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Article

Robot Motion in Radial Mass Density Field †

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Abstract: Control of autonomous robot motion in radial mass density field is presented. In that sense the robot motion is described as the function of the radial mass density parameters. The radial mass density field is between the maximal and the minimal radial mass density values. Between these two limited values one can use n points ($n = 1, 2, . . . n_{max}$) and calculate the related radial mass density for each point. The radial mass density is maximal at the minimal gravitational radius and minimal at the maximal gravitational radius. This conclusion is valid for Planck scale, but also for the scales that are less or higher of that one. Using the ratio of the Planck mass and Planck radius it is generated energy conservation constant with value $\kappa = 0.99993392118$. Further, in this theory it is possible to connect Planck's and gravitational parameters as functions of the maximal (or minimal) radial mass density. In that sense the autonomous robot motion in radial mass density field is important for the control of the robot motion in macro, micro and nano scales.

Keywords: robot motion control; maximal (minimal) radial mass density; energy conservation constant; macro (micro; nano) robot motion; radial mass density field

1. Introduction

The autonomous robots have a very large application area. The first one is the application in the precise production processes. The second one is in the micro and nano scales as it is in medicine for cell manipulation, drug delivery, medical image acquisition and non-invasive intervention. For that application, one can use the electrical, or chemical actuated robots [1,2-5]. The magnetic soft robots have the advantages because of the fast response, unlimited endurance, and no obstruction restrictions [6]. Here, the motion of the autonomous robots is described in the radial mass density field. This field is in the region from the minimal radius (with the maximal radial mass density, $\rho_{r\,max}$) and maximal radius (with the minimal radial mass density, $\rho_{r\,min}$). Between these two limited values one can chose n points ($n=1,2,..n_{max}$). In the case of the precise robot motiion the number n_{max} should be biger. Contrarery, for the less precise robot motion, the number n_{max} may be smaller.

The very important consequence of the solution of the field equations by including gravitational energy-momentum tensor (*EMT*) on the right side of the field equations [7-10] is that the gravitational field exhibit repulsive (positive) and attractive (negative) gravitational forces. The time transition between quantum states in gravitational field is present in [11]. In order to precisely follow the desired trajectory of the autonomous robot motion one can include the new Relativistic Radial Density Theory (RRDT) [12]. The particle transition and correlation in quantum mechanics is discussed in [13]

Independent position control of two identical magnetic micro-robots in a plane using permanent magnets and magnetically powereful micro robots is presented in [14]. This represents the new approach to the medical revolution epohe. Magnetically powered micro-robots are discussed in [15,16]. Further, the robust control of micro-robot motion is presented in [17]. The global positioning of robot manipulators with mixed revolute and prismatic joints is discussed in [18]. In the case of vehicle dynamics control, a conjugate gradient-based BPTT-like optimal control algorithm can be

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applied [19,20]. The same algorithm can also be adapted to control of autonomous robot (micro-nano robot) motion in combined electromagnetic and gravitational fields. A robust motion control with antiwindup scheme for electromagnetic actuated microrobot using time-delay estimation is presented in [21]. Further, the quantization of the electromagnetic and gravitational fields is pointed out in [22]. The two indipendet position control of two indentical microrobots motion in a plane are realized by using rotating permanent magnets [23]. Magnetically powered microrobots and the robust motion control, with antiwindup scheme for electromagnetic actuated microrobots, are presented in [24] and [25], respectively. Robotic assisted minimally invasive surgery is ilustrated in [26]. Design of a novel haptic joystick for the teleoperation of continuum-mechanism-based medical robots is presented in [27]. In this reference a novel mechanism with series of coupled gears, that aims for the control of continuum robots for medical applications is pointed out.

Positioning control of robotic manipulators subject to excitation from non-ideal sources is discused in [28]. Further, tractor-robot cooperation is ilustrated in [29]. Indor using positionings systems of mobile robots is presented in [30].

