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Article

RLC Electrical Modelling of Black Hole and Early Universe. Generalization of Boltzmann's Constant in Curved Space-Time

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Abstract: Here we mathematically model black holes and the early universe following dynamics similar to RLC electrical models, focusing on their similarities at the singularity. We use this mathematical modelling to hypothesize the evolution of an expanding universe as the result of a black hole collapse followed by its evaporation. Our model consists of several steps defined by: (1) the formation of a black hole following general relativity equations; (2) growth of the black hole modelled as a resistance-capacitance-like electrical circuit; (3) expansion of space-time following the disintegration of the black hole, modelled by RLC-like dynamics. In updating this article, version 2, we will expand by presenting three additional examples related to the theory of the generalization Boltzmann's constant in curved space-time and the theory of general and especial relativity. We will calculate the critical mass to produce a black hole in the LHC, the existence of a high temperature Bose-Einstein condensate and finally we will demonstrate the existence of a tangential force to the repulsion force in the disintegration of subatomic particles.

Keywords: RLC electrical model; RC electrical model; cosmology; astronomy; astrophysics; background radiation; Hubble's law; Boltzmann's constant; dark energy; dark matter; black hole; Big Bang; cosmic inflation; early universe; quantum gravity; CERN; LHC; Fermilab; general relativity; particle physics; condensed matter physics

1. RC ELECTRICAL MODEL FOR A BLACK HOLE

If considering electric charge and mass as fundamental properties of matter.

From the point of view of electric charge, we know that a capacitor stores electrical energy and we can represent it as an RC circuit.

Analogously, from the mass point of view, we can consider a black hole as a capacitor that stores gravitational potential energy.

Continuing with the analogy, the space-time that surrounds a black hole can be represented as the inductance L.

from this simple conceptual idea was born RLC electrical modelling of black hole and early universe.

RC electrical model for a Black Hole:

Here we put forward the hypothesis of a black hole growth in analogy to an RC electrical circuit that grows according to a constant Tau being defined as:

$$\tau = RC \quad (1)$$

First, we will consider the total mass of a black hole to consist of the sum of baryonic mass and dark matter mass (equation 2), considering dark matter as an imaginary number.

$$M = m - i\delta \quad (2)$$

where M is the total mass of a black hole, m is the baryonic mass; δ corresponds to dark matter and I is the irrational number $\sqrt{-1}$. This equation is in analogy to impedance of an RC circuit.

$$Z = R - iX_c \quad (3)$$

where z represents impedance; R represents resistance and X_c represents reactance.

If proper accelerations for the masses are introduced in equation (2) we obtain the following:

$$F = f - i\varphi \quad (4)$$

where F is the total force, f is the force associated to baryonic mass, and $i\varphi$ is the force associated to dark mass. In analogy to a phasor diagram for an RC circuit, in which the reactance phasor lags the resistance phasor R by $\frac{\pi}{2}$, we can represent the two forces associated to barionic matter and dark matter as two orthogonal vectors (Figure 1).

Vector diagram of forces in a black hole for circular motion with constant acceleration:

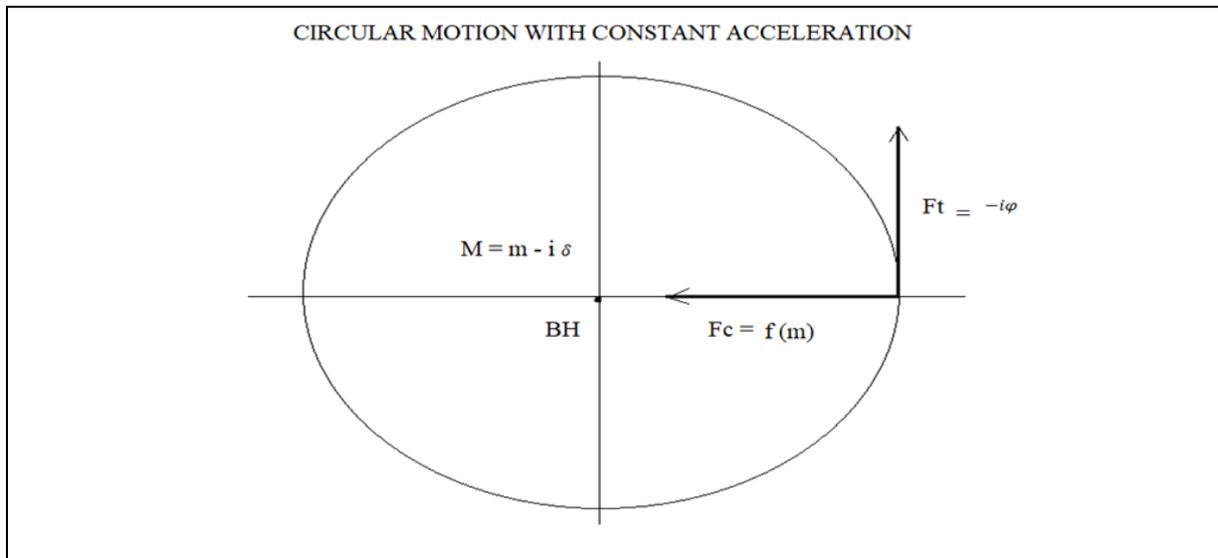


Figure 1. Vector representation of the forces in a black hole. $F_c = f$, represents the force towards the interior of the black hole generated by the mass m and $F_t = -i\varphi$, is a tangential force that retards F_c by 90 degrees, generated by the mass δ .

taking into account Newton's equation of universal gravitation:

$$F = - (G M_1 M_2) / r^2$$

The sign (-) of the equation means that the force F_c is at 180 degrees with respect to the resistance R and the force F_t is also at 180 degrees from the reactance X_c .

It is important to make clear the physical interpretation of the imaginary mass, it is simply telling us that the force F_t due to the mass δ lag the force f_c by 90 degrees, that lag is represented by the imaginary number i . Later we will determine that the mass δ , is the result of $v > c$ inside a black hole.

Where v is the speed of a massless particle and c is the speed of light in a vacuum.

Figure 1 is represented for a circular motion with constant acceleration simply because the tangential velocity of a particle is proportional to the radius from the centre of the black hole multiplied by the average angular frequency.

$$V_t = r \omega$$

The contribution of (F_t, V_t) is what makes the speed of the galaxy remain constant as the radius of the galaxy grows.

Where V_t represents the tangential velocity of a galaxy, r is the radius from the galaxy, and ω is the average angular velocity of the rotation of the galaxy.

Circular motion with constant acceleration tells us that the mass input into a black hole is negligible with respect to the black hole's own mass.

The growth of a black hole according to the tau constant is an intrinsic property of a black hole and is independent of the amount of matter that enters a black hole.

To calculate the total energy associated to the black hole, we can introduce its total mass (equation 2) into:

$$E^2 = c^2 p^2 + c^4 M^2 \quad (5)$$

where E is energy; c represents the speed of light and m represents the mass. This lead to:

$$E^2 = c^2p^2 + (m^2 - \delta^2)c^4 - 2im\delta c^4. \quad (6)$$

We can assume that during the big bang inflation phase baryonic matter was overrepresented compared to dark matter together with an infinitesimal momentum, which would give us from equation (6) the following:

$$E^2 = -\delta^2 c^4 ; E = (+/-)\delta c^2 i \quad (7)$$

As expected, this result corresponds to the total energy of the universe at the big bang if we consider it to be made of dark matter represented as a reactance in an RC circuit.

The positive value of E is determined by matter, there is no antimatter inside a black hole.

If we consider charge as a fundamental property of matter, $E = (+)\delta c^2 i$, represents the amount of relativistic dark matter inside the black hole at the time of disintegration.

If we consider mass as a fundamental property of matter, $E = (-)\delta c^2$, represents the amount of relativistic dark matter inside a black hole, which exerts a repulsive gravitational force at the moment of disintegration. This repulsive gravitational force is what generates the dark energy after the Big Bang.

At time T_0 , when the black hole disintegrates and the Big Bang occurs, roughly all matter was dark matter.

We could also consider a universe at infinity proper time in which baryonic matter is dominant over dark matter, which would transform equation (6) back into equation (5) but with baryonic matter.

$$E^2 = c^2p^2 + m^2c^4. \quad (8)$$

2. RLC ELECTRICAL MODEL OF THE UNIVERSE

We will analyse the Dirac delta function $\delta(t)$.

$$\delta(t) = \{\infty, t = 0\} \wedge \{0, t \neq 0\}$$

If we perform the Fourier transform of the function $\delta(t)$ and analyse the amplitude spectrum, we observe that the frequency content is infinite.

If we perform the Fourier transform of the function $\delta(t)$ and analyse the phase spectrum, we observe that the phase spectrum is zero for all frequencies.

We say that it is a non-causal zero phase system.

The most important thing to emphasize in this system is that an infinite impulse has an infinite frequency content.

When we work in seismic prospecting looking for gas or oil, using explosives, the detonations produce an energy peak that generates a frequency spectrum that propagates in the layers of the earth. The energy produced in the detonation is not instantly transferred to the ground, a time delay occurs, it is said to be causal system of minimum phase.

In analogy, we are going to suppose that the Big Bang also behaves like a causal system of minimum phase.

Here we put forward the hypothesis that the big bang is the convolution of the energy released by disintegration of the black hole with the space-time surrounding the black hole, being defined as:

$$(m - i\delta) * \epsilon \quad (9)$$

where $m - i\delta$, is the total mass of a black hole, ϵ is the space-time surrounding the black hole and $*$ is the convolution symbol.

Equation (9) can be simplified and considered analogous to an RLC circuit.

Where RC represents a black hole and L represents the space-time around a black hole

$$RC = m - i\delta \quad (10)$$

$$L = \epsilon \quad (11)$$

the resolution of the quadratic equation of the RLC circuit will determine how space-time will expand after the Big Bang and the bandwidth of the equation will give us the spectrum of gravitational waves that originated during the Big Bang.

3. COSMIC INFLATION

From the following equation:

$$ds^2 = - \left(1 - \left(\frac{2MG}{Rc^2} \right) \right) c^2 dt^2 + \left(1 / \left(1 - \frac{2MG}{Rc^2} \right) \right) dR^2 + R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2 \quad (12)$$

We will analyse the Schwarzschild solution for a punctual object in which mass and gravity are introduced.

$$R_s = 2GM / c^2, \text{ is the Schwarzschild's radius.} \quad (13)$$

where M is the mass of a black hole, c is the speed of light, and G is the gravitational constant.

if we consider $d\theta = 0$; and $d\phi = 0$; that is, we move in the direction of dR. (14)

$$R = R_s, ds = 0, \text{ let's analyse this specific situation.} \quad (15)$$

Replacing the conditions given in (13), (14) and (15) in equation (12), we have:

$$(dR / dt)^2 = v^2 = c^2 (1 - (2MG/Rc^2))^2$$

$$R = R_s, v = 0; ds^2 = 0; R_s \text{ is the Schwarzschild's radius.} \quad (16)$$

$$R > R_s, v < c; ds < 0, \text{ time type trajectory.} \quad (17)$$

$$R < R_s, v > c; ds > 0, \text{ space type trajectory.} \quad (18)$$

Condition (18) is very important because to the extent that $R < R_s, v > c$ is fulfilled, it is precisely this speed difference that generates the imaginary mass in a black hole given by $-i\delta$.

Planck length equation:

$$L_p = \sqrt{(h G / c^3)} \quad (19)$$

where h is Planck's constant, G is the gravitational constant, and c is the speed of light.

If we consider condition (18) and equation (19), to the extent that $R < R_s$ and $v > c$, are fulfilled, we deduce that the Planck length decreases in value.

We define the following:

$L_{p\epsilon} = L_p = 1.616199 \cdot 10^{-35}$ m; electromagnetic Planck length.

L_{pG} = gravitational Planck length.

Always holds:

$$L_{pG} < L_{p\epsilon}$$

Here we put forward the hypothesis that cosmic inflation is the expansion of space-time that is given by L_{pG} that tends to reach its normal value $L_{p\epsilon}$ after a black hole disintegrates.

If we consider the Planck length $L_{p\epsilon}$, the minimum length of space-time, like a spring and due to the action of $v > c$ (300,000 km/s), this length decreases in values of L_{pG} , that is, $L_{pG} < L_{p\epsilon}$, allowing us to imagine the immense forces involved in compressing space-time of length $L_{p\epsilon}$ into smaller values of space-time L_{pG} . The immense energy stored and released in the spring of length L_{pG} , to recover its initial length $L_{p\epsilon}$, is the cause of the exponential expansion of space-time in the first moments of the Big Bang.

At time T_0 , when the black hole disintegrates and the Big Bang occurs, roughly all matter was dark matter, relativistic dark matter.

4. GENERALIZATION OF THE BOLTZMANN'S CONSTANT IN CURVED SPACE-TIME

Equation of state of an ideal gas as a function of the Boltzmann constant.

$$P V = N K B T \quad (20)$$

where, P is the absolute pressure, V is the volume, N is the number of particles, KB is Boltzmann's constant, and T is the absolute temperature.

Boltzmann's constant is defined for 1 mole of carbon 12 and corresponds to $6.0221 \cdot 10^{23}$ atoms.

Equation (20) applies for atoms, molecules and for normal conditions of pressure, volume and temperature.

We will analyse what happens with equation (20) when we work in a degenerate state of matter.

We will consider an ideal neutron star, only for neutrons.

We will analyse the condition:

$$(P V) / T = N K B = \text{constant} \quad (21)$$

This condition tells us that the number of particles remains constant, under normal conditions of pressure, volume and temperature

However, in an ideal neutron star, the smallest units of particles are neutrons and not atoms.

This leads us to suppose that number of neutrons would fit in the volume of a carbon 12 atom, this amount can be represented by the symbol D_n .

In an ideal neutron star,

$$(P V) / T = D_n N K B \quad (22)$$

where D_n represents the number of neutrons in a carbon 12 atom.

