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A Unified Spinor Description of the Photon and Electron Relativistic Fields

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Abstract: We propose a description of the electromagnetic field in the form of a four-component complex spinor, from which a vector of electromagnetic potential with two degrees of freedom, calibrated by two conditions - zero length and zero component along the y-axis - is obtained by using Pauli matrices. A similar approach is applied to the field of a fermion, in particular, the electron. It is known that the quantum field of the electron and the electron itself is a four-component complex spinor, so, existing in the Minkowski vector space, we cannot observe it directly. But with the help of Pauli matrices a vector is formed from the electron spinor, which is known to us as an electric current vector, and this current vector describes exactly a single particle. As a vector, it is available to us for observation in our vector space. Similarly, the electromagnetic field and its photon particle is also a four-component spinor, from which the universal formula using Pauli matrices produces a vector, it is known to us as the electromagnetic potential vector, and it too describes even a single photon. All the differences in the properties of the current vector and the electromagnetic potential vector, and hence the electron and the electromagnetic field, are due only to a slight difference in the structures of their four-component spinors and inextricable linked to them momentum spinors and coordinate spinors. Expressions for the electric and magnetic fields of a photon during its interaction with an electron, including the Pauli matrices and momentum spinors of these particles, are presented. Thus, a unified way to describe bosons and fermions in spinor space is proposed. The consequences of the Dirac equation for the electron spinor are considered and the existence of a similar first-order equation for the photon spinor is assumed. Each spinor using the same formula corresponds to a vector, in the case of a fermion it is a current vector, in the case of a boson it is a vector, for example, of the electromagnetic potential. Each spinor of a field is matched with a spinor of coordinates and a spinor of momentum, which are transformed by the same Lorentz transformations and which have the same structure as their corresponding field spinor, that is, the momentum and coordinates of boson have a bosonic spinor structure, while momentum and coordinates of fermion are a spinor with a fermionic structure. Field, coordinates and momentum vectors of boson automatically have a zero length, while in the case of fermion they all have a nonzero length, so the fermion, in contrast to the boson, has a nonzero mass, nonzero charge and moves with a sub light speed. The presented approach, in the long run, makes it possible to carry out calculations of the interaction of particles in two-dimensional spinor space, and to interpret in terms of the Minkowski vector space only the final results. While quantum mechanics treats probability as a real number, quantum field theory deals with probability as a four-dimensional real vector. The place of the probability amplitude, which in quantum mechanics is a complex number, in quantum field theory is taken by a complex spinor.

Keywords: quantum field theory; Minkowski space; electromagnetic potential calibration; Lorentz force; Casimir operators; wave equation; Dirac equation

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1. Natural calibration of the electromagnetic potential

This paper uses a description of electrodynamics, relying as much as possible on group transformations of coordinates and relativistic fields. We use the Minkowski space with the metric $\eta_{\mu\nu} = \eta^{\mu\nu}$ and the signature (+---). Let us denote the contravariant and covariant coordinates with the speed of light equal to one as

$$X^\mu = (t, X, Y, Z) \equiv (X^0, X^1, X^2, X^3)$$
$$X_\mu = (X_0, -X_1, -X_2, -X_3)$$

$$X_\mu = \eta_{\mu\nu} X^\mu$$

Let us denote the real components of the electromagnetic potential as

$$A^\mu = (A_t, A_x, A_y, A_z) \equiv (A^0, A^1, A^2, A^3)$$

Consider the linear homogeneous Lorentz transformation of the Minkowski space coordinates

$$X'^\mu = \Lambda^\mu{}_\alpha X^\alpha$$

where the matrix $\Lambda^\mu{}_\alpha$ with coefficients independent of coordinates has the property

$$\eta_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = \eta_{\mu\nu}$$

The Lorentz transformations form a Lie group; we will use its identity (attached) representation in the form of the Lorentz transformation matrices themselves. We will also use the Poincaré group and its operator representation.

Suppose that at a point in space with coordinates x^μ, the potential is described by contravariant quantities A^μ. We are interested in the magnitude of the potential at a point with transformed coordinates $\Lambda^\mu{}_\alpha x^\alpha$. The key assumption we will further rely on is that the vector of potential at the transformed point can be obtained from the vector of potential at the original point using the same transformation matrix with which the transformed coordinates are obtained from the original coordinates

$$A^\mu(\Lambda^\delta{}_\gamma X^\gamma) = \Lambda^\mu{}_\alpha A^\alpha(X^\gamma)$$

In this case we will call A^μ a relativistic field. There are other definitions of the relativistic field that use the coordinate system change procedure, the principles of symmetry, invariance, and covariance. We do not use mental transitions to other reference systems, but work in one chosen one. We define the antisymmetric covariant tensor of the electromagnetic field as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

where

$$\partial_\mu \equiv \frac{\partial}{\partial X^\mu}$$

The electric and magnetic field components are components of this tensor

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

It is known [1] that $F_{\mu\nu}$ is transformed as a tensor even though A_μ may not be transformed as a vector. In fact, it will be shown further that the vector potential is a vector indeed. The assumptions made are already sufficient to calculate electric and magnetic fields in practical situations and to check in experiments the validity of the assumption made about the properties of the relativistic field. The transformation law of any tensor has the form

$$F_{\mu\nu}(\Lambda^\delta{}_\gamma X^\gamma) = \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu F_{\alpha\beta}(X^\gamma)$$

with this transformation, we can find the electric and magnetic fields at the transformed point from the known values of the fields at the starting point.

The Lorentz transformation group includes spatial rotations and boosts. Let us know the fields at the stationary point at the origin of the used reference system at zero moment of time. Let us choose for the example the simplest Lorentz transformation in the form of a boost in the x-axis direction with velocity v with the corresponding boost parameter in the form of an angle

$$\beta = \mathrm{arcth}(v/c)$$

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$$\Lambda = \begin{pmatrix} \cosh(\beta) & \sinh(\beta) & 0 & 0 \\ \sinh(\beta) & \cosh(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transformed point will still be at the origin of coordinates at zero moment of time, but will have velocity v along the x-axis. The transformed electromagnetic field tensor will contain new components of electric and magnetic fields. If we perform all necessary transformations to the tensor, we should obtain well-known field transformation formulas which, for the general case of an arbitrary velocity of the observation point $\mathbf{v}$, look like

$$\hat{\mathbf{E}} = \gamma(\mathbf{E} - (\mathbf{v} \times \mathbf{B})) - \Gamma\mathbf{v}(\mathbf{v} \cdot \mathbf{E})$$

$$\hat{\mathbf{B}} = \gamma\big(\mathbf{B} + (\mathbf{v} \times \mathbf{E})\big) - \Gamma\mathbf{v}(\mathbf{v} \cdot \mathbf{B})$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}} \qquad \Gamma = \frac{\gamma - 1}{v^2}$$

If a charge is placed in this point, it will be subject to a force from the electric field to which a contribution equal to the vector product of velocity by the value of the initial magnetic field at the stationary point is added during the transformation. And this force from the electric field will be the only force acting on the charge. There will be no effect on the charge from the magnetic field with the new value, additional accounting of the effect on the charge of the transformed magnetic field will lead to the total effect, contradicting the experiment. This example illustrates the fact that a charge, whether stationary or moving, is only affected by an electric field and never by a magnetic one. The concept of a Lorentz force acting on a moving charge from the magnetic field is superfluous; all movements of the charge are accounted with the Lorentz transformations of the electromagnetic field and give a magnitude of force from the transformed electric field which is consistent with experiment.

As we can see, to recalculate the electric and magnetic fields in the Lorentz group transformations, we do not need to know the values of the electromagnetic potential and do not need to transform it. But let us consider the more complicated case when the angles of rotations and boosts determining the Lorentz transformations are not constants but depend on coordinates. Let us again consider the simplest case of the boost on the x-axis. The transformation in this case has the form

$$\Lambda(\beta(x^\mu)) = \begin{pmatrix} \cosh(\beta) & \sinh(\beta) & 0 & 0 \\ \sinh(\beta) & \cosh(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \exp\left(\beta(x^\mu)\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right)$$

Since the angle β depends on the coordinates, the corresponding velocity

$$v/c = \mathrm{th}(\beta)$$

depends on the coordinates, which means that the derivatives of the velocity along all four coordinates are, generally speaking, not equal to zero, i.e., the observation point moves accelerated after the transformation. To calculate the electric and magnetic fields in such an accelerated moving point, we already have to refer to the definition of the electromagnetic field tensor

$$F_{\mu\nu}\left(\Lambda^{\delta}{}_{\gamma}X^{\gamma}\right) = \Lambda^{\alpha}{}_{\mu}\partial_{\alpha}\left[\Lambda(X^{\gamma})^{\beta}{}_{\nu}A_{\beta}\right] - \Lambda^{\beta}{}_{\nu}\partial_{\beta}\left[\Lambda(X^{\gamma})^{\alpha}{}_{\mu}A_{\alpha}\right] =$$

$$= \Lambda^{\alpha}{}_{\mu}\Lambda(X^{\gamma})^{\beta}{}_{\nu}\partial_{\alpha}\left[A_{\beta}\right] - \Lambda^{\beta}{}_{\nu}\Lambda(X^{\gamma})^{\alpha}{}_{\mu}\partial_{\beta}\left[A_{\alpha}\right] + \Lambda^{\alpha}{}_{\mu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\alpha}}A_{\beta}\right]$$

$$- \Lambda^{\beta}{}_{\nu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\beta}}A_{\alpha}\right]$$

$$= \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\partial_{\alpha}A_{\beta} - \Lambda^{\beta}{}_{\nu}\Lambda^{\alpha}{}_{\mu}\partial_{\beta}A_{\alpha} + \Lambda^{\alpha}{}_{\mu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\alpha}}A_{\beta}\right] - \Lambda^{\beta}{}_{\nu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\beta}}A_{\alpha}\right]$$

$$= \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}F_{\alpha\beta}(X^{\gamma}) + \Lambda^{\alpha}{}_{\mu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\alpha}}A_{\beta}\right] - \Lambda^{\beta}{}_{\nu}\left[\frac{\partial\Lambda(X^{\gamma})^{\beta}{}_{\nu}}{\partial X^{\beta}}A_{\alpha}\right]$$

Now, because of the dependence of the boost angle on the coordinates, the expression for the transformed tensor contains additional terms that depend on derivatives of the form $\frac{\beta(x^{\mu})}{\partial x^{\alpha}}$.

Since we now need to know the values of the electromagnetic potential explicitly, we turn to the question of its calibration. Calibration introduces constraints that prevent the potential from taking arbitrary values, while the coordinate vector has no such constraints and can take any values. Hence, there are doubts whether the vector of potential with imposed restrictions can transform according to the same law as the coordinates without any restrictions imposed on them. To eliminate this discrepancy, we propose to switch from considering real four-dimensional vectors to two-dimensional complex spinors. Let us take an arbitrary two-dimensional complex spinor and call it a coordinate spinor

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Let us compare to each coordinate spinor the spinor of the electromagnetic potential

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Transformations from the SL(2,C) group can be applied to the coordinate spinors. Let the matrix M belongs to this group and perform the spinor transformation

$$\mathbf{x}' = M\mathbf{x}$$

Let us know the values of the components of the field spinor $\mathbf{a}$ associated with the coordinate spinor $\mathbf{x}$, and we want to know their values for the field spinor corresponding to the transformed coordinate spinor. As in the case of four-dimensional vectors, we will assume that when the coordinate spinor is transformed, the field spinor is transformed by the same law

$$\mathbf{a}(M\mathbf{x}) = M\mathbf{a}(\mathbf{x})$$

The coordinate spinor and the field spinor can take arbitrary values, so the coincidence of their transformation laws looks more natural.

From spinors we want to go to the real coordinate space of vectors and vector potentials. This can be done by means of Pauli matrices

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which allow us to determine four components of the coordinate vector

$$X_\mu = \mathbf{x}^\dagger \sigma_\mu \, \mathbf{x}$$

and the field vector

$$A_\mu = \mathbf{a}^\dagger \sigma_\mu \, \mathbf{a}$$

So defined vectors have real components and zero lengths in Minkowski space, that is

$$\eta_{\alpha\beta} X^\alpha X^\beta = X_\beta X^\beta = 0$$

$$\eta_{\alpha\beta} A^\alpha A^\beta = A_\beta A^\beta = 0$$

The same real vector with zero length from an arbitrary complex spinor can also be obtained in another way, by forming a tensor by direct product of the spinor on itself with a complex conjugation and taking the trace of its product with the corresponding Pauli matrix

$$A_\mu = \mathrm{Tr}[(\mathbf{a}\mathbf{a}^\dagger)\sigma_\mu]$$

We can go in the opposite direction and first construct a spintensor from the potential vector using Pauli matrices

$$S = A_\mu \sigma_\mu$$

and then impose a zero-length requirement on this vector

$$A_\beta A^\beta = 0$$

In this case, the determinant of the spintensor will be zero

$$det\ S = 0$$

and then S can be represented as a direct product of two arbitrary and generally different complex spinors. In the case of nonzero vector length, the derived spinor cannot be represented in this form, and when transformed using the elements of the SL(2,C) group, it is transformed as a combination of left and right spinors having different conversion laws. Thus, the calibration of the potential in the form of a zero vector length requirement entails a simplification of the structure of the spintensor and its representation as a direct product of arbitrