box, heat energy flows continuously causing cylindrical rolls of fluid to be created by the thermal currents. The rolls conduct a continuous flow of energy from high temperature to low. We can associate the paths of helium molecules with action minima as described by Equation (1).

A slight increase of heat causes a wobble to appear and then the period doubles, or "bifurcates" as a second oscillation is added. The bifurcation, which increases energy flow by adding a second dissipation center, may be described as a four-dimensional localization of energy. The flows do not interfere with each other so the superposition is continuous. However, the appearance of the new dissipation center occurs discontinuously. Continuous increases of energy are followed by discrete transitions to a new center of flow and a heightened flow level overall. It supports the hypothesis that the time evolution of discrete energy states described in 2.0 has universal value since it can be applied to both microscopic and macroscopic phenomena.

3.7. Pendulum connected to an external drive

We can also observe bifurcations graphically by tracing a pendulum's motion with respect to equally spaced points in time [16]. In the Figures 1 and 2) we see the time-asymptotic phase-space orbit of a pendulum as it transitions from period one motion to period two motion in response to an external drive. In period one motion the pendulum repeats the same motion over and over. In period two motion the cycles are distinguishable from each other and are repeated on alternate periods of the external drive. The transition from period one to period two occurs unpredictably at a single moment in time.

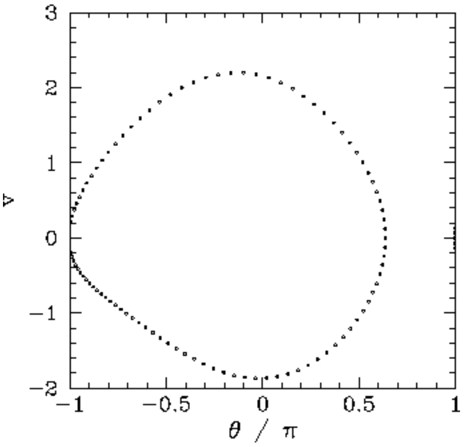

Figure 1. Period one motion.

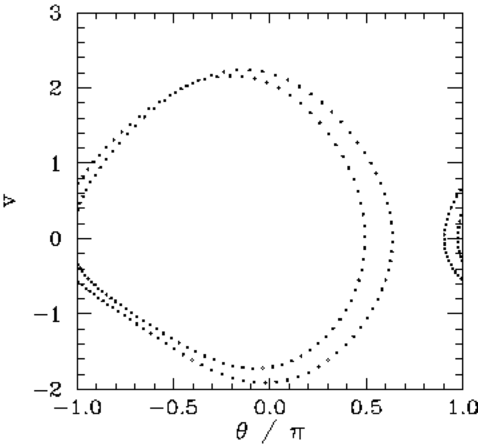

Figure 2. Period two motion.

The diagrams describe the time evolution between classical energy states. To compare them with the time evolution of a quantum state we apply Hamilton's principle which expresses the meaning of the entire set of differential equations describing the paths. The symmetry of the experiment calls for generalized coordinates in the plane of the orbits just as generalized coordinates on electron shells is called for by the atomic transitions described in Equations (1) and (2). Rather than deriving differential equations of motion by using the gravitational potential and calculating the positions and velocities of the pendulum as in Newtonian mechanics; we apply Hamilton's principle, which states that the path actually taken is the path for which the action is minimized, where the action S[q(t)] is given by the time integral of a Lagrangian.

$$S[q(t)]=\int_{t_1}^{t_2} L(\dot{q}(t),q(t),t)dt \qquad 6)$$

We describe the energy state of period one motion in 6) *relativistically* by the action S=Ldt integrated over the period of an orbit $t_2 - t_1$. The energy state of period 2 motion (two oscillations) is slightly higher than twice the energy of period 1 motion. The transition between period one and period two motion, referred to as a "bifurcation", occurs discretely. As in the case of quantum states, it is impossible to describe the transition explicitly by means of a differential equation of motion.

A *non-relativistic* description of pendulum motion would seek to predict the energy difference between period 1 motion and period 2 motion. This is because the Schrödinger equation describes transition between energy states (the eigenvalues) as time-independent values measured at single points in time. It disregards the continuous motion of the pendulum and instead attempts to determine the measurable value of a discrete energy transition. The same practice is used to describe electron transitions between energy states (see 2.0). Thus transition energy is non-relativistic if it is measured as a discrete value in absolute time, and relativistic if a continuous increase of energy is followed by a discrete transition relative to the background time. As noted in 3.4, whether an equation of motion is relativistic or non-relativistic is determined by whether it is influenced by changes in gravitational potential. The pendulum is influenced by gravity both in its continuous oscillations and its discrete transitions.

4.0. Conclusion

The Hamiltonian formulation of quantum mechanics provides the best available method for predicting experimental findings at the microscopic level. Although it gives us our most accurate description of microscopic phenomena, it does not give us our most complete description for it neglects what cannot be observed or measured precisely, the absorption energy. Absorption energy is critically important in quantum mechanics for it must precede emission energy in order to satisfy energy conservation. The wave function represents two events, absorption and emission, and the first (absorption) must be carried out to completion before the second one (emission) can begin. Each one is governed by the uncertainty principle and each one follows a path determined by the action minimum ħ. Because the wave function includes twice the action minimum it does not describe paths or particles, rather it represents the complete quantum system.

An improved description of the time evolution of material systems is obtained with the Lagrangian formulation in three steps: continuous absorption, four-dimensional localization, and the discrete release of energy. The differences between a non-relativistic Hamiltonian formulation and the more fundamental Lagrangian formulation becomes especially apparent when macroscopic phenomena are considered. In macroscopic, classically defined experiments such as period doubling absorptions are prominent, are easily measured, and are an integral part of the time evolution of isolated systems. Thus changes of state of any type, quantum or classical, are more accurately described by means of Lagrangian methods which isolates the system from environmental influence and applies Equations (1) and (2) to show a gradual increase of energy followed by a discrete transformation and emission in the form of localized field geometries.

This new physical model describing the time evolution of energy states has heuristic value in the search for a more complete understanding of nature. The gradually increasing energy of a material system followed by discrete changes in structure is a pattern that is repreated countless times. We find it in the extremely small, the emission of a photon by an atom; the extremely complex, period doubling phenomena in chaos theory; and the extremely large, the gravitational collapse of a star that results in the formation of a black hole. The same energy transformation pattern is even found in a theory of evolutionary biology. Periods of uniformity in the fossil record are often followed by sudden structural change in a process referred to as "punctuated equilibrium" [17]. Once the correct mathematical foundations are established in 2.0 and understood, it is possible to appreciate that quantum mechanics is a truly universal theory of nature touching every aspect of the world around us and within us, both microscopic and macroscopic.

References

1. Lanczos, C. *The Variational Principles of Mechanics* 4th edn. (Dover, 1970)
2. Dirac, P.A.M., "The Lagrangian in quantum mechanics" *Phys Zeit Sow* **3,** 1933, p.1.
3. Oldani R. "The Conservation Laws in Quantum Mechanics" https://www.preprints.org/manuscript/202303.0334/v4
4. Einstein, A. "On the quantum theory of radiation" *Phys Z* **18**, 121 1917, p. 63.
5. Ritz, W. and Einstein, A. "On the present status of the radiation problem" *Phys Z* **10** (1909) 323-4.
6. *Brown, L.S.; Gabrielse, G. (1986). "Geonium theory: physics of a single electron or ion in a Penning trap"*Reviews of Modern Physics*. **58** (1): 233–311.* doi:10.1103/RevModPhys.58.233.

7. T. Bothwell *et al*. "Resolving the gravitational red shift within a millimeter atomic sample"*Nature* **602**, Issue 7897, 420 (2022) arXiv:2109.12238
8. Braginsky, V.B., Vorontsov, Y.I.., Thorne, K.S. "Quantum nondemolition measurements" *Science* **209** 4456 (1980). doi: 10.1126/science.209.4456.547
9. Oldani, R. "Galactic Symmetry". Preprints **2020**, 2020090215. https://doi.org/10.20944/preprints202009.0215.v6
10. [Landsberg, P. and Matsas, G. "The impossibility of a universal relativistic temperature transformation" *Physica A* **340** (1-3) (2004), 92-94. doi: https://doi.org/10.1016/j.physa.2004.03.081
11. Einstein, A. "On a heuristic point of view about the creation and conversion of light" *Annalen der Physik* **17** (1905).
12. Einstein, A. "On the present status of the radiation problem" *Phys Z* **10** (1909) 185-93.
13. W. Heisenberg *Z Phys* **33** (1) (1925), in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics* (Amsterdam, 1967).
14. Pauli, W. *Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg*. Vol 1 Hermann, A., von Meyenn, K., & Weisskopf, V. Eds. (NY: Springer, 1979), p. 251.
15. A. Libchaber in Nonlinear Phenomena at Phase Transitions and Instabilities, ed. T. Riste (NY: Plenum, 1982), p. 259.
16. Fitzpatrick, R. *Introduction to Computational Physics* (University of Texas), p. 155, 164.
17. Eldredge, N.; Niles, S.J. (1972). "Punctuated equilibria: An alternative to phyletic gradualism" In Schopf, T.J.M. (ed.). *Models in Paleobiology*. San Francisco, CA: Freeman Cooper. pp. 82–115.