However, equation (22) is not constant, with respect to equation (21), the number of particles increased by a factor D_n , to make it constant again, I must divide it by the factor D_n .

$$(P V) / T = D_n N K B / D_n \quad (23)$$

$$(P V) / T = N' K B' = \text{constant} \quad (24)$$

where $N' = (D_n N)$, is the new number of particles if we take neutrons into account and not atoms as the fundamental unit.

where $K B' = (K B / D_n)$, is the new Boltzmann's constant if we take neutrons into account and not atoms as the fundamental unit.

We can say that equation (21) is equal to equation (24), equal to a constant

Generalizing, it is the state in which matter is found that will determine Boltzmann's constant.

A white dwarf star will have a Boltzmann's constant $K B_e$, a neutron star will have a Boltzmann's constant $K B_n$, and a black hole will have a Boltzmann's constant $K B_q$.

There is a Boltzmann's constant $K B$ that we all know for normal conditions of pressure, volume and temperature, for a flat space-time.

There is an effective Boltzmann's constant, which will depend on the state of matter, for curved space-time.

The theory of general relativity tells us that in the presence of mass or energy space-time curves but it does not tell us how to quantify the curvature of space-time.

Here we put forward the hypothesis that there is an effective Boltzmann' constant that depends on the state of matter and through the value that the Boltzmann' constant takes we can measure or quantify the curvature of space-time.

Quantifying space-time, considering the variable Boltzmann constant, is also quantizing gravitational waves and, as with the electromagnetic spectrum, we will determine that there is a spectrum of gravitational waves.

These analogies to represent the gravitational and electromagnetic wave equations are achieved thanks to the ADS/CFT correspondence.

We can determine the equations of electromagnetic and gravitational waves as shown below.

Electromagnetic wave spectrum for flat space-time:

$$E_\epsilon = h \times f_\epsilon$$

$$C_\epsilon = \lambda_\epsilon \times f_\epsilon$$

$$E_\epsilon = h \times C_\epsilon / \lambda_\epsilon$$

$$E_\epsilon = K_{B\epsilon} \times T_\epsilon$$

$$K_{B\epsilon} = 1.38 \cdot 10^{-23} \text{ J/K}$$

Gravitational wave spectrum for curved spacetime:

$$E_G = h \times f_G$$

$$C_G = \lambda_G \times f_G$$

$$E_G = h \times C_G / \lambda_G$$

$$E_G = K_{B_G} \times T_G$$

$$K_{B_G} = 1.38 \cdot 10^{-23} \text{ J/K to } 1.78 \cdot 10^{-43} \text{ J/K.}$$

where the subscript ϵ means electromagnetic and the subscript g means gravitational.

It can be seen that there is an electromagnetic and a gravitational frequency as well as an electromagnetic and a gravitational temperature.

The maximum curvature of space-time occurs for an effective Boltzmann's constant of $k_B = 1.78 \cdot 10^{-43}$ J/K, given by the ADS/CFT correspondence in which a black hole is equivalent to the plasma of quarks and gluons to calculate the viscosity of the plasma of quarks and gluons.

Once a black hole is formed and the maximum curvature of space-time is reached, as a black hole grows following the tau growth law analogous to an RC circuit, as v grows fulfilling the relationship $v > c$, it happens that the gravitational Planck length becomes less than the electromagnetic Planck length, it holds that $L_{pg} < L_{pe}$.

5. BLACK HOLE'S RADIATION

Equation (2) defines the mass of a black hole, as shown below:

$$M = m - i\delta \quad (25)$$

where M is the total mass of a black hole, m is the baryonic mass; δ corresponds to dark matter and i is the irrational number $\sqrt{-1}$.

Also, here we put forward the hypothesis of a black hole growth in analogy to an RC electrical circuit that grows according to a constant τ being defined as:

$$\tau = RC \quad (26)$$

If we consider the black hole's radiation that produces pairs of particles and antiparticles at the event horizon.

Here we put forward the hypothesis:

the HR (matter) particle, with frequency ω and energy $h\omega$, falls into the black hole and adds to m and δ increasing the mass of the black hole, that is, it adds mass.

This is defined with the assumption that a black hole grows according to the tau constant just like an RC circuit.

The P particle (antimatter), with frequency ω and energy $-h\omega$, moves away from the black hole in the form of a gravitational wave.

According to the proposed hypothesis, a black hole always grows, following the curve of the tau constant in analogy to an RC electrical circuit.

For a black hole of 3 solar masses, the stationary frequency would be approximately $2.6 \cdot 10^3$ Hz.

6. APPLICATION OF THE MODEL AND RESULTS

6.1. Additional calculations. Growth of a black hole in analogy to the tau growth curve of an RC circuit

In the ADS/CFT correspondence to calculate the viscosity of quark-gluon plasma, the following assumption is used, a black hole is equivalent to quark-gluon plasma.

We consider the temperature of a black hole equal to the temperature of the quark-gluon plasma, equal to $T = 10^{13}$ K.

Another way of interpreting it is as follows:

When a star collapses, a white dwarf star, a neutron star, or a black hole is formed.

A white dwarf star has a temperature of about 10^6 K, a neutron star has a temperature of about 10^{11} K. If we consider that a black hole is a plasma of quarks and gluons, its temperature is expected to be higher than 10^{11} K.

Hypothesis: the temperature of a black hole is 10^{13} K.

We will make the following approximation:

$$T = 0.00000000000001\tau, T = 10^{13}\tau$$

$$\tau = 10^{26} \text{ K}$$

$$C_g(T) = C_{gmax} (1 - e^{-(T/\tau)})$$

$$C_g(T) = C_{gmax} (1 - e^{-0.00000000000001(\tau/\tau)})$$

$$C_G(T) = C_{Gmax} (1 - e^{-0.000000000000001})$$

$$C_G(T) = C_{Gmax} (1 - e^{-1/10^{13}})$$

$$C_G(T) = C_{Gmax} (1 - 1/e^{(1/10^{13})})$$

$$C_G(T) = C_{Gmax} (1 - 0.999999999999999)$$

$$C_G(T) = C_{Gmax} \times 10^{-13}$$

$$C_{Gmax} = C_G(T) / 10^{-13} = 3 \times 10^8 \text{ m/s} \times 10^{13}$$

$$C_{Gmax} \equiv 3 \times 10^{21} \text{ m/s.}$$

where T is the absolute temperature, τ represents the growth constant tau, $C_G = v$ represents the speed of a massless particle greater than the speed of light and C_{Gmax} represents the maximum speed that C_G can take.

With the following equations we obtain the following graphs, represented by Table 1 and Figure 2:

Parametric equations:

$$C_G(T) = C_{Gmax} (1 - e^{-(T/\tau)})$$

T (kelvin) = $\{(h c^3) / (8 \pi \times K_B \times G \times M)\}$, Hawking's equation for the temperature of a black hole.

$R_s = (2 \times G \times M) / c^2$, Schwarzschild's radius.

$IMI = K |mI$, where K is a constant.

$$IMI = I \delta I$$

$K_{BQ} = 1.78 \times 10^{-43} \text{ J/K}$, Boltzmann's constant for black hole.

- a) In item 1 of the Table 1, for the following parameters, $T = 10^{13} \text{ K}$, $C_G = C = 310^8 \text{ m/s}$, calculating we get the following values:

$m = 6 \times 10^{30} \text{ kg}$, baryonic mass.

$\delta = 0$, dark matter mass.

$M = m = 6 \times 10^{30} \text{ kg}$

$R_s = 8,89 \times 10^3 \text{ m}$, Schwarzschild radius.

- b) In item 9 of the Table 1, for the following parameters, $T = 5 \times 10^{26} \text{ K}$, $C_G = 3 \times 10^{21} \text{ m/s}$, $C = 310^8 \text{ m/s}$, calculating we get the following values:

$m = 1.20 \times 10^{56} \text{ kg}$, baryonic mass.

$\delta = 1.20 \times 10^{82} \text{ kg}$, dark matter mass.

$M = \delta = 1.20 \times 10^{82} \text{ kg}$

$R_s = 1.77 \times 10^{29} \text{ m}$, Schwarzschild radius.

- c) It is important to emphasize, for the time t equal to 5τ , at the moment the disintegration of the black hole occurs, the big bang originates, the total baryonic mass of the universe corresponds to $m = 10^{56} \text{ kg}$.
- d) Figure 2 shows the growth of the tau (τ) constant, as a function of speed vs. temperature.

Table 1. Represents values of $|mI$, baryonic mass; $|\delta I$, dark matter mass; IMI , mass of baryonic matter plus the mass of dark matter; $IEmI$, energy of baryonic matter; $IE\delta I$, dark matter energy; IEI , Sum of the energy of baryonic matter plus the energy of dark matter and R_s , Schwarzschild's radius, as a function of, c , speed of light; C_G , speed greater than the speed of light; T , temperature in Kelvin; using the parametric equations.

Item	T	CG	C	I m I	I δ I	I M I	I Em I	I E δ I	I E I	Rs
0	kelvin	m/s	m/s	kg	kg	kg	Joule	Joule	Joule	m
1	10^{13}	$3 \cdot 10^8$	$3 \cdot 10^8$	$6.00 \cdot 10^{30}$	0	$6.00 \cdot 10^{30}$	$5.40 \cdot 10^{47}$	0	$5.40 \cdot 10^{47}$	$8.89 \cdot 10^3$
2	10^{14}	$3 \cdot 10^{10}$	$3 \cdot 10^8$	$6.00 \cdot 10^{35}$	$6.00 \cdot 10^{39}$	$6.00 \cdot 10^{39}$	$5.40 \cdot 10^{52}$	$5.40 \cdot 10^{56}$	$5.40 \cdot 10^{56}$	$8.89 \cdot 10^8$
3	10^{17}	$3 \cdot 10^{13}$	$3 \cdot 10^8$	$6.00 \cdot 10^{41}$	$6.00 \cdot 10^{51}$	$6.00 \cdot 10^{51}$	$5.40 \cdot 10^{58}$	$5.40 \cdot 10^{68}$	$5.40 \cdot 10^{68}$	$8.89 \cdot 10^{14}$
4	10^{21}	$3 \cdot 10^{15}$	$3 \cdot 10^8$	$6.00 \cdot 10^{43}$	$6.00 \cdot 10^{57}$	$6.00 \cdot 10^{57}$	$5.40 \cdot 10^{60}$	$5.40 \cdot 10^{74}$	$5.40 \cdot 10^{74}$	$8.89 \cdot 10^{16}$
5	$1 \cdot 10^{26}$	$3 \cdot 10^{17}$	$3 \cdot 10^8$	$6.00 \cdot 10^{44}$	$6.00 \cdot 10^{62}$	$6.00 \cdot 10^{62}$	$5.40 \cdot 10^{61}$	$5.40 \cdot 10^{79}$	$5.40 \cdot 10^{79}$	$8.89 \cdot 10^{17}$
6	$2 \cdot 10^{26}$	$3 \cdot 10^{18}$	$3 \cdot 10^8$	$3.00 \cdot 10^{47}$	$3.00 \cdot 10^{67}$	$3.00 \cdot 10^{67}$	$2.70 \cdot 10^{64}$	$2.70 \cdot 10^{84}$	$2.70 \cdot 10^{84}$	$4.44 \cdot 10^{20}$
7	$3 \cdot 10^{26}$	$3 \cdot 10^{20}$	$3 \cdot 10^8$	$2.00 \cdot 10^{53}$	$2.00 \cdot 10^{77}$	$2.00 \cdot 10^{77}$	$1.80 \cdot 10^{70}$	$1.80 \cdot 10^{94}$	$1.80 \cdot 10^{94}$	$2.96 \cdot 10^{26}$
8	$4 \cdot 10^{26}$	$9 \cdot 10^{20}$	$3 \cdot 10^8$	$4.05 \cdot 10^{54}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{71}$	$3.28 \cdot 10^{96}$	$3.28 \cdot 10^{96}$	$6.00 \cdot 10^{27}$
9	$5 \cdot 10^{26}$	$3 \cdot 10^{21}$	$3 \cdot 10^8$	$1.20 \cdot 10^{56}$	$1.20 \cdot 10^{82}$	$1.20 \cdot 10^{82}$	$1.08 \cdot 10^{73}$	$1.08 \cdot 10^{99}$	$1.08 \cdot 10^{99}$	$1.77 \cdot 10^{29}$

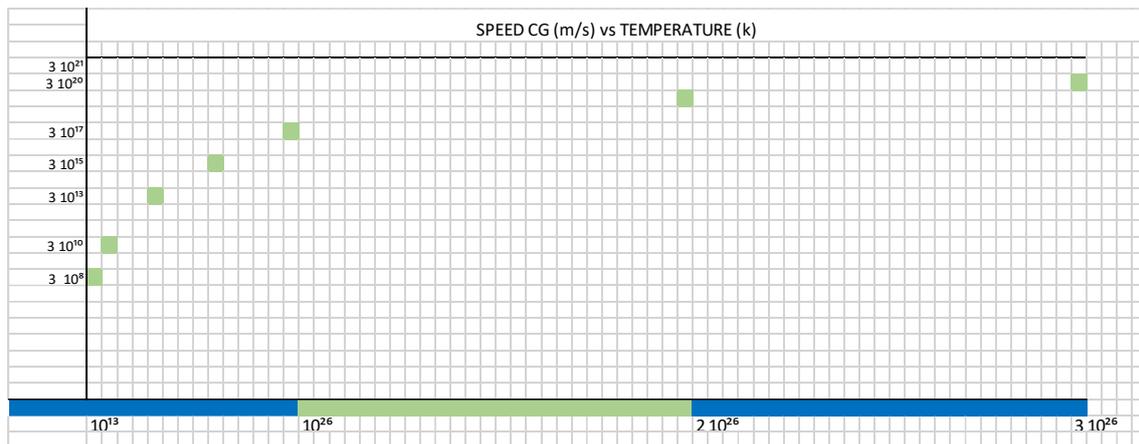


Figure 2. Represents the variation of speed C_G , as a function of temperature T , inside a black hole.

6.2. Calculation of the amount of dark matter that exists in the Milky Way

Mass and Schwarzschild's radius of the Sagittarius A* black hole:

$$m = 4.5 \cdot 10^6 M_s = 4.5 \cdot 10^6 \times 1.98 \cdot 10^{30} \text{ kg}$$

Where M_s is the mass of the sun.

$$m = 8.1 \times 10^{36} \text{ kg}$$

$$R_s = 6 \text{ million kilometres}$$

where R_s is the Schwarzschild's radius of the Sagittarius A*.

$$R_s = 6 \times 10^9 \text{ m}$$

If we look at Figure 2, for $m = 8.1 \times 10^{36} \text{ kg}$ and $R_s = 6 \times 10^9 \text{ m}$, extrapolating we have approximately that $T = 3 \cdot 10^{14} \text{ K}$.

To calculate the speed C_G we are going to use the Hawking temperature equation:

$$T = hc^3 / (8\pi \times KB \times G \times M)$$

where h is Boltzmann's constant, c is the speed inside a black hole, KB is Boltzmann's constant, G is the universal constant of gravity, and M is the mass of the black hole.

Substituting the values and calculating the value of C we have:

$$C_G = 10.30 \cdot 10^{10} \text{ m/s}$$

If we look at Figure 3, we see that this value corresponds approximately to the calculated value.

With the value of C_G we calculate δ and M :

$$E = M C^2$$

where E is energy, M is mass, and C is the speed of light.

$$E_G = M C_G$$

$$E_G = K M C^2$$

$$C_G^2 = k C^2$$

where K is a constant.

Calculation of the constant K:

$$C = 3 \cdot 10^8 \text{ m/s,}$$

$$C_G = 10.30 \cdot 10^{10} \text{ m/s,}$$

$$M = 8.1 \cdot 10^{36} \text{ kg}$$

$$E = 8.1 \cdot 10^{36} \text{ kg} \times 9 \cdot 10^{16} \text{ m}^2/\text{s}^2$$

$$E_G = 8.1 \cdot 10^{36} \times (10.30 \cdot 10^{10})^2 = 8.1 \cdot 10^{36} \times 106 \cdot 10^{20}$$

$$E_G = (106 / 9) \cdot 10^4 \times 8.1 \cdot 10^{36} \times 9 \cdot 10^{16}$$

$$E_G = K E$$

$$K = 11.77 \cdot 10^4$$

Calculation of the total mass M:

$$M = K m$$

$$M = (11.77 \cdot 10^4) \times (8.1 \cdot 10^{36} \text{ kg})$$

$$M = 9.54 \cdot 10^{41} \text{ kg, Total mass of black hole Sagittarius A*}$$

$$m = 8.1 \times 10^{36} \text{ kg, total baryonic mass inside the black hole Sagittarius A*}$$

Calculation of the mass of dark matter δ :

$$M = \delta$$

$$\delta = 9.54 \cdot 10^{41} \text{ kg, total dark matter inside the black hole Sagittarius A*}$$

Calculation of the ratio of the mass of dark matter and the mass of the Milky Way

$$M_{vl} = 1.7 \cdot 10^{41} \text{ kg, mass of the milky way}$$

$$\delta = 9.54 \cdot 10^{41} \text{ kg, total dark matter inside the black hole Sagittarius A*}$$

$$\delta / M_{vl} = (9.54 \cdot 10^{41} \text{ kg} / 1.7 \cdot 10^{41} \text{ kg})$$

$$\delta / M_{vl} = 5.61, \text{ ratio of the mass of dark matter and the mass of the Milky Way}$$

$$\delta = 5.61 M_{vl}$$

The total dark matter δ is 5.61 times greater than the measured amount of baryonic mass of the Milky Way M_{vl} .

6.3. Calculation of the variations of the Planck length, Planck time and Planck temperature as a consequence of the fact that the velocity v varies from $3 \cdot 10^8 \text{ m/s}$ to $3 \cdot 10^{21} \text{ m/s}$

We define the following:

$$C_\epsilon = 3 \cdot 10^8 \text{ m/s} \quad (27)$$

$$C_{gmax} = 3 \cdot 10^{21} \text{ m/s} \quad (28)$$

$$C_\epsilon < C_g < C_{gmax}$$

where ϵ stands for electromagnetic, g stands for gravitational, and max stands for maximum.

Planck's length equation:

$$L_p = \sqrt{(h G / C^3)} \quad (29)$$

Planck's time equation:

$$t_p = \sqrt{(h G / C^5)} \quad (30)$$

Planck's temperature equation:

$$T_p = \sqrt{\{h C^5 / (G K_B^2)\}} \quad (31)$$

where L_p represents the Planck's length, t_p represents the Planck's time, and T_p represents the Planck's temperature.

where h stands for Planck's constant, C for the speed of light, G for the universal constant of gravity, and K_B for Boltzmann's constant.

Substituting the values of (27) and (28) in equations (29), (30) and (31) we obtain:

Electromagnetic Planck constants:

$$C_{\epsilon} = 3 \times 10^8 \text{ m/s}$$

$$L_{p\epsilon} = 1.61 \times 10^{-35} \text{ m}$$

$$t_{p\epsilon} = 5.39 \times 10^{-44} \text{ s}$$

$$T_{p\epsilon} = 1.41 \times 10^{32} \text{ K}$$

Gravitational Planck constants:

$$C_G = 3 \times 10^8 \text{ m/s to } 3 \times 10^{21} \text{ m/s}$$

$$L_p = 1.61 \times 10^{-35} \text{ m to } 1.28 \times 10^{-54} \text{ m}$$

$$t_p = 5.39 \times 10^{-44} \text{ s to } 0.426 \times 10^{-75} \text{ s}$$

$$T_p = 1.41 \times 10^{32} \text{ K to } 0.62 \times 10^{90} \text{ K}$$

Table 2. we represent the range of variation of the velocity C, the Planck's length, the Planck's time and the Planck's temperature.

Range	Minimum value	Maximum value	Units
Velocity C_G	3×10^8	3×10^{21}	m/s
Planck's length L_p	1.28×10^{-54}	1.61×10^{-35}	m
Planck's time t_p	0.42×10^{-75}	5.39×10^{-44}	s
Planck's temperature T	1.41×10^{32}	0.62×10^{90}	K

6.4. The observation of the 1919 solar eclipse in Brazil and Africa provided the first experimental proof of the validity of Albert Einstein's theory of relativity. We will calculate the Boltzmann constant for the sun and show how it adjusts to the deviation found.

No solar eclipse has had as much impact in the history of science as that of May 29, 1919, photographed and analysed at the same time by two teams of British astronomers. One of them was sent to the city of Sobral, Brazil, in the interior of Ceará; the other to the island of Principe, then a Portuguese territory off the coast of West Africa. The goal was to see if the path of starlight would deviate when passing through a region with a strong gravitational field, in this case the surroundings of the Sun, and by how much this change would be if the phenomenon was measured.

Einstein introduced the idea that gravity was not a force exchanged between matter, as Newton said, but a kind of secondary effect of a property of energy: that of deforming space-time and everything that propagates over it, including waves like light. "For Newton, space was flat. For Einstein, with general relativity, it curves near bodies with great energy or mass", comments physicist George Matsas, from the Institute of Theoretical Physics of the São Paulo State University (IFT-Unesp). With curved space-time, Einstein's calculated value of light deflection nearly doubled, reaching 1.75 arcseconds.

The greatest weight should be given to those obtained with the 4-inch lens in Sobral. The result was a deflection of 1.61 arc seconds, with a margin of error of 0.30 arc seconds, slightly less than Einstein's prediction.

Demonstration:

- i) Let us calculate the Boltzmann's constant for the Sun, K_{BS} , curved spacetime.

Hawking's temperature equation:

$$K_{BS} = (h \times c^3) / (8 \times \pi \times T_s \times G \times M_s)$$

Where K_{BS} is the Boltzmann constant for the sun, T_s is the temperature of the sun's core, G is the universal constant of gravity, and M_s is the mass of the sun.

$$K_{BS} = (6.62 \times 10^{-34} \times 27 \times 10^{24}) / (8 \times 3.14 \times 1.5 \times 10^7 \times 6.67 \times 10^{-11} \times 1.98 \times 10^{30})$$

$$K_{BS} = 3.59 \times 10^{-37} \text{ J/K, Boltzmann's constant of the sun.}$$

We use the following equation:

$$E_s = K_{BS} \times T_s$$

$$E_s = 3.59 \cdot 10^{-37} \times 1.5 \cdot 10^7$$

$$E_s = 5.38 \cdot 10^{-30} \text{ J/K}$$

We use the following equation:

$$E_s = h \times f_s$$

$$f_s = E_s / h$$

$$f_s = 5.38 \cdot 10^{-30} / 6.62 \cdot 10^{-34} = 0.81 \cdot 10^4 = 8.1 \cdot 10^3 \text{ Hz}$$

$$f_s = 8.1 \cdot 10^3 \text{ Hz}$$

We use the following equation:

$$c = \lambda_s \times f_s$$

$$\lambda_s = c / f_s$$

$$\lambda_s = 3 \cdot 10^8 / 8.1 \cdot 10^3$$

$$\lambda_s = 3.7 \cdot 10^4 \text{ m}$$

We use the following equation:

$$\text{Degree} = \lambda_s / 360$$

$$\text{Degree} = 102.77 \text{ m}$$

We use the following equation:

$$\text{Arcsecond} = \text{degree} / 3600$$

$$\text{Arcsecond} = 102.77 \text{ m} / 3600 = 0.0285 \text{ m}$$

$$1.61 \text{ arcsecond} = 0.0458 \text{ m}$$

$$1 \text{ inch} = 0.0254 \text{ m}$$

$$4 \text{ inch} = 0.1016 \text{ m}$$

With a 4-inch lens, we can measure the deflection produced by the 1.61 arcsecond curvature of space-time, which was predicted by Albert Einstein's theory of general relativity, and corresponds to a wavelength $\lambda_s = 3.7 \cdot 10^4 \text{ m}$, a frequency $f_s = 8.1 \cdot 10^3 \text{ Hz}$, for an effective Boltzmann constant of the sun $K_{BS} = 3.59 \cdot 10^{-37} \text{ J/K}$.

ii) We will carry out the same calculations for $K_B = 1.38 \cdot 10^{-23} \text{ J/K}$, flat space-time.

$$K_B = 1.38 \cdot 10^{-23} \text{ J/K}$$

We use the following equation:

$$E = K_B \times T_s$$

$$E = 1.38 \cdot 10^{-23} \times 1.5 \cdot 10^7$$

$$E = 2.07 \cdot 10^{-16} \text{ J/K}$$

We use the following equation:

$$E = h \times f$$

$$f = E / h = 2.07 \cdot 10^{-16} / 6.62 \cdot 10^{-34}$$

$$f = 3.12 \cdot 10^{17} \text{ Hz}$$

We use the following equation:

$$c = \lambda \times f$$

$$\lambda = c / f$$

$$\lambda = 3 \cdot 10^8 / 0.312 \cdot 10^{18}$$

$$\lambda = 9.61 \cdot 10^{-10} \text{ m}$$

We use the following equation:

$$\text{Degree} = \lambda / 360$$

Degree = $0.02669 \cdot 10^{-10}$ m

We use the following equation:

Arcsecond = degree / 3600

Arcsecond = $7.41 \cdot 10^{-16}$ m

Using the Boltzmann constant $K_B = 1.38 \cdot 10^{-23}$ J/K, we cannot correctly predict by mathematical calculations the deflection of light given by Albert Einstein's general theory of relativity, to be measured in the telescope at Sobral.

Through the example given, we can conclude that the Boltzmann's constant $K_{Bs} = 3.59 \cdot 10^{-37}$ J/K fits the calculations of the deflection of light in curved space-time.

6.5. Dark energy and its relationship with the wave equation of the universe produced by the big bang and the generalization of Boltzmann's constant for curved space-time.

a) Calculation of the wave equation of the universe for the time T_0 when the Big Bang occurs:

we will use the Table 1.

Table 1. Represents values of ImI , baryonic mass; $I\delta I$, dark matter mass; IMI , mass of baryonic matter plus the mass of dark matter; $IEmI$, energy of baryonic matter; $IE\delta I$, dark matter energy; IEI , Sum of the energy of baryonic matter plus the energy of dark matter and R_s , Schwarzschild's radius, as a function of, c , speed of light; C_g , speed greater than the speed of light; T , temperature in Kelvin; using the parametric equations.

Item	T	CG	C	ImI	IδI	IMI	IEmI	IEδI	IEI	Rs
0	kelvin	m/s	m/s	kg	kg	kg	Joule	Joule	Joule	m
1	10^{23}	$3 \cdot 10^8$	$3 \cdot 10^8$	$6.00 \cdot 10^{20}$	0	$6.00 \cdot 10^{20}$	$5.40 \cdot 10^{47}$	0	$5.40 \cdot 10^{47}$	$8.89 \cdot 10^2$
2	10^{24}	$3 \cdot 10^{10}$	$3 \cdot 10^8$	$6.00 \cdot 10^{25}$	$6.00 \cdot 10^{26}$	$6.00 \cdot 10^{26}$	$5.40 \cdot 10^{52}$	$5.40 \cdot 10^{52}$	$5.40 \cdot 10^{52}$	$8.89 \cdot 10^4$
3	10^{27}	$3 \cdot 10^{13}$	$3 \cdot 10^8$	$6.00 \cdot 10^{41}$	$6.00 \cdot 10^{51}$	$6.00 \cdot 10^{51}$	$5.40 \cdot 10^{98}$	$5.40 \cdot 10^{98}$	$5.40 \cdot 10^{98}$	$8.89 \cdot 10^{14}$
4	10^{27}	$3 \cdot 10^{15}$	$3 \cdot 10^8$	$6.00 \cdot 10^{45}$	$6.00 \cdot 10^{57}$	$6.00 \cdot 10^{57}$	$5.40 \cdot 10^{98}$	$5.40 \cdot 10^{98}$	$5.40 \cdot 10^{98}$	$8.89 \cdot 10^{16}$
5	$1 \cdot 10^{28}$	$3 \cdot 10^{17}$	$3 \cdot 10^8$	$6.00 \cdot 10^{44}$	$6.00 \cdot 10^{62}$	$6.00 \cdot 10^{62}$	$5.40 \cdot 10^{91}$	$5.40 \cdot 10^{92}$	$5.40 \cdot 10^{92}$	$8.89 \cdot 10^{17}$
6	$2 \cdot 10^{28}$	$3 \cdot 10^{18}$	$3 \cdot 10^8$	$3.00 \cdot 10^{47}$	$3.00 \cdot 10^{67}$	$3.00 \cdot 10^{67}$	$2.70 \cdot 10^{84}$	$2.70 \cdot 10^{84}$	$2.70 \cdot 10^{84}$	$4.44 \cdot 10^{20}$
7	$3 \cdot 10^{28}$	$3 \cdot 10^{20}$	$3 \cdot 10^8$	$2.00 \cdot 10^{55}$	$2.00 \cdot 10^{77}$	$2.00 \cdot 10^{77}$	$1.80 \cdot 10^{75}$	$1.80 \cdot 10^{74}$	$1.80 \cdot 10^{74}$	$2.96 \cdot 10^{25}$
8	$4 \cdot 10^{28}$	$9 \cdot 10^{20}$	$3 \cdot 10^8$	$4.05 \cdot 10^{54}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{71}$	$3.28 \cdot 10^{75}$	$3.28 \cdot 10^{75}$	$6.00 \cdot 10^{27}$
9	$5 \cdot 10^{28}$	$3 \cdot 10^{21}$	$3 \cdot 10^8$	$1.20 \cdot 10^{56}$	$1.20 \cdot 10^{82}$	$1.20 \cdot 10^{82}$	$1.08 \cdot 10^{73}$	$1.08 \cdot 10^{79}$	$1.08 \cdot 10^{79}$	$1.77 \cdot 10^{28}$

We are going to consider that at the instant $t=0^-$, the black hole is about to disintegrate.

Calculation of gravitational waves for a damped parallel RLC circuit ($\alpha > \omega_0$).

Initial conditions:

$V(0)^- = 1.08 \cdot 10^{73}$ V, V is equivalent to E

$I(0)^r = I(0)^c = 3 \cdot 10^{21}$ A, I is equivalent to C

Calculation of the value of the wavelength λ .

$\lambda = 1.000.000$ light years = $10^6 \times 9.46 \cdot 10^{15}$ m

$\lambda = 9.46 \cdot 10^{21}$ m

Calculation of the value of the frequency f:

$C = \lambda \times f$

$f = C / \lambda$

$f = 3 \cdot 10^{21} / 9.46 \cdot 10^{21}$

$f = 0.317$ Hz

Calculation of the value of the angular frequency ω :

$\omega = 2 \pi f$

$\omega = 2.00$ rad/s

Calculation of the value of the resistor R:

$V(0) = I(0) \times R$

$R = V(0) / I(0) = 1.08 \cdot 10^{73} / 3 \cdot 10^{21}$

$$R = 3.60 \cdot 10^{51} \text{ Ohms}$$

Calculation of the number of seconds in 380,000 years:

$$t = 11.81 \cdot 10^{12} \text{ s}$$

Let's consider $\alpha = 55 \cdot 10^9 \omega_0$

$$\alpha = 110 \cdot 10^9$$

let's define:

$\omega = \omega_0 = 2.00 \text{ rad/s}$; the fundamental frequency is equal to the resonant frequency.

$$\alpha = 1 / 2RC$$

Calculation of the value of capacitance C:

$$C = 1 / 2R\alpha$$

$$C = 1 / (2 \times 3.60 \cdot 10^{51} \times 110 \cdot 10^9)$$

$$C = 1.26 \cdot 10^{-63} \text{ F}$$

Calculation of the value of inductance L:

$$\omega_0^2 = 1/LC$$

$$L = 1/(\omega_0^2 \times C) = 1 / (4 \times 1.26 \cdot 10^{-63})$$

$$L = 1.98 \cdot 10^{62} \text{ Hy}$$

Calculation of the value of S1:

$$S1 = -\alpha + \sqrt{(\alpha^2 - \omega_0^2)}$$

$$S1 = -1.81 \cdot 10^{-11}$$

Calculation of the value of S2:

$$S2 = -\alpha - \sqrt{(\alpha^2 - \omega_0^2)}$$

$$S2 = -2.19 \cdot 10^{11}$$

With these calculated values we have the following equation:

$$V(t) = A1 e^{(-1.81 \cdot 10^{-11}t)} + A2 e^{(-2.19 \cdot 10^{11}t)} \quad (32)$$

Calculations of the constant A1 and A2:

First condition V(0):

$$V(t) = A1 e^{(-1.81 \cdot 10^{-11}t)} + A2 e^{(-2.19 \cdot 10^{11}t)}$$

$$V(0) = A1 + A2 = 0$$

$$A1 = -A2 \quad (33)$$

Second condition $dV(0) / dt$:

$$V(t) = A1 e^{(-1.81 \cdot 10^{-11}t)} + A2 e^{(-2.19 \cdot 10^{11}t)}$$

$$dV(t) / dt = d(A1 e^{(-1.81 \cdot 10^{-11}t)} + A2 e^{(-2.19 \cdot 10^{11}t)})$$

$$dV(t)/dt = -1.81 \cdot 10^{-11} \times A1 \times e^{-1.81 \cdot 10^{-11}t} - 2.19 \cdot 10^{11} \times A2 \times e^{-2.19 \cdot 10^{11}t}$$

$$dV(0)/dt = -1.81 \cdot 10^{-11} \times A1 - 2.19 \cdot 10^{11} \times A2 \quad (34)$$

Let's calculate $dV(0)/dt = ?$

Third condition:

$IR + IC + IL = 0$; but for $t = 0$, $IL = 0$ then it remains

$$V/R + CdV(t)/dt = 0$$

$$dV(0)/dt = V/RC$$

$$dV(0)/dt = 2.38 \cdot 10^{84} \quad (35)$$

Combining the equations (33), (34) and (35) we obtain the following values for A1 and A2:

$$A1 = +1.086 \cdot 10^{73}$$

$$A2 = -1.086 \cdot 10^{73}$$

Substituting the values of A1 and A2 in equation (32) we obtain the equation of gravitational waves of the Big Bang for the time T_0 .

$$E(t) = 1.08 \cdot 10^{73} \{e^{-(1.81 \cdot 10^{-11}t)}\} - 1.08 \cdot 10^{73} \{e^{-(2.19 \cdot 10^{11}t)}\} + E_0 \quad (36)$$

where $E(t)$ represents the energy of gravitational waves and E_0 represents the energy that corresponds to the temperature of 2.7K.

The spectrum of amplitude and phase as a function of frequency (Figure 3) is the Fourier transform from ideal similar equation (36).

The amplitude spectrum shows us the frequency content as a function of the magnitude. The phase spectrum shows the frequency content as a function of the angle, but we have to remember by Fourier that the angle is a function of time, therefore a variation of angle implies a variation in displacement and it is precisely this very important characteristic, which we can relate to dark energy.

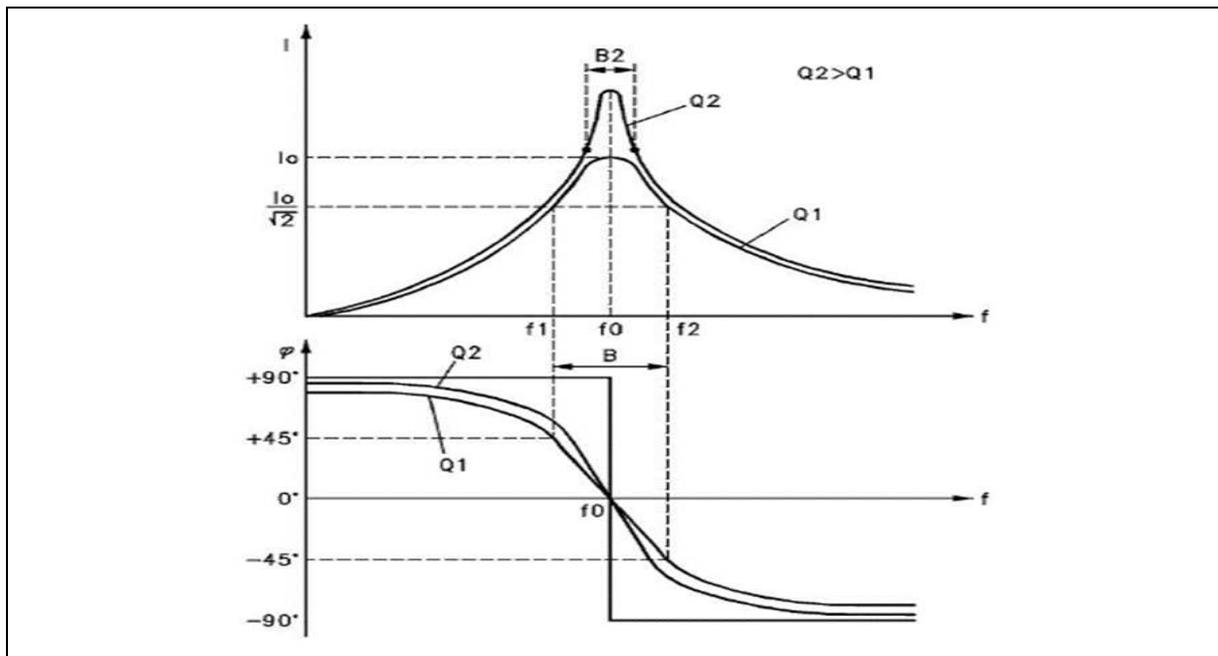


Figure 3. shows the amplitude spectrum as a function of frequency at the top and the phase spectrum as a function of frequency at the bottom.

Here we put forward the hypothesis that dark energy is the expansion of space-time that is produced by a spectrum of gravitational waves whose produced frequencies are a function of time, when the disintegration of a black hole (big bang) occurs.

Here we put forward the hypothesis that dark energy is the result of relativistic dark matter that propagates when the black hole disintegrates (Big Bang).

Therefore, dark energy is the result of the combination of the spectrum of gravitational waves whose frequency content is a function of time added to the relativistic dark matter, both propagate with the disintegration of the black hole (Big Bang).

Additional calculations

Calculation of the temperature of the universe for a time $t = 380,000$ years:

Let's calculate $E(t)$ for $t = 11.81 \cdot 10^{12}$ s, (380,000 years)

$$E(t) = 1.08 \cdot 10^{73} \{e^{-1.81 \cdot 10^{11}t}\} - 1.08 \cdot 10^{73} \{e^{-2.19 \cdot 10^{11}t}\}$$

$$E(t) = 1.08 \cdot 10^{73} \{e^{-213}\}$$

$$E(t) = 0.33 \cdot 10^{-19} \text{ Joules}$$

$$T = E/K_B$$

$$T = 2390 \text{ k}$$

Approximately the temperature of the cosmic microwave background.

Calculation of the time t for when the universe stabilizes and reaches the temperature of 2.7 K

$$2.7 \text{ K} = 3.72 \cdot 10^{-23} \text{ J} \quad (37)$$

Substituting (37) in equation (36) we have:

$$3.72 \cdot 10^{-23} = 1.08 \cdot 10^{73} e^{-1.81 \cdot 10^{-11}t}$$

$$e^{-1.81 \cdot 10^{-11}t} = 0.290 \cdot 10^{96}$$

$$1.81 \cdot 10^{-11}t = \ln(0.290 \cdot 10^{96})$$

$$t = 1.22 \cdot 10^{13} \text{ s}$$

In that time t the space-time travels the following distance:

$$e = v \times t$$

Where e is space, v is velocity, and t is time.

$$\begin{aligned} e &= 3 \cdot 10^{21} \text{ m/s} \times 1.22 \cdot 10^{13} \text{ s} \\ e &= 3.66 \cdot 10^{34} \text{ m} \end{aligned} \quad (38)$$

If we calculate the Fourier transform of equation (32), that is, $E(\omega)$.

All the frequencies that make up the frequency spectrum have to travel the distance given by equation (38), that is, $3.66 \cdot 10^{34} \text{ m}$.

Therefore, the influence of the spectrum of gravitational waves in the expansion of space-time will be twice as long, that is, $2.44 \cdot 10^{26} \text{ s}$

If we divide by power of 10, logarithmic scale, we have approximately 26 steps.

Let's calculate the time t today.

$$t = 4.35 \cdot 10^{17} \text{ s, correspond to 17 steps.}$$

$$(17,5 / 26) \times 100 = 67.3\%, \text{ this is similar to the dark energy content of the universe.}$$

$$100\% - 67.3 = 32.7\%, \text{ this is similar to the dark matter content of the universe.}$$

Calculation of the number of seconds in 380,000 years:

$$t = 11.81 \cdot 10^{12} \text{ s} \quad (39)$$

Calculation of the number of seconds for when the universe stabilizes and reaches the temperature of 2.7 K

$$t = 1.22 \cdot 10^{13} \text{ s} \quad (40)$$

We divide the time t , given by (39) by the time t , in (40), we get:

$$(11.81 \cdot 10^{12} \text{ s} / 1.22 \cdot 10^{13} \text{ s}) \times 100 = 96.72\%$$

$$100\% - 96,72\% = 3.28\%, \text{ this is similar to the baryonic matter content in the universe.}$$

The true interpretation of this result is the following, the fundamental wavelength that corresponds to $\lambda = 1,000,000$ light years, represents the fundamental peak of the CMB sound spectrum, has convolved 96% with the space-time of the universe and still needs to be convolved 4%.

All these calculations are referenced to a time $t = 11.81 \cdot 10^{12} \text{ s}$, which correspond to the CMB.

b) Dark energy and the relationship that exists with the generalization theory of Boltzmann's constant and curved space-time

The formation of a black hole produces a contraction of space-time.

For the sun, the contraction would be in the following order:

$$R = 696,340 \text{ km, Sun radius.}$$

$$R_s = 3 \text{ km, Schwarzschild's radius of the sun.}$$

Equation of volume of a sphere:

$$(4/3) \pi R^3$$

Calculation of the volume of the sun:

$$V = (4/3) \pi R^3$$

$$V = (4/3) \times 3.14 \times (6.9610^8)^3$$

$$V = 1411.54 \cdot 10^{24}$$

Calculation of the volume of the equivalent black hole of the sun:

$$V_s = (4/3) \pi R_s^3$$

$$V_s = (4/3) \times 3.14 \times (3 \cdot 10^3)^3$$

$$V_s = 113.04 \cdot 10^9$$

Calculation of the V / V_s ratio:

$$V / V_s = 1411.54 \cdot 10^{24} / 113.04 \cdot 10^9$$

$$V / V_s = 12.48 \cdot 10^{15}$$

In three dimensions the space-time contraction factor is 10^{15} times.

In one dimension the space-time contraction factor is 10^5 times.

We can call it the contraction factor of space-time or the compactification factor of matter.

Another way to calculate the factor of contraction of space-time or compactification of matter is the following:

Boltzmann's constant for flat space-time, is defined for 1 mole of carbon 12 and corresponds to $6.0221 \cdot 10^{23}$ atoms.

We assume the ratio of the quark given by the German accelerator HERA (Hadron-Elektron-Ringanlage) in the year of 2016, whose article is published following the right of the internet (21).

$R_{C12} = 0.75 \cdot 10^{-10}$ m, Radius of the atom carbon 12

$R_q = 0.43 \cdot 10^{-18}$ m, radius of the quark

Equation of volume of a sphere:

$$(4/3) \pi R^3$$

where R is the radius of the sphere.

Calculation of the volume of the atom carbon 12:

$$V_{aC12} = (4/3) \times 3.14 \times (0.75 \cdot 10^{-10})^3$$

$V_{aC12} = 1.76 \cdot 10^{-30}$ m³, volume of C12 atom.

Calculate the volume of a quark:

$R_q = 0.43 \cdot 10^{-18}$ m, radius of the quark

$$V_q = (4/3) \times 3.14 \times (0.43 \cdot 10^{-18})^3$$

$$V_q = 0.33 \cdot 10^{-54}$$
 m³

Calculation of the contraction factor V_{aC12} / V_q :

$$V_{aC12} / V_q = 1.76 \cdot 10^{-30} \text{ m}^3 / 0.33 \cdot 10^{-54} \text{ m}^3$$

$$V_{aC12} / V_q = 5.33 \cdot 10^{24}$$

In three dimensions the space-time contraction factor is 10^{24} times.

In one dimension the space-time contraction factor is 10^8 times.

In both examples, we can relate the contraction of space-time to the Boltzmann's constant as follows:

There is a Boltzmann's constant KB that we all know for normal conditions of pressure, volume and temperature, for a flat space-time.

There is an effective Boltzmann's constant, which will depend on the state of matter, for curved space-time.

Knowing that Boltzmann's constant is defined between the following limits

$$1.38 \cdot 10^{-23} \text{ J/K} > KB > 1.78 \cdot 10^{-43} \text{ J/K}$$

Through the variation of the Boltzmann's constant we can quantify the curvature of space-time.

Analysing we can conclude the following:

In both examples, there is a contraction of spacetime which is related to the curvature of space-time.

According to our theory, the Big Bang is born from the disintegration of a black hole.

Generalizing, let's define dark energy:

Here we put forward the hypothesis that dark energy is the expansion of space-time that is produced by a spectrum of gravitational waves whose produced frequencies are a function of time, when the disintegration of a black hole (big bang) occurs.

Here we put forward the hypothesis that dark energy is the result of relativistic dark matter that propagates when the black hole disintegrates (Big Bang).

Here we put forward the hypothesis that dark energy is the expansion of space-time produced by a curved space-time ($KB = 1.78 \cdot 10^{-43} \text{ J/K}$) that tends to reach its normal state, flat space-time ($KB = 1.38 \cdot 10^{-23} \text{ J/K}$)

Therefore, dark energy is the result of the combination of the spectrum of gravitational waves whose frequency content is a function of time, added to the relativistic dark matter, both propagate with the disintegration of the black hole (Big Bang); added to the expansion of space-time produced by a curved space-time ($KB = 1.78 \cdot 10^{-43} \text{ J/K}$) that tends to reach its normal state, flat space-time ($KB = 1.38 \cdot 10^{-23} \text{ J/K}$).

Dark energy is a combination of events already mentioned, which determine the expansion of space-time in our universe.

6.6. Calculation of the density parameter of the universe $\Omega_{M,0}$

1) Calculation of $\Omega_{M,0}$

$\Omega_{M,0}$: relationship of density of the universe today

$$\Omega_{M,0} = \rho_0 / \rho_{cr,0}$$

ρ_0 , density of the universe today

$\rho_{cr,0}$, critical density of the universe today, UFSC data.

$$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3$$

Today, a time $t = 4.35 \cdot 10^{17} \text{ s}$, is considered.

In the following table:

Table 1. Represents values of $I_m I$, baryonic mass; $I_\delta I$, dark matter mass; $I M I$, mass of baryonic matter plus the mass of dark matter; $I E_m I$, energy of baryonic matter; $I E_\delta I$, dark matter energy; $I E I$, Sum of the energy of baryonic matter plus the energy of dark matter and R_s , Schwarzschild's radius, as a function of, c , speed of light; C_G , speed greater than the speed of light; T , temperature in Kelvin; using the parametric equations.

Item	T	CG	C	I m I	I δ I	I M I	I E m I	I E δ I	I E I	R _s
0	kelvin	m/s	m/s	kg	kg	kg	Joule	Joule	Joule	m
1	10^{13}	$3 \cdot 10^8$	$3 \cdot 10^8$	$6.00 \cdot 10^{20}$	0	$6.00 \cdot 10^{20}$	$5.40 \cdot 10^{47}$	0	$5.40 \cdot 10^{47}$	$8.89 \cdot 10^2$
2	10^{14}	$3 \cdot 10^{10}$	$3 \cdot 10^8$	$6.00 \cdot 10^{25}$	$6.00 \cdot 10^{25}$	$6.00 \cdot 10^{25}$	$5.40 \cdot 10^{52}$	$5.40 \cdot 10^{52}$	$5.40 \cdot 10^{52}$	$8.89 \cdot 10^5$
3	10^{17}	$3 \cdot 10^{13}$	$3 \cdot 10^8$	$6.00 \cdot 10^{41}$	$6.00 \cdot 10^{41}$	$6.00 \cdot 10^{41}$	$5.40 \cdot 10^{88}$	$5.40 \cdot 10^{88}$	$5.40 \cdot 10^{88}$	$8.89 \cdot 10^{14}$
4	10^{21}	$3 \cdot 10^{15}$	$3 \cdot 10^8$	$6.00 \cdot 10^{63}$	$6.00 \cdot 10^{63}$	$6.00 \cdot 10^{63}$	$5.40 \cdot 10^{126}$	$5.40 \cdot 10^{126}$	$5.40 \cdot 10^{126}$	$8.89 \cdot 10^{18}$
5	$1 \cdot 10^{25}$	$3 \cdot 10^{17}$	$3 \cdot 10^8$	$6.00 \cdot 10^{85}$	$6.00 \cdot 10^{85}$	$6.00 \cdot 10^{85}$	$5.40 \cdot 10^{170}$	$5.40 \cdot 10^{170}$	$5.40 \cdot 10^{170}$	$8.89 \cdot 10^{22}$
6	$2 \cdot 10^{28}$	$3 \cdot 10^{18}$	$3 \cdot 10^8$	$3.00 \cdot 10^{87}$	$3.00 \cdot 10^{87}$	$3.00 \cdot 10^{87}$	$2.70 \cdot 10^{174}$	$2.70 \cdot 10^{174}$	$2.70 \cdot 10^{174}$	$4.44 \cdot 10^{25}$
7	$3 \cdot 10^{28}$	$3 \cdot 10^{20}$	$3 \cdot 10^8$	$2.00 \cdot 10^{85}$	$2.00 \cdot 10^{77}$	$2.00 \cdot 10^{77}$	$1.80 \cdot 10^{154}$	$1.80 \cdot 10^{74}$	$1.80 \cdot 10^{74}$	$2.96 \cdot 10^{28}$
8	$4 \cdot 10^{28}$	$9 \cdot 10^{20}$	$3 \cdot 10^8$	$4.05 \cdot 10^{84}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{79}$	$3.64 \cdot 10^{71}$	$3.28 \cdot 10^{74}$	$3.28 \cdot 10^{74}$	$6.00 \cdot 10^{27}$
9	$5 \cdot 10^{28}$	$3 \cdot 10^{21}$	$3 \cdot 10^8$	$1.20 \cdot 10^{88}$	$1.20 \cdot 10^{82}$	$1.20 \cdot 10^{82}$	$1.08 \cdot 10^{164}$	$1.08 \cdot 10^{82}$	$1.08 \cdot 10^{82}$	$1.77 \cdot 10^{29}$

$m = 1.20 \cdot 10^{56} \text{ kg}$, total baryonic mass

$\delta = 1.20 \cdot 10^{82} \text{ kg}$, total mass of dark matter

It is very important to make it clear, the expansion of the universe is a function of frequency, each frequency has a certain expansion.

The calculations that we are going to carry out are referenced to the fundamental frequency.

In the spectrum of sound waves of the CMB, the fundamental frequency corresponds to the peak of greatest amplitude or first peak.

$\omega = 2.0 \text{ rad/s}$, fundamental angular frequency

$f = 0.317 \text{ Hz}$, fundamental frequency

$\lambda = 1.000.000 \text{ light years}$

$\lambda = 9.46 \cdot 10^{21} \text{ m}$

$c_1 = 3 \cdot 10^{21} \text{ m/s}$

$t_1 = 1.22 \cdot 10^{13} \text{ s}$

Calculation of the expansion of space-time to today:

Distance travelled 1:

where e_1 is the distance travelled 1, $c_1 = 3 \cdot 10^{21} \text{ m/s}$ and $t_1 = 1.22 \cdot 10^{13} \text{ s}$:

$$e1 = c1 \times t1$$

$$e1 = 3 \times 10^{21} \text{ m/s} \times t = 1.22 \times 10^{13} \text{ s}$$

$$e1 = 3.66 \times 10^{34} \text{ m}$$

Distance travelled 2:

where $e2$ is the distance travelled 2, $c2 = 3 \times 10^8 \text{ m/s}$ and $t2 = 4.35 \times 10^{17} \text{ s}$:

$$e2 = c2 \times t2$$

$$e2 = 3 \times 10^8 \text{ m/s} \times 4.35 \times 10^{17} \text{ s}$$

$$e2 = 1.30 \times 10^{26} \text{ m}$$

Total distance travelled:

$$e = e1 + e2$$

$$e = 3.66 \times 10^{34} \text{ m} + 1.30 \times 10^{26} \text{ m}$$

We know that the bandwidth of the spectrum goes from 10^{-13} s to approximately 10^{13} s .

If we consider the time 10^{-1} s , close to the fundamental frequency, important for its contribution, we can increase the space e , a power of 10.

Figure 4 represents the sound spectrum of the CMB, the fundamental frequency is defined by the first peak or the peak with the highest amplitude.

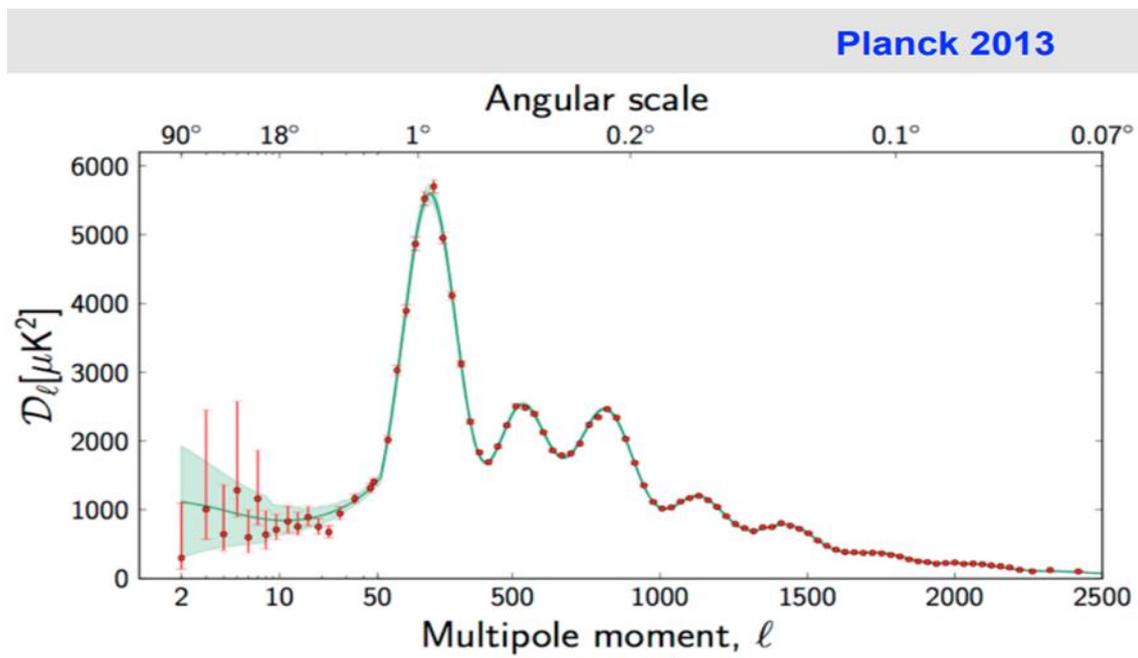


Figure 4. CMB Power Spectrum.

Although we have considered the contribution of the first peak to the right, we note that it is important to consider the contribution of the first peak to the left, that is why we consider the frequency content 10^{-1} s before the fundamental frequency.

Therefore, the total distance covered will be:

$$e = 3.66 \times 10^{35} \text{ m}$$

In one dimension, the universe will have the following radius:

R_u , radius of the universe:

$$R_u = 3.66 \times 10^{35} \text{ m}$$

$$1 \text{ light-year} = 9.46 \times 10^{15} \text{ m}$$

$$R_u = 3.66 \times 10^{35} / 9.46 \times 10^{15}$$

$$R_u = 3.86 \times 10^{19} \text{ light-year}$$

Knowing the radius of the universe, we will calculate the density.

Density equation:

$$\rho = m / v$$

Where ρ is density, m is mass, and v is volume.

$$v = 4/3 \times \pi \times R^3$$

$$\rho = m / (4/3 \times \pi \times R^3)$$

$$\rho = 1.20 \cdot 10^{82} / (1.33 \times 3.14 \times 49.02 \cdot 10^{105})$$

$$\rho = 5.86 \cdot 10^{-26} \text{ kg/m}^3$$

Density of the universe today.

$$\rho_0 = 5.86 \cdot 10^{-29} \text{ g/cm}^3$$

Critical density of the universe today.

$$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3$$

Calculation of $\Omega_{M,0}$:

$$\Omega_{M,0} = \rho_0 / \rho_{cr,0}$$

$$\Omega_{M,0} = 5.86 \cdot 10^{-29} / 3.84 \cdot 10^{-29}$$

According to the calculations:

$$\Omega_{M,0} = 1.52; \text{ most probable value.}$$

II) Another way to calculate $\Omega_{M,0}$:

$$\Omega_{M,0} = \rho_0 / \rho_{cr,0}$$

ρ_0 , density of the universe today

$\rho_{cr,0}$; critical density of the universe today

$$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3, \text{ UFSC data.}$$

look at Figure 7

In item 9, the Schwarzschild's radius corresponds to:

$$R_s = 1.77 \cdot 10^{29} \text{ m}$$

We can call it the contraction factor of space-time or the compactification factor of matter.

$$R_{c12} = 0.75 \cdot 10^{-10} \text{ m, Radius of the atom carbon 12}$$

$$R_q = 0.43 \cdot 10^{-16} \text{ m, 100 times the radius of the quark.}$$

Equation of volume of a sphere:

$$(4/3) \pi R^3$$

Where R is the radius of the sphere.

Calculation of the volume of the atom carbon 12:

$$V_{aC12} = (4/3) \times 3.14 \times (0.75 \cdot 10^{-10})^3$$

$$V_{aC12} = 1.76 \cdot 10^{-30} \text{ m}^3, \text{ volume of C12 atom.}$$

Calculate the volume of a 100-quark:

$$R_q = 0.43 \cdot 10^{-16} \text{ m, 100 times the radius of the quark}$$

$$V_q = (4/3) \times 3.14 \times (0.43 \cdot 10^{-16})^3$$

$$V_q = 0.33 \cdot 10^{-48} \text{ m}^3$$

Calculation of the contraction factor V_{aC12} / V_q :

$$V_{aC12} / V_q = 1.76 \cdot 10^{-30} \text{ m}^3 / 0.33 \cdot 10^{-48} \text{ m}^3$$

$$V_{aC12} / V_q = 5.33 \cdot 10^{18}$$

In three dimensions the space-time contraction factor is $5.33 \cdot 10^{18}$ times.

In one dimension the space-time contraction factor is $1.74 \cdot 10^6$ times.

$$F_c = 1.74 \cdot 10^6$$

The approximate expansion of space-time will be equal to the Schwarzschild radius multiplied the contraction factor of space-time in one dimension.

In one dimension, the universe will have the following radius:

R_u , radius of the universe:

$$R_u = R_s \times F_c$$

R_s , Schwarzschild radius.

F_c , contraction factor of space-time in one dimension.

$$R_u = 1.77 \cdot 10^{29} \text{ m} \times 1.74 \cdot 10^6$$

$$R_u = 3.09 \cdot 10^{35} \text{ m}$$

Knowing the radius of the universe, we will calculate the density.

Density equation:

$$\rho = m / v$$

Where ρ is density, m is mass, and v is volume.

$$v = 4/3 \times \pi \times R^3$$

$$\rho = m / (4/3 \times \pi \times R^3)$$

$$\rho = 0.00971 \cdot 10^{-23}$$

$$\rho = 9.71 \cdot 10^{-26} \text{ kg/m}^3$$

$$\rho = 9.71 \cdot 10^{-29} \text{ g/cm}^3$$

Density of the universe today.

$$\rho_0 = 9.71 \cdot 10^{-29} \text{ g/cm}^3$$

Critical density of the universe today.

$$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3$$

Calculation of $\Omega_{M,0}$:

$$\Omega_{M,0} = \rho_0 / \rho_{cr,0}$$

$$\Omega_{M,0} = 9.71 \cdot 10^{-29} \text{ g/cm}^3 / 3.84 \cdot 10^{-29} \text{ g/cm}^3$$

According to the calculations:

$$\Omega_{M,0} = 2.52$$

III) Calculate $\Omega_{M,\infty}$; for $t \rightarrow \infty$:

$$\Omega_{M,\infty} = \rho_{,\infty} / \rho_{cr,0}$$

$\rho_{,\infty}$; density of the universe for $t \rightarrow \infty$

$\rho_{cr,0}$; critical density of the universe today

$$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3, \text{ UFSC data.}$$

look at Figure 7

In item 9, the Schwarzschild's radius corresponds to:

$$R_s = 1.77 \cdot 10^{29} \text{ m}$$

We can call it the contraction factor of space-time or the compactification factor of matter.

Boltzmann's constant for flat space-time, is defined for 1 mole of carbon 12 and corresponds to $6.0221 \cdot 10^{23}$ atoms.

We assume the ratio of the quark given by the German accelerator HERA (Hadron-Elektron-Ringanlage) in the year of 2016, whose article is published following the right of the internet (21).

$$R_{c12} = 0.75 \cdot 10^{-10} \text{ m, Radius of the atom carbon 12}$$

$$R_q = 0.43 \cdot 10^{-18} \text{ m, radius of the quark}$$

Equation of volume of a sphere:

$$(4/3) \pi R^3$$

Where R is the radius of the sphere.

Calculation of the volume of the atom carbon 12:

$$V_{aC12} = (4/3) \times 3.14 \times (0.75 \cdot 10^{-10})^3$$

$V_{aC12} = 1.76 \cdot 10^{-30} \text{ m}^3$, volume of C12 atom.

Calculate the volume of a quark:

$R_q = 0.43 \cdot 10^{-18} \text{ m}$, radius of the quark

$V_q = (4/3) \times 3.14 \times (0.43 \cdot 10^{-18})^3$

$V_q = 0.33 \cdot 10^{-54} \text{ m}^3$

Calculation of the contraction factor V_{aC12} / V_q :

$V_{aC12} / V_q = 1.76 \cdot 10^{-30} \text{ m}^3 / 0.33 \cdot 10^{-54} \text{ m}^3$

$V_{aC12} / V_q = 5.33 \cdot 10^{24}$

In three dimensions the space-time contraction factor is $5.33 \cdot 10^{24}$ times.

In one dimension the space-time contraction factor is $1.74 \cdot 10^8$ times.

$F_c = 1.74 \cdot 10^8$

The approximate expansion of space-time will be equal to the Schwarzschild radius multiplied the contraction factor of space-time in one dimension.

In one dimension, the universe will have the following radius:

R_u , radius of the universe:

$R_u = R_s \times F_c$

R_s , Schwarzschild radius.

F_c , contraction factor of space-time in one dimension.

$R_u = 1.77 \cdot 10^{29} \text{ m} \times 1.74 \cdot 10^8 \text{ m}$

$R_u = 3.07 \cdot 10^{37} \text{ m}$

Knowing the radius of the universe, we will calculate the density.

Density equation:

$\rho = m / v$

Where ρ is density, m is mass, and v is volume.

$v = 4/3 \times \pi \times R^3$

$\rho_{,\infty} = m / (4/3 \times \pi \times R^3)$

$\rho_{,\infty} = 0.00971 \cdot 10^{-29}$

$\rho_{,\infty} = 9.71 \cdot 10^{-32} \text{ kg/m}^3$

$\rho_{,\infty} = 9.71 \cdot 10^{-35} \text{ g/cm}^3$

Density of the universe for $t \rightarrow \infty$.

$\rho_{,\infty} = 9.71 \cdot 10^{-35} \text{ g/cm}^3$

Critical density of the universe today.

$\rho_{cr,0} = 3.84 \cdot 10^{-29} \text{ g/cm}^3$

Calculation of $\Omega_{M,\infty}$:

$\Omega_{M,\infty} = \rho_{,\infty} / \rho_{cr,0}$

$\Omega_{M,\infty} = 9.71 \cdot 10^{-35} \text{ g/cm}^3 / 3.84 \cdot 10^{-29} \text{ g/cm}^3$

According to the calculations:

$\Omega_{M,\infty} = 2.52 \cdot 10^{-6}$ for $t \rightarrow \infty$

6.7. We will demonstrate how the expansion of space-time as a function of frequency is asymmetry, that is, a variation in time gives us a variation in displacement.

In the damped RLC model, the fundamental frequency is the resonant frequency.

$\lambda = \lambda_0 = 1,000,000 \text{ light years}$

$\lambda = \lambda_0 = 9.46 \cdot 10^{21} \text{ m}$

$\omega = \omega_0 = 2 \text{ rad /s.}$

$f_0 = 0.31 \text{ Hz}$

low cut-off frequency calculation

$$\omega_1 = 1.81 \cdot 10^{-11} \text{ rad/s}$$

$$f_1 = 2.88 \cdot 10^{-11} \text{ Hz; low cut-off frequency}$$

$$\lambda_1 = 1.08 \cdot 10^{33} \text{ m}$$

High cut-off frequency calculation:

$$\omega_2 = 2.19 \cdot 10^{11} \text{ rad/s}$$

$$f_2 = 0.348 \cdot 10^{11} \text{ Hz; high cut-off frequency}$$

$$\lambda_2 = 8.60 \cdot 10^{10} \text{ m}$$

For the low cut-off frequency, it is fulfilled:

$$E(\omega_1) = 0.707 E_{max} \quad (41)$$

$$E(t) = 1.08 \cdot 10^{73} \{e^{\wedge} - (1.81 \cdot 10^{-11}t)\} - 1.08 \cdot 10^{73} \{e^{\wedge} - (2.19 \cdot 10^{11}t)\} + E_0 \quad (42)$$

If we replace (41) in (42)

$$0.707 = 1 / e^{- (1.81 \cdot 10^{-11}t)}$$

$$t = \ln(1.41) / 1.81 \cdot 10^{-11}$$

$$t = 0.3467 / 1.81 \cdot 10^{-11}$$

$$t_1 = 1.915 \cdot 10^{10} \text{ s}$$

For the high cut-off frequency, it is fulfilled:

$$E(\omega_2) = 0.707 E_{max} \quad (43)$$

If we replace (43) in (42)

$$0.707 = 1 / e^{- (2.19 \cdot 10^{11}t)}$$

$$t = \ln(1.41) / 2.19 \cdot 10^{11}$$

$$t = 0.3467 / 2.19 \cdot 10^{11}$$

$$t_2 = 0.158 \cdot 10^{-11} \text{ s}$$

Observe Figure 5, we are going to calculate the time variation $I\Delta t$ between the frequency ω_2 and ω_1 .

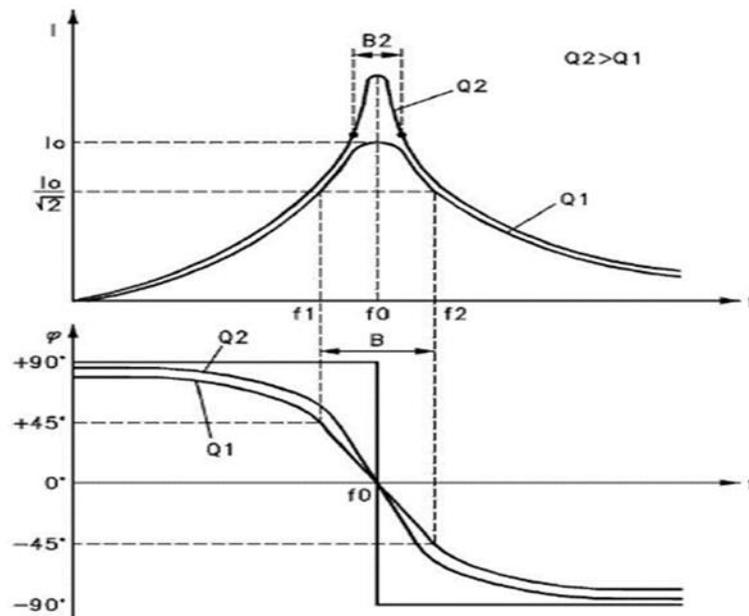


Figure 5. For a frequency difference given by ω_2 and ω_1 , we can observe that there is a phase difference, a time difference and therefore a displacement difference $I\Delta t$.

$$I\Delta t = I t_2 - t_1 I$$

$$I\Delta t = I 0.158 \cdot 10^{-11} \text{ s} - (-1.915 \cdot 10^{10} \text{ s}) I$$

Consider that t_1 originates much earlier than t_2

$$|\Delta t| = 1.915 \cdot 10^{10} \text{ s}$$

This variation of time occurs within the interval of expansion of space-time, inside the bandwidth of the equation of gravitational waves, therefore its speed corresponds to $3 \cdot 10^{21} \text{ m/s}$.

We will calculate the displacement variation $|\Delta X|$ for a variation of $|\Delta t| = 1.915 \cdot 10^{10} \text{ s}$.

$$|\Delta X| = v \times t$$

$$|\Delta X| = 3 \cdot 10^{21} \times 1.915 \cdot 10^{10}$$

$$|\Delta X| = 5.745 \cdot 10^{31} \text{ m.}$$

For the instant at which ω_2 occurs, ω_1 advances ω_2 by 90 degrees and this corresponds to a time difference $|\Delta t| = 1.915 \cdot 10^{10} \text{ s}$, and a difference in displacement $|\Delta X| = 5.745 \cdot 10^{31} \text{ m}$.

We show how space-time, as a function of frequency, expands asymmetrically.

6.8. Calculation of the critical mass to produce a black hole in the LHC applying the theory of the generalization of Boltzmann constant in curved spacetime

Method No. 1:

let us consider the average thermal energy equal to the gravitational potential energy of a black hole of radius R_s .

$$E_\varepsilon = E_{pg} \quad (44)$$

E_ε , Average thermal energy of the plasma of quarks and gluons.

E_{pg} , Gravitational potential energy of a black hole of three solar masses and radius $R_s = 8.89 \cdot 10^3 \text{ m}$.

$$N \times K_B \times T_\varepsilon = (G \times M \times M_1) / R_s \quad (45)$$

where N , number of elementary particles; K_B , Boltzmann 'constant; T_ε , electromagnetic temperature; G , gravitational constant; M , mass of a black hole of three solar masses; M_1 , unit mass and R_s , Schwarzschild's radius.

We consider the mass M_1 as a unitary body of mass 1 kg that orbits around a black hole of mass M and radius R_s .

$$(M/m) \times K_B \times T_\varepsilon = (G \times M \times M_1) / R_s \quad (46)$$

where M is the mass of a volume V and m , the amount of mass of a particle in the volume V .

We divide everything by the volume V in do

$$(M/V) \times m^{-1} \times K_B \times T_\varepsilon = G \times (M/V) \times M_1 \times R_s^{-1} \quad (47)$$

We consider on both sides that the densities $\rho = M/V$ are equal in (47):

$$(K_B \times T_\varepsilon) / m = (G \times M_1) / R_s \quad (48)$$

$$mc = (K_B \times T_\varepsilon \times R_s) / (G \times M_1) \quad (49)$$

Critical mass equation to create a black hole in the LHC

Calculation of the value of the critical mass mc :

$$K_B = 1.78 \cdot 10^{-43} \text{ J/K}$$

$$T_\varepsilon = 10^{13} \text{ K}$$

$$R_s = 8.89 \cdot 10^3 \text{ m}$$

$$G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}$$

$$M_1 = 1 \text{ kg}$$

replacing in (49), we have:

$$mc = 2.37 \cdot 10^{-16} \text{ kg}$$

$$mc = 13.33 \cdot 10^{10} \text{ GeV}/c^2$$

Method No. 2:

Let's start from the following assumption:

$$Em = E_{pg} \quad (50)$$

where E_m , energy of the mass at rest and E_{pg} , gravitational potential energy of a black hole of mass M and radius R_s .

$$m \times c^2 = G \times M \times M1 \times R_s^{-1} \quad (51)$$

where m , mass corresponding to a volume V ; c , speed of light; G , gravitational constant; M , mass of a black hole of three solar masses; $M1$, unit mass and R_s , Schwarzschild's radius.

Remember that energy is also defined as follows:

For a photon of wavelength λ ,

$$E = (h \times c) / \lambda \quad (52)$$

where E , energy of a photon; h , Planck's constant; c , speed of light and λ , wavelength of a photon.

replacing we have:

$$(N \times h \times c) / \lambda = G \times M \times M1 / R_s \quad (53)$$

where N is the number of photons

If we consider the following conditions:

$$\lambda / 2 = 2 \times \pi \times R_s \quad (54)$$

We get the following equation:

$$N \times h \times c = 4 \pi \times G \times M \times M1 \quad (55)$$

M is the mass of a volume V .

m , the amount of unit mass equivalent in volume V .

$$N = M/m \quad (56)$$

replacing we have:

$$(M/m) \times h \times c = 4 \times \pi \times G \times M \times M1 \quad (57)$$

We divide everything by the volume V , we have:

$$((M/V) \times h \times c) / m = 4 \times \pi \times G \times (M/V) \times M1 \quad (58)$$

If we consider that the density (M/V) is the same on both members, we obtain:

$$(h \times c) / m = 4 \times \pi \times G \times M1 \quad (59)$$

$$mc = h \times c / (4 \times \pi \times G \times M1) \quad (60)$$

Planck's critical mass equation

Calculation of the value of the critical mass mc :

$$c = 3 \times 10^8 \text{ m/s}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}$$

$$M1 = 1 \text{ kg}$$

$$h = 6.62 \times 10^{-34} \text{ J.s}$$

replacing in (60), we have:

$$mc = 2.37 \times 10^{-16} \text{ kg}$$

$$mc = 13.33 \times 10^{10} \text{ GeV}/C^2$$

We have verified that both methods to calculate the critical mass to produce a black hole in the LHC reach the same result.

- Planck's mass $m_p = 1.2 \times 10^{19} \text{ GeV}/C^2$; it is not the limit to form a black hole in a particle accelerator.
- $13.33 \times 10^{10} \text{ GeV}/C^2$. It should be considered a critical or limit value and we should not work with higher values in the LHC.

Using two different methods, we must emphasize how the results of the equations shown below are practically the same for $K_B = 1.78 \times 10^{-43} \text{ J/K}$.

Here we put forward the hypothesis that there is a critical mass mc to produce a black hole at the LHC, given by the following equations:

- $mc = (K_B \times T \epsilon \times R_s) / G \times M_1$; $mc = 13.33 \cdot 10^{10} \text{ GeV}/C^2$.
- $mc = h \times c / (2\pi \times G \times M_1)$; $mc = 13.33 \cdot 10^{10} \text{ GeV}/C^2$.
- This equality is given for $K_B = 1.78 \cdot 10^{-43} \text{ J/K}$

Example:

Currently, the CERN particle accelerator is working with energies of the order of 14 TeV.

If we consider that the LHC works with proton packages of 100,000 10^6 protons (22), we have:

$$M_p = 100,000 \cdot 10^6 \times m_p$$

Where M_p , total mass of the collision and m_p , proton mass.

$$M_p = 10^{11} \times 1.672 \cdot 10^{-27} \text{ kg} = 1.672 \cdot 10^{-16} \text{ kg}$$

$$M_p = 1.672 \cdot 10^{-16} \text{ kg}$$

$M_p \approx mc$, we are working on the order of the critical mass to produce a black hole at the LHC.

6.9. Calculation of the critical temperature T_c of the high-temperature Bose-Einstein Condensate applying the theory of the generalization of Boltzmann's constant in curved space-time.

Bose-Einstein Condensate:

In quantum mechanics, all particles have an associated wave and at normal temperature this wave is very small, much smaller than the typical distance between atoms in a gas, but as the temperature decreases the wavelength increases.

Below a critical temperature, the particle waves begin to overlap and interact, interfering and forming a joint wave. This is what is called a Bose-Einstein condensate, a state where atoms act together, coherently, with a single wave function for all atoms, where the quantum wave properties of matter are visible, we can't talk about individual atoms, but of the whole as a single atom; like a big atom.

This can be understood as follows:

In a physical system, particles can be established at various energy levels, discrete levels, at a certain temperature, atoms are randomly distributed among all available energy levels. Now, if I decrease the temperature, the discrete levels of energy available go down, so the lowest energy levels are taken. If the temperature is too low, passing a critical threshold, only the minimum level of energy, the fundamental level, is available. Here we clarify that the Bosons are all characterized by being able to occupy the same energy level, the fundamental one; then they become indistinguishable, they behave like a single atom, a big atom, and their quantum properties become macroscopic, visible.

Plasma of quarks and gluons – confinement:

Quark-gluon plasma is created in RHIC or LHC particle collisions, colliding gold or lead particles at relativistic speeds with energies on the order of Tera electron-volts. The quark-gluon plasma is a perfect liquid and its temperature is of the order of 10^{12} kelvin.

The temperature generated in these collisions is so high that the quarks and gluons that were confined within the protons and neutrons in atomic nuclei break apart, and for a short time, move freely.

Quarks and gluons have a property called confinement, and under normal temperature conditions, they form protons and neutrons (baryons). This means that if we consider the color of the quarks (red, green or blue), they must always form states of white color and cannot escape from the ball of neutrons or protons, that is, they are confined.

White color states can also be formed with a quark and an antiquark, called mesons.

The confinement tells us that the force of interaction experienced by the quarks is a function of energy. At very high energy, the interaction between quarks and gluons is very weak, a weak coupling. At low energy the interaction is strong, quarks cannot escape from confinement because the interaction of quarks and gluons is strong, a strong coupling.

Analysis of an isolated quark

Let's see what happens if we drop a quark into the quantum vacuum. We know that gluons have charges, they are charged colored dipoles, they are carriers of the strong force.

Now, if we throw a quark and a gluon into the quantum vacuum, we observe that the gluon is oriented with the same color charge in the direction of the quarks, this is called antiscreening, this causes the quark's strength to increase.

So, when we drop a quark into a vacuum, the gluons start accumulating, forming an infinite cascade with infinite energy. An isolated quark has infinite energy.

This would explain why in nature quarks are not found in an isolated state, except inside a black hole.

Here we put forward the hypothesis that, when the baryons that make up the matter we know reach a limit of temperature and energy density, the Hagedorn temperature, a phase change occurs in the matter, the quarks and gluons inside it no longer belong to the nuclei of protons and neutrons, begin to move freely, isolated, they are no longer confined, forming a plasma of quarks and gluons. If we continue to increase the temperature and energy density, we reach another critical threshold T_c , at which the plasma of quarks and gluons, a superfluid as a whole, behaves like a single quark whose gluons begin to stack up in an orderly fashion, forming a cascade of infinite energy, that is, a black hole is created, which makes it the most energetic state of existing matter, with almost infinite energy.

Calculation of the critical temperature T_c of the high-temperature Bose-Einstein Condensate:

Equation that defines T_c for a Bose-Einstein condensate

$$T_c = \left(\frac{n}{\zeta(3/2)} \right)^{2/3} \frac{2\pi\hbar^2}{mk_B} \approx 3.3125 \frac{\hbar^2 n^{2/3}}{mk_B}$$

where:

T_c is the critical temperature,

n the particle density,

m the mass per boson,

\hbar the reduced Planck constant,

k_B the Boltzmann constant and

ζ the Riemann zeta function; $\zeta(3/2) \approx 2.6124$.

According to the information of (24), Cauê Muraro - Agência USP - 10/30/2007, the temperature of a Bose-Einstein condensate for 100,000 rubidium atoms corresponds to $T_{cmin} = 180$ nK.

where T_{cmin} , low temperature Bose-Einstein condensate.

For $K_B = 1.38 \cdot 10^{-23}$ J/K and rubidium atoms corresponds:

$T_{cmin} = 180$ nk

Approximate critical temperature of the Bose-Einstein condensate for low temperatures, with rubidium atoms.

Let's calculate T_{cmax} , for $K_B = 1.78 \cdot 10^{-43}$ J/K

Where T_{cmax} , High temperature Bose-Einstein condensate.

T_{cmax} , we are going to calculate considering the relationship between the Boltzmann constant $K_B = 1.38 \cdot 10^{-23}$ J/K, for flat space-time and $K_B = 1.78 \cdot 10^{-43}$ J/K for curved space-time.

For $K_B = 1.78 \cdot 10^{-43}$ J/K and rubidium atoms corresponds:

$T_{cmax} = 180$ nk / $1,78 \cdot 10^{-20} = 1,01 \cdot 10^{13}$ K

$T_{cmax} = 1.01 \cdot 10^{13}$ K

Critical temperature of the Bose-Einstein condensate for high temperatures with rubidium atoms.

Here we put forward the hypothesis that for an effective Boltzmann's constant $K_B = 1.78 \cdot 10^{-43}$ J/K, there is a temperature T_{cmax} , that corresponds to a high temperature Bose Einstein condensate.

For a temperature of approximately $1.01 \cdot 10^{13}$ K, in a plasma of quarks and gluons, a phase transition occurs that gives rise to a Bosonic-Fermionic condensate, at high temperatures, which is characterized by being very energetic.

We can interpret it as follows, when a star collapses and a black hole is formed, we can affirm that a high-temperature Bose-Einstein condensate exists inside a black hole.

In analogy with the properties of materials at very low temperatures, super fluids and superconductivity; quark-gluon plasma achieves similar exotic properties, but not with atoms and molecules as we normally know; these properties are achieved for the quark-gluon plasma, a superfluid or super solid, the main property of which makes this liquid or solid behave like isolated quarks, allowing the gluons to stack up neatly in an infinite cascade of energy, making it the most energetic matter in the universe. We also said that quarks are fermions and gluons are bosons, but in black holes, by analogy with what happens with superconducting materials, and super fluids and super solids, the plasma of quarks and gluons as a whole act as a Bose-Einstein condensate, as a single atom whose macroscopic properties are unique.

6.10. Intuitively we will demonstrate the existence of a tangential force to the repulsive force in subatomic decays analogous to the tangential force f_t that lags the force f_c by 90 degrees inside black holes.

We will analyse the following equation:

$$E^2 = m^2c^4 + P^2c^2 \quad (61)$$

where E is energy, m is mass, and c is the speed of light in a vacuum.

If we consider 0 the moment P of a particle, $P = 0$, we have:

$$E^2 = m^2c^4$$

$$E = (+/-) mc^2$$

If we consider mass as a fundamental property of matter we have:

$E = + mc^2$, positive energy, (+ m), gravity attracts.

$E = - mc^2$, negative energy (- m), gravity repels.

According to the equation $E = (+/-) mc^2$, we have that gravity acts in two ways, (+m) as an attractive force or (-m) as a repulsive force.

METRIC FOR TIME TYPE TRAJECTORIES.

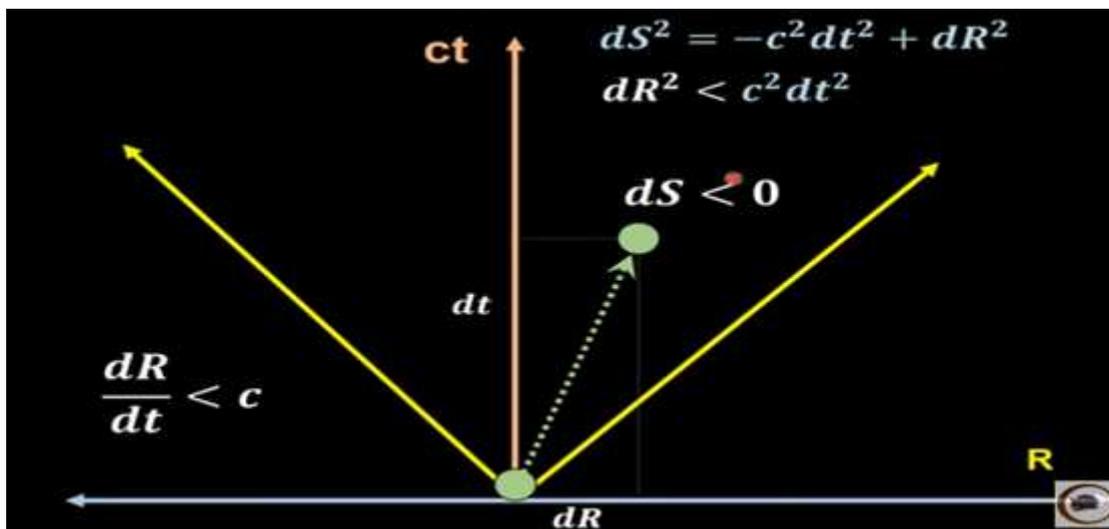


Figure 6. Time-like trajectory, light cone, $ds < 0$.

Let's write the metric:

$$d\tau^2 = dt^2 - (dx/c^2 + dy/c^2 + dz/c^2) > 0 \quad (62)$$

This metric is defined for speeds less than light, $v < c$.

We skip the mathematical steps and with this metric we calculate the moment P and the energy E.

$$P = mv / \sqrt{1 - \left(\frac{v^2}{c^2}\right)}, \text{moment of a particle.} \quad (63)$$

$$E = mc^2 / \sqrt{1 - \left(\frac{v^2}{c^2}\right)}, \text{energy of a particle.} \quad (64)$$

If we analyse the energy, we see that when the particle is at rest the energy corresponds to $E = mc^2$; when the speed tends to c , the energy tends to infinity.

$$v = 0, E = mc^2$$

$$v \rightarrow c, E \rightarrow \infty$$

Now we are going to perform the following mathematical trick, although the metric does not allow us to do this because it is not defined for speeds greater than light, $v > c$, we are going to see the consequences of the following mathematical operation.

$$E = mc^2 / \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

multiplying the numerator and denominator by the imaginary number i :

$$E = -i mc^2 / \left(\sqrt{\left(\frac{v^2}{c^2}\right) - 1}\right) \quad (65)$$

we see that the terms $-i$ appear.

If we compare with the mass of a black hole:

$$M = m - i\delta \quad (66)$$

m , baryonic mass.

$$-i mc^2 / \left(\sqrt{\left(\frac{v^2}{c^2}\right) - 1}\right) = -i\delta, \text{ for } v > c; \text{ mass of dark matter.}$$

How can we interpret this, what meaning does it have?

Although the metric we use is not defined for particles that move at a speed greater than that of light, there are massless, tachyonic particles that can cross this barrier and travel at a speed greater than that of light.

These tachyonic particles produce a tangential force F_t to the attractive force F_c of gravity and as the speed increases with respect to the speed of light, they generate dark matter. It must be made clear that these particles are inside the black hole.

METRIC FOR SPACE TYPE TRAJECTORIES

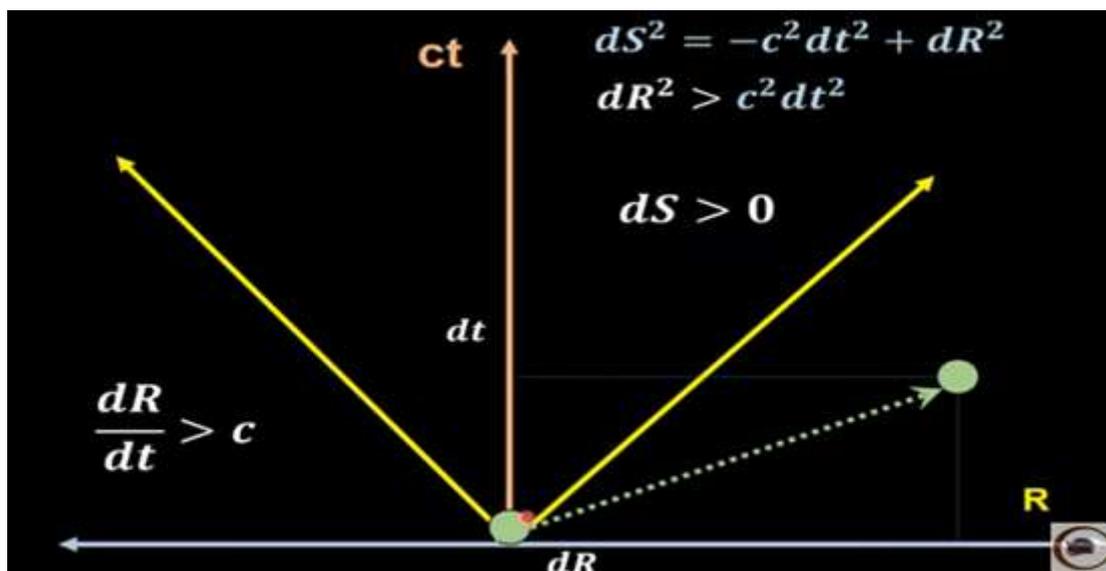


Figure 7. Space-like trajectory, light cone, $ds > 0$.

Let's write the metric:

$$d\tau^2 = -dt^2 + (dx/c^2 + dy/c^2 + dz/c^2) < 0 \quad (67)$$

We skip the mathematical steps and with this metric we calculate the moment P and the energy E.

$$P = -mv / \sqrt{(v^2/c^2) - 1} \quad (68)$$

$$E = -mc^2 / \sqrt{(v^2/c^2) - 1} \quad (69)$$

These equations for momentum P and energy E are valid for speeds greater than light and can never reach speeds of light.

$$v \rightarrow \infty, E = 0$$

$$v \rightarrow c, E = -\infty$$

How can we interpret this, what does it mean?

We are going to relate the equations of P and E with the electrical model RLC of the universe, at the moment that the black hole explodes, let us remember that the space-time that was compressed begins to expand and generates a well of gravitational potential of negative energy analogous to the equation $E = -mc^2 / \sqrt{(v^2/c^2) - 1}$, in other words, a spectrum of gravitational waves is produced that produce a repulsive force that gives rise to the expansion of space-time. In this case, tachyons are related to gravitons, in which, during the period of cosmic inflation, they travel at a speed greater than that of light.

Now we are going to perform the following mathematical trick, although the metric does not allow us to do this because it is not defined for speeds less than light, $v < c$, we are going to see what happens if a particle exceeds the limit for speeds less than c.

$$E = -mc^2 / \sqrt{(v^2/c^2) - 1}$$

$$E = -mc^2 / \sqrt{-1} \sqrt{(1 - v^2/c^2)}$$

Multiplying and dividing by the imaginary number i.

$$E = i mc^2 / (\sqrt{(1 - (v^2/c^2))}) \quad (70)$$

If we compare with the mass of a black hole:

$$M = m - i\delta \quad (71)$$

m , baryonic mass.

$$-M = -m + i\delta$$

$$i mc^2 / \sqrt{(1 - v^2/c^2)} = i\delta \quad (72)$$

where $-M$ represents the negative total mass, the minus sign indicates that it is repulsive, $-m$ represents negative baryonic mass, the minus sign indicates that it is repulsive and $i\delta$ is a mass tangential to the mass $-m$ and leads 90 degrees to the mass $-m$.

The subatomic disintegrations that occur in particle accelerators represent a clear example.

Here we put forward the hypothesis that, for $v < c$, there is an additional force that corresponds to the mass $i\delta$ that leads 90 degrees to the force given by the mass $-m$, in other words, when the subatomic disintegration of particles occurs, two forces act, a repulsive force given by the mass $-m$ and a tangential force that leads 90 degrees to the force given by $-m$, resulting from the mass $i\delta$.

In the following tables we will define the statement.

Table 3. From left to right represented by the numbers 1,2 and 3; We describe the forces that act on matter. In phase 1, for $v < c$, only an attractive force acts; in phase 2, for $v = c$, only an attractive force acts; in phase 3, for $v > c$, inside a black hole, we can see that two forces act, an attractive force and a tangential force that delays the attractive force by 90 degrees.

TIME TYPE PATH	LIGHT TYPE PATH	SPACE TYPE PATH
1	2	3
$ds < 0$	$ds = 0$	$ds > 0$

$v < c$	$v = c$	$v > c$
m	m	$M = m - i\delta$
attraction	attraction	attraction
$Lp = Lpe$	$Lpg = Lpe = Lp$	$Lpg < Lpe$
$E = m c^2 / \sqrt{1 - (v^2/c^2)}$	Phase change	$E = -i \delta c^2 / \sqrt{(v^2/c^2) - 1}$

Table 4. From right to left, represented by the numbers 1,2 and 3, we will describe the forces that act on matter. In phase 1, for $v > c$, we see that a repulsive force acts, in phase 2, for $v = c$, we see that a repulsive force acts; in phase 3, for $v < c$, we see that two forces act, a repulsive force and a tangential force that leads the repulsive force by 90 degrees.

TIME TYPE PATH	LIGHT TYPE PATH	SPACE TYPE PATH
3	2	1
$ds < 0$	$ds = 0$	$ds > 0$
$v < c$	$v = c$	$v > c$
$-M = -m + i\delta$	$-m$	$-m$
Repulsion	Repulsion	Repulsion
$Lp = Lpe$	$Lpg = Lpe = Lp$	$Lpg < Lpe$
$E = i\delta c^2 / \sqrt{1 - (v^2/c^2)}$	Phase change	$E = -m c^2 / \sqrt{(v^2/c^2) - 1}$

FORCES AT WORK IN THE DISINTEGRATION OF SUBATOMICAL PARTICLES

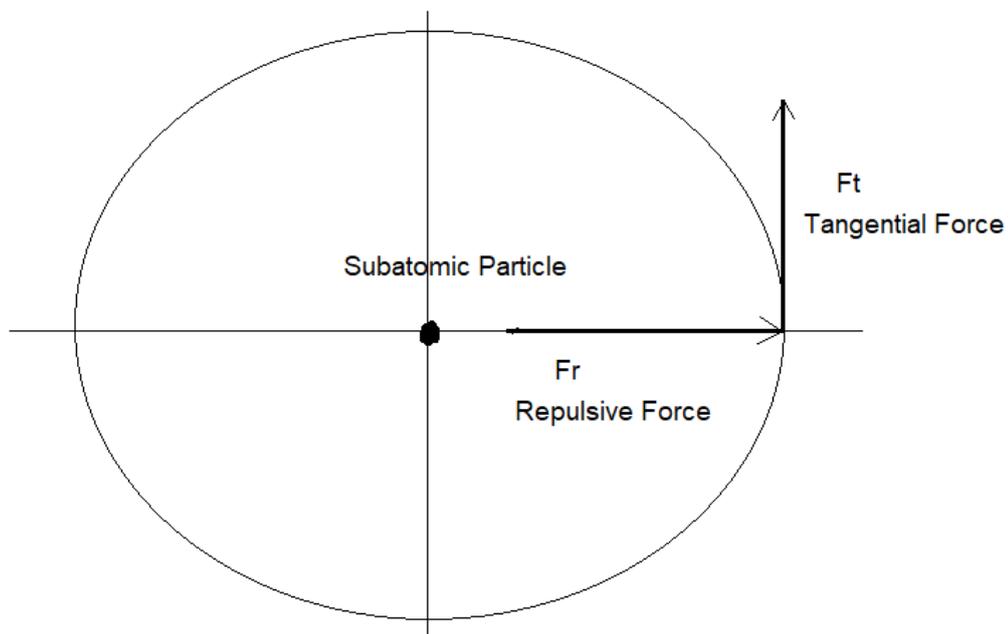


Figure 8. Diagram of forces that act in the disintegration of subatomic particles.

7. Conclusions:

Through the calculations and analysis carried out in item 6), it is possible to demonstrate that the theory proposed in items 1) to 5), is a theory that complements the Lambda-CDM model, in which several of the important calculations that predict the theory of the RLC electric model of the universe conforms to the predictions of the Lambda CDM model.

It was also possible to demonstrate, how the curvature of space-time is measured, by means of the theory of the generalization of space-time for curved space-time, fundamental, for the theory of the RLC electrical model of the universe to work.

The RLC electric model theory of the universe generalizes the Lambda-CDM model theory and predicts the origin of the universe, the origin of cosmic inflation, the origin of dark matter and the origin of dark energy. It also solves the horizon problem and demonstrates how the universe is flat and uniform.

In the update of this article, version 2, we calculated the existence of the critical mass to produce a black hole in the LHC and proved the existence of a high-temperature Bose-Einstein condensate, applying the theory of generalization of the constant Boltzmann in curved spacetime.

We also show intuitively that in the disintegrations of subatomic particles, two forces act, a repulsive force whose direction is opposite to the attractive force and a tangential force that leads the repulsive force by 90 degrees. If we manage to demonstrate the existence of this tangential force in subatomic disintegrations, we can infer the existence of the tangential force inside black holes.

About the author

HECTOR GERARDO FLORES (ARGENTINA, 1971). I studied Electrical Engineering with an electronic orientation at UNT (Argentina); I worked and continue to work in oil companies looking for gas and oil for more than 25 years, as a maintenance engineer for seismic equipment in companies such as Western Atlas, Baker Hughes, Schlumberger, Geokinetics, etc.

Since 2010, I study theoretical physics in a self-taught way.

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Conflicts of Interests: The author declares that there are no conflicts of interest.

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