In this article, the problem of the nonlinear control of the robot motion is solved by using the well known concept of the external linearization. Recent trends in robot learning and evolution for swarm robotics is presented in [31]. Multi robot task scheduling for consensus based fault resilient intelligent behaviour in smart factories is discussed in [32]. A new single leg lower limb rehabilitation robot with design, analysis and experimental evolution is evaluted in [33]. Some problems in connection with IoT based vision and remote control of a compact mobile robots is presented in [34]. It is also important to now how the portable surveillance robots can be used in IoT apllication [35]. The recent trends in robot learnung and evolution for swarm robotics is presented in[36]. Further in this article it is introduced the new notion named as "the radial mass density" as the ratio of the mass and related gravitational radius. This is very important value, because the most of the physical items can be described by the radial mass density. There exist the maximal and the minimal radial mass densities. The maximal radial mass density for related mass is at the minimal radius. On the other hand, the minimal radial mass density is happened at the maximal radius. The maximal and minimal radial mass densities are constants for the all amounts of masses. The larger masses have the larger minimal and maximal radiuses. Of course, the smaller masses have the smaller minimal and maximal radiuses. Since the Planck's mass is not the smallest mass in the space-time, the Planck's length (radius) is not the smallest length (radius) in it. Here it is started by presentation of the dynamics and control of the robot motiion in general case. After that the dynamics and control of the autonomous robot motion in two-potential electromagnetic end gravitational radial mass density field is calculated. Finally, it is presented some other methods of applications of the maximal radial mass density theory to the robot control.

2. General Case of Dynamics of Autonomous Robot Motion in Radial Mass Density Field

The problem of the nonlinear control of autonomous robot motion here is discussed as the function of the maximal radial mass density vaslue. In order to simplifay the relared calculation, here it is started with he concept of the external linearization of the nonlinear control of the robot motion in the radial mass density field. In that case, in the closed regulation loop, one obtains the linear behavior of the hole-system. Thus, the problem of the robot position control in the radial mass density field can be started by the calculation of the control of the error vector, $e(t)$. This vector is a function of the radial mass density, ρ_r, and can be presented by the relations:

$$e = X_w - X,\ \ \frac{d^2 e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c} N F_I\right],\ \ r_w(t) = \frac{d^2 X_w}{dt^2} = \frac{1/n}{\rho_{r\,max} r_{min}}\left[F_{pw} + F_{tw} + \frac{1}{c} N F_{Iw}\right]. \tag{1}$$

Here $n=1,2,..,n_{max}$ and $n_{max} = \rho_{r\,max} / \rho_{r\,min}$. Thus in (1) the subscript w denotes the desired robot motion, while the variables without this subscript present the real autonomous robot motion. Further, F_p is a potential force, F_t is a time - variation force, F_i is interaction force and N is the related connection parameter. At the same time the relations (1) also describes the canonical differential equations of autonomous robot motion in the combination of the electromagnetic and gravitational

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fields. Vector $r_w(t)$ is the desired (nominal) acceleration of the autonomous robot motion in the radial mass density field. Now following the idea of the external linearization, one can introduce the following substitution:

$$u(t)=\frac{d^2e}{dt^2}=r_w(t)-\frac{n}{\rho_{r\,max}r_{min}}\left[F_p+F_t+\frac{1}{c}NF_I\right],\quad u(t)=(u_x(t)\,u_y(t)\,u_z(t))^T. \tag{2}$$

Here $u(t)$ is the internal control vector of autonomous robot motion in radial mass density field. Further, applying the phase state-space variables, $(z_1\,z_2\,z_3)^T$, we obtain from (1) the related state-space model of the robot motion in the radial mass density field:

$$e=(e_x\,e_y\,e_z)^T=Z_I=(z_1\,z_2\,z_3)^T,\ \frac{de}{dt}=(\frac{de_x}{dt}\,\frac{de_y}{dt}\,\frac{de_z}{dt})^T=Z_{II}=(z_4\,z_5\,z_6)^T, \tag{3}$$

and

$$dZ/dt=AZ(t)+Bu(t),\quad A=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix},\ B=\begin{bmatrix}0\\ I\end{bmatrix},\ I=diag[1,\,1,\,1]. \tag{4}$$

In (4), parameters A and B are constant matrices with dimension (6x6) and (6x3), respectively. Here, it is supposed that the disturbances in state-space model of the robot motion in the radial mass density field (3) and (4) are of the initial condition types. In order to eliminate the control error of the autonomous robot motion in the radial mass density field, caused by the disturbances, one can introduce the following internal control:

$$u(t)=-KZ\,,\qquad F_p=\frac{1}{n}\rho_{r\,max}r_{min}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_t+\frac{N}{c}F_I\right]. \tag{5}$$

Here, K is a state space controller, Z is control eror, F_p is the potential force, F_t is time variable force, F_I is an interaction force, N is a constant and c is the speed of the light in vacuum. Including the internal control relations (3) and (4) into (5), one obtains the related equation of the potential force as function of radial mass density value in the linear form:

$$F_p=\frac{1}{n}\rho_{r\,max}r_{min}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_t+\frac{N}{c}F_I\right]. \tag{6}$$

Now starting from the previous relations one can generate the new equations of the potential forces F_p :

$$F_{p_x}=-\left(\frac{\partial\Sigma U_j}{\partial x}+\frac{\partial U_c}{\partial x}\right),\quad F_{p_y}=-\left(\frac{\partial\Sigma U_j}{\partial y}+\frac{\partial U_c}{\partial y}\right),\quad F_{p_z}=-\left(\frac{\partial\Sigma U_j}{\partial z}+\frac{\partial U_c}{\partial z}\right). \tag{7}$$

It is followed by the inclusion of the control potential force, F_{cp} , that is derived from the artificial control field with potential control energy U_c . After inclusion of the relation (7) into the relation (6), one obtains the nonlinear control of the autonomous robot (micro – nano robot) motion in the multi-potential field as the function of the maximal radial mass density $\rho_{r\,max}$:

$$F_{cp}=\frac{1}{n}\rho_{r\,max}r_{min}\left[r(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_{dp}+\mathrm{F}_t+\frac{N}{c}F_I\right]. \tag{8}$$

Now, using (8), the control of the nonlinear system is solved by employing the concept of the external linearization in the radial mass density field as the function of the radial mass density value.

3. Dynamics of Autonomous Robot Motion in Two-Potential Electromagnetic end Gravitational Radial Mass Density Field

The general approach to control of the dynamics of the autonomous robot motion in radial mass density field for more potential fields, given in (8), can also be applied to the two-potential electromagnetic and gravitational field. In this sense, let an autonomous robot be an electric charged particle with charge q and rest mass m_0 that is moving with a non-relativistic velocity ($v \ll c$) in combined electromagnetic and gravitational potential fields. Further it is also assumed that the gravitational field is produced by the spherically symmetric non-charged body with mass M. In that case, the total potential energy U of the autonomous robot motion in the two potential radial mass

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density fields is described by the relation:

$$U = qV_e + m_0 V_g = qV_e + m_0\left(-\frac{GM}{r}\right),\ \rho_{r\,max} = \frac{m_0}{r_{min}},\ U = qV_e + \frac{1}{n}\rho_{r\,max} r_{min}\left(-\frac{GM}{r}\right). \quad (9)$$

Here V_e and V_g are the related scalar potentials of the electromagnetic and gravitational radial mass density fields, respectively. Parameter G is the gravitational constant and r is the radius as distance between the autonomous robot and center of the mass M and $n=1,2,..,n_{max}$, $n_{max} = \rho_{r\,max} / \rho_{r\,min}$. Now applying (9) and using the notations, (E_e,H_e) for an electromagnetic field and (E_g,H_g) for the gravitational field, one can generate the vector equation as the explicit functions of the Lorentz forces:

$$\frac{1}{n}\rho_{r\,max} r_{min}\frac{d^2X}{dt^2} = q\left(E_e + \frac{1}{c}v\times H_e\right) + \frac{1}{n}\rho_{r\,max} r_{min}\left(E_g + \frac{1}{c}v\times H_g\right). \quad (10)$$

The parameters E_e, E_g, H_e and H_g are vectors described by the relations:

$$E_e = \begin{bmatrix} E_{e_x} \\ E_{e_y} \\ E_{e_z} \end{bmatrix},\quad E_g = \begin{bmatrix} E_{g_x} \\ E_{g_y} \\ E_{g_z} \end{bmatrix},\quad H_e = \begin{bmatrix} H_{e_x} \\ H_{e_y} \\ H_{e_z} \end{bmatrix},\quad H_g = \begin{bmatrix} H_{g_x} \\ H_{g_y} \\ H_{g_z} \end{bmatrix}. \quad (11)$$

In this example, an autonomous robot is a particle with charge q and rest mass m_0 and therefore, the autonomous robot interacts with both electromagnetic and gravitational radial mass density fields. In that sence the relations (10) and (11) describe the dynamic of the autonomous robot motion in two-potential electromagnetic and gravitational field. The components of the vector E_e and E_g can be calculated by using the following equations:

$$E_{e_x} = -\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t},\quad E_{g_x} = -\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t},\quad E_{e_y} = -\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t}, \quad (12)$$

and

$$E_{g_y} = -\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t},\quad E_{e_z} = -\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t},\quad E_{g_z} = -\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}. \quad (13)$$

The components of vectors A_e, A_g, H_e and H_g in (12) and (13) are given by the relations:

$$A_{e_i} = \left(\frac{v_i V_e}{c}\right),\quad A_{g_i} = \left(\frac{v_i V_g}{c}\right),\quad i = x,y,z,\ H_{e_x} = \frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z},\quad H_{g_x} = \frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}, \quad = \quad (14)$$

and

$$H_{e_y} = \frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x},\ H_{g_y} = \frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x},\ H_{e_z} = \frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y},\ H_{g_z} = \frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}. \quad (15)$$

Applying (14) and (15) to the canonical differential equations of the autonomous robot motion in the two-potential radial mass density field, and using $m_0 = \rho_{r\,max} r_{min}$ one obtains the control error model of the autonomous robot motion as a function of the maximal radial mass density:

$$\ddot{e}(t) = r_w(t) - \frac{q}{\frac{1}{n}\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}\mathrm{v}\times H_e\right) - \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right), \quad (16)$$

and

$$r_w(t) = \frac{q}{\frac{1}{n}\rho_{r\,max} r_{min}}\left(E_{e_w} + \frac{1}{c}\mathrm{v}_w\times H_{e_w}\right) - \left(E_{g_w} + \frac{1}{c}\mathrm{v}_w\times H_{g_w}\right). \quad (17)$$

In (17) $r_w(t)$ is the vector of desired acceleration of the autonomous robot motion. The subscript w denotes desired values of the related variables. The next step is the application of the concept of the external linearization in order to transform the equation in (16) into the new relation:

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$$u(t)=r_w(t)-\frac{q}{\frac{1}{n}\rho_{r\,max}r_{min}}\left(E_e+\frac{1}{c}v\times H_e\right)-\left(E_g+\frac{1}{c}v\times H_g\right). \tag{18}$$

Here $u(t)$ is the internal control vector and $n=1,2,..n_{max}$ is the number of the robot steps from the minimal to the maximal rediuses in radial mass density field. From (17) and (18), one obtains the related equivalent of the linear control error model of the autonomous robot motion in the combined electromagnetic and gravitational radial mass density field, given by (14) and (15). The phase state-space variables of the system (2) are determined by applying (3). The related state-space model of an autonomous robot motion is given in matrix form (6). In order to eliminate the control error of an autonomous robot motion, caused by disturbances of the initial condition types, one can introduce internal control in the form of (7). Applying (7) to (18), one obtains the new relation as the function of the maximal radial mass density in the form:

$$E_e=\frac{\rho_{r\,max}r_{min}}{nq}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left(\frac{1}{c}\mathrm{v}\times H_e\right)-\frac{\rho_{r\,max}r_{min}}{nq}\left(E_g+\frac{1}{c}\mathrm{v}\times H_g\right). \tag{19}$$

Now, let the electric field E_e is consisting of the two electric components $E_e = E_{de} + E_{ce}$. Here E_{de} is a disturbance electric field that is caused by the influence of a two-potential field on the motion of an autonomous robot in radial mass density field. The component E_{ce} is an artificial electric control field that should control autonomous robot motion in the two potential field. Including E_e into (19), one obtains the nonlinear electric control of the autonomous robot motion in the two-potential radial mass density field as the function of the maximal radial mass density $\rho_{r\,max}$:

$$E_{ce}=\frac{\rho_{r\,max}r_{min}}{nq}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left(E_{de}+\frac{1}{c}\mathrm{v}\times H_e\right)-\frac{\rho_{r\,max}r_{min}}{nq}\left(E_g+\frac{1}{c}\mathrm{v}\times H_g\right). \tag{20}$$

Taking into account the relation (10), the canonical differential equations of the autonomous robot motion, in the two-potential radial mass density field, can be rewritten as a function of the maximal radial mass density:

$$\frac{d^2X}{dt^2}=\frac{nq}{\rho_{r\,max}r_{min}}\left(E_{de}+E_{ce}+\frac{1}{c}\mathrm{v}\times H_e\right)+\left(E_g+\frac{1}{c}\mathrm{v}\times H_g\right). \tag{21}$$

Applying the nonlinear control E_{ce} from (20) to the nonlinear dynamical model of the autonomous robot motion (21), one obtains the closed-loop system of the linear form:

$$\frac{d^2X}{dt^2}=r_w(t)+K_IZ_I+K_{II}Z_{II}\,. \tag{22}$$

Thus, the equation (20) is the nonlinear control, which in the closed loop with a nonlinear canonical differential equations of autonomous robot motion (21), results in linear behavior of the hole system (22). On that way the problem of control of the autonomous robot motion in the combination of an electromagnetic and gravitational radial mass density field, has been solved by employing the so-called concept of the external linearization. This is very important for application of micro and nano robots in the drag delivery across the human body. 4. The other methods of applications of the maximal radial mass density theory to the robot control Global positioning of robot manipulators with mixed revolute and prismatic joints is presented in [19]. In this section it is illustrated how one can apply the maximal radial mass density theory to the mentioned class of robots. In that sense the dynamic model of the robot with n-link rigid body can be described as the function of the maximal radial mass density:

$$M(q)\frac{d^2q}{dt^2}+C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)=U,\ M(q)=\rho_{r\,max}r_{min},$$
$$\rho_{r\,max}r_{min}(q)\frac{d^2q}{dt^2}+C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)=U. \tag{23}$$

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Here q is ($nx1$) vector of robot joints coordinates, dq/dt is the related vector of joints velocities, U is a vector of applied joint torques and forces, $M(q)$ is (nxn) inertia matrix, $C(q,dq/dt)dq/dt$ is ($nx1$) vector of centrifugal and Coriolis torques. Further $q(q)$ is the vector of gravitational torques and forces and $\rho_{r\,max}$ is the maximal radial mass density at the minimal radius. If the robot (23) has the closed loop with the nonlinear *PID* controler described by the relation:

$$U(t) = -(K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}). \tag{24}$$

than the closed loop sistem of the relations (23) and (24) is resulted in the following form that is the function of the maximal radial mass density:

$$\rho_{r\,max} r_{min}(q) \frac{d^2q}{dt^2} + C(q, \frac{dq}{dt}) \frac{dq}{dt} + q(q) = -(K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}). \tag{25}$$

The relation (25) can be applied in the region n =1,2,..., $\rho_{r\,max} / \rho_{r\,min}$. Now one can use the related ion of the robot (25) can be calculated by using the relation:

$$\frac{d^2q}{dt^2} = -\frac{n}{\rho_{r\,max} r_{min}(q)} (C(q, \frac{dq}{dt}) \frac{dq}{dt} + q(q) + K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}). \tag{26}$$

Thus, using the relation (26) it is possible to control of the robot's acceleration by changing of the numerical parameter n. The dynamics of the robot motion can also be described as the function of the alpha field parameters derived in the Relativistic Alpha Field Theory (RAFT) [7]. In this theory one can start with the potential energy of the robot (particle) in electromagnetic field, U_e, and gravitational field, U_g. Now let q, m, V_e, and V_g are robot (particle) charge, mass, electrical potential and gravitational potential, respectively. With G is denoted the gravitational constant, M is mass of the gravitational field and c is the speed of the light in vacuum. The potential energy of the robot in combination of the electromagnetic and gravitational fields is given by the relations:

$$U = U_e + U_g = \pm q V_e - \frac{mGM}{r}, \quad \frac{U}{mc^2} = \pm \frac{qV_e}{mc^2} - \frac{GM}{rc^2}, \tag{27}$$

and $$m = \rho_{r\,max} r_{min}, \quad \frac{U}{\rho_{r\,max} r_{min} c^2} = \pm \frac{qV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}, \tag{28}$$

The relation (28) can also be described as the function of the parameter n :

$$\frac{nU}{\rho_{r\,max} r_{min} c^2} = \pm \frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}, \quad n = 1 ..., n_{max}. \tag{29}$$

If one wants to use RAF theory in the robotics than it requires the introduction of the related alpha field parameters. The solutions of the field parameters for an electron in the two-potential electromagnetic and gravitational field are given as folows:

$$\alpha_1 = 1 + i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \quad \alpha'_1 = 1 - i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \tag{30}$$

and

$$\alpha_2 = 1 - i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \quad \alpha'_2 = 1 + i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \tag{31}$$

and

$$\alpha_3 = -1 + i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \quad \alpha'_3 = -1 - i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \tag{32}$$

and

$$\alpha_4 = -1 - i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \quad \alpha'_4 = -1 + i\sqrt{\frac{nq\,V_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}. \tag{33}$$

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Now one can introduce the generalized Lorentz-Einstein parameters, for an electron in a two-potential electromagnetic and gravitational field. These parameters are described by the following equations:

$$H_{1,2}=\left[1-\frac{v^2}{c^2+\dfrac{nq\,V_e}{\rho_{r\,max}r_{min}}-\dfrac{GM}{r}}\pm\frac{2\,i\sqrt{\dfrac{nq\,V_e}{\rho_{r\,max}r_{min}c^2}-\dfrac{GM}{rc^2}}\;c\cdot v}{c^2+\dfrac{nq\,V_e}{\rho_{r\,max}r_{min}}-\dfrac{GM}{r}}\right]^{-1/2},\quad H_{3,4}=H_{1,2}.$$

(34)

From the relation (34) one can see that the solutions $H_{3,4}$ are symmetric to the solutions of the parameters $H_{1,2}$. The previous presented two potrential field can be generalized by the application of the multi potential field as the function of the field parameters α and α'. Now, for derivation of a four-potential vector A of the related potential field one can recall the general Hamilton function, H, for the weak potential fields:

$$H=-c\zeta_1\left(p_x-\frac{U_p v_x}{c^2}\right)-c\zeta_2\left(p_y-\frac{U_p v_y}{c^2}\right)-c\zeta_3\left(p_z-\frac{U_p v_z}{c^2}\right)-\frac{\beta\,\rho_{r\,max}r_{min}}{n}c^2+U_p. \quad (35)$$

Here U_p is a potential energy, (p_x, p_y, p_z) is a three-momentum vector, (v_x, v_y, v_z) is a three-velocity vector and ζ_1, ζ_2, ζ_3 and β are the well known Dirac's matrices. If an electron is moving with a constant velocity v << c in an electromagnetic field with a scalar potential V, then one should use the following relations

$$\frac{U_p v_x}{c^2}=\frac{q}{c}\frac{V\,v_x}{c}=\frac{q}{c}A_x\,,\quad \frac{U_p v_y}{c^2}=\frac{q}{c}\frac{V\,v_y}{c}=\frac{q}{c}A_y\,,\quad \frac{U_p v_z}{c^2}=\frac{q}{c}\frac{V\,v_z}{c}=\frac{q}{c}A_z\,. \quad (36)$$

Here q is an electric charge of an electron and (A_x , A_y , A_z) is a three-potential vector of the electromagnetic field. Including (36) into the equation (35), one obtains the well known Hamilton function for Dirac's electron in an electromagnetic field:

$$H=-c\zeta_1\left(P_x-\frac{q}{c}A_x\right)-c\zeta_2\left(P_y-\frac{q}{c}A_y\right)-c\zeta_3\left(P_z-\frac{q}{c}A_z\right)-\frac{\beta\,\rho_{r\,max}r_{min}}{n}c^2+qV. \quad (37)$$

On the other hand, if a robot (particle) is moving with constant velocity v << c in a gravitational field, then, according to the previous procedure, one should use the following relations:

$$U_p=-\frac{\rho_{r\,max}r_{min}GM}{nr}=\frac{\rho_{r\,max}r_{min}}{n}V_g\,,\quad \frac{U_p v_x}{c^2}=\frac{\rho_{r\,max}r_{min}}{nc}\frac{V_g v_x}{c}=\frac{\rho_{r\,max}r_{min}}{nc}A_{g_x}\,, \quad (38)$$

and

$$\frac{U_p v_y}{c^2}=\frac{\rho_{r\,max}r_{min}}{nc}\frac{V_g v_y}{c}=\frac{\rho_{r\,max}r_{min}}{nc}A_{g_y}\,,\quad \frac{U_p v_z}{c^2}=\frac{\rho_{r\,max}r_{min}}{nc}\frac{V_g v_z}{c}=\frac{\rho_{r\,max}r_{min}}{nc}A_{g_z}\,. \quad (39)$$

In the relations (38) and (39) G is a gravitational constant, M is a gravitational mass, V_g is a gravitational scalar potential and (A_{gx} , A_{gy} , A_{gz}) is a three-potential vector of the gravitational field. Including (39) into the equation (37), one obtains the Hamilton function H_g for the particle in a gravitational field:

$$\begin{aligned}H_g=&-c\zeta_1\left(P_x-\frac{\rho_{r\,max}r_{min}}{nc}A_{g_x}\right)-c\zeta_2\left(P_y-\frac{\rho_{r\,max}r_{min}}{nc}A_{g_y}\right)\\&-c\zeta_3\left(P_z-\frac{\rho_{r\,max}r_{min}}{nc}A_{g_z}\right)-\beta\frac{\rho_{r\,max}r_{min}}{n}c^2+\frac{\rho_{r\,max}r_{min}}{n}V_g.\end{aligned} \quad (40)$$

Generally, if the robot velocity v in a potential field is constant, then the four-potential vector A can be derived as a function of the field parameters α and α' :

$$A=\left[A^0,A^1,A^2,A^3\right],\quad A^0=\frac{c^2}{\eta}\left(\alpha\alpha'-1\right),\quad A^1=A_x=A^0\frac{v_x}{c}, \quad (41)$$

and

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$$A^2 = A_y = A^0 \frac{v_y}{c}, \quad A^3 = A_z = A^0 \frac{v_z}{c}. \tag{42}$$

Now, the components of the field tensor F_{ij} of the potential field can be calculated by using relations (41) and (42) and the well known procedure:

$$F_{ij} = \frac{\partial A^j}{\partial x^i} - \frac{\partial A^i}{\partial x^j}, \quad i,j = 0,1,2,3, \quad X = \left[x^0, x^1, x^2, x^3\right] = \left[ct, x, y, z\right]. \tag{43}$$

As the result of this calculation one obtains the anti-symmetric tensor F_{ij} of the potential field in the following form:

$$F_{ij} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ F_{10} & 0 & F_{12} & F_{13} \\ F_{20} & F_{21} & 0 & F_{23} \\ F_{30} & F_{31} & F_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ -F_{01} & 0 & F_{12} & -F_{13} \\ -F_{02} & -F_{12} & 0 & F_{23} \\ -F_{03} & F_{13} & -F_{23} & 0 \end{bmatrix}. \tag{44}$$

This tensor can be employed for derivation of the related Maxwell's like equations in a vacuum. Following the previous consideration one can introduce the normalized scalar potential A_m^0 of a multi-potential field in the dimension of specific potential energy:

$$A_m^0 = \Sigma\left(\eta_j A_j^0\right) = \left(\alpha\alpha'_j - 1\right)c^2, \qquad j = 1,2,..,n, \tag{45}$$

where term $\alpha\alpha'$ has to be calculated by employing the relations (30) to (33):

$$\left(\alpha\alpha'\right) = \left(1 + \frac{\Sigma U_{p_j}}{\frac{\rho_{r\,max} r_{min}}{n} c^2}\right), \qquad j = 1,2,..,n. \tag{46}$$

The relations (45) and (46) tell us what the normalized scalar potential A_m^0 really is:

$$A_m^0 = \frac{\Sigma U_{p_j}}{\frac{\rho_{r\,max} r_{min}}{n}}, \qquad j = 1,2,..,n. \tag{47}$$

In recent decades, it has been created a wide range of robotic systems mostly inspired by the animals. In that sense engineers have created a wide range of robotic systems like a four - legged robots, snake robots, insect robots and fish robots [37]. The motion of the fish robot is controlled by the central pattern generator (CPG) system.

5. The Numerical Calculation of the Robot Motion in the Radial Mass Density Field

Gravitational field with the mass M_g has the maximal and minimal gravitational radial mass densities given in [12]:

$$\rho_{rm\,\max} = \frac{M_g}{r_{\min}} = \frac{(1+\kappa)c^2}{G} = 2.693182 \cdot 10^{27}\, kg/m, \tag{48}$$

and

$$\rho_{rm\,\min} = \frac{M_g}{r_{\max}} = \frac{(1-\kappa)c^2}{G} = 0.888779 \cdot 10^{23}\, kg/m. \tag{49}$$

The numerical values in (48) and (49) are constant and are valued for all amounts of the gravitational mases M_g. In the relations (48) and (49) the parameter κ is the energy conservation constant that has been calculated in the reference [12] by using Planck mass and Planck length:

$$L_p = \frac{2GM_p}{(1+\kappa)c^2}, \quad \kappa = \frac{2GM_p}{L_p c^2} - 1 = 0.99993392118. \tag{50}$$

Using the combination of the equations (21) and (23) one obtains the canonical differential equations of the autonomous robot motion at the minimal gravitational radius by the maximal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{2.693182\cdot 10^{27}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right) m/kg. \tag{51}$$

On the other hand, using the combination of (21) and (24) one obtains the canonical differential equations of the autonomous robot motion at the maximal gravitational radius but with the minimal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{0.888779\cdot 10^{23}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right) m/kg. \tag{52}$$

Now, one can calculate the ratio between the maximal and minimal radial mass densities:

$$n_{\max} = \frac{\rho_{rm\max}}{\rho_{rm\min}} = \frac{2.693182\cdot 10^{27}}{0.888779\cdot 10^{23}} = 3.030204\cdot 10^4. \tag{53}$$

This ratio is the constant and is valued for the all amounts of the gravitational masses. Following the previous equations one can calculate of the maximal steps, n_{step} , between maximal and minimal radiuses in gravitational field. For the calculation of the precise motion of the autonomous robots in the gravitational radial direction one can introduce the variable step of the robot motion, n_{var}. In that case it is possible to select the scale of the desirable step of the robot motion in the radial mass density field. Let the variable step of the robot motion in the radial direction is given by the amount n_{var} = 100. In that case the number of the robot steps from the minimal to the maximal radiuses has the value:

$$n_{step} = \frac{n_{\max}}{n_{\mathrm{var}}} = \frac{303.0204\cdot 10^2}{100} = 303.0204 \tag{54}$$

In this calculation a robot neds cca 303 steps of the motion from the minimal to the maximal radiuses in the radial direction. In the case that the robot motion is not in the radial direction then one should use the related projection of the radial trasjectory to the desired robot trajectory. Now, if one wants to introduce $U_{g\ max}$ and $U_{g\ min}$ as the potential energies at the minimal and maximal gravitational radiuses, respectively, then it is possible to calculate the mini- mal and the maximal radial lengths, L_{gmin} *and* L_{gmax} , respectively, by using the relations:

$$M_g = \rho_{rm\max} r_{\min}, \quad L_{g\min} = \frac{2m_0 G \rho_{rm\max} r_{\min}}{U_{g\min}} = \frac{2m_0 G \rho_{rm\max} r_{p\min}}{(1+\kappa)c^2}, \tag{55}$$

and

$$L_{g\max} = \frac{2m_0 G \rho_{rm\min} r_{\max}}{U_{g\max}} = \frac{2m_0 G \rho_{rm\min} r_{p\min}}{(1-\kappa)c^2}, \tag{56}$$

From the relations (55) and (56) one can see how the potential energies in a gravitational field can influence to the autonomous robot motion in the two-potential radial mass density field.

6. Conclusions

This article is based on the new Relativistic Radial Density Theory (*RRDT*) that has been applied to the control of the robot motion in potential fields. This is calculated as the radial motion from the minimal to the maximal gravitational radiuses and vice-verse. In that sense the Planck's and gravitational parameters are described as the functions of the radial mass density values. It is shown that the maximal radial mass density occurs at the minimal gravitational radius of the related mass. On the other hand, the minimal radial mass density is happened at the maximal radius of the related mass. Farther, the both maximal and the minimal radial mass densities can also be described as functions of the energy conservation constant κ. In that sense, the gravitational length, time, energy, and temperature can be represented as functions of Planck length, time, energy, and temperature,

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respectively. In some of the examples it is necessary to transform the radial motion into the rectangular coordinates by using related projection. Finally, it is shown that the rate of the precise motion of the autonomous robots in the gravitational radial direction can be controlled by the introduction of the variable step of the robot motion.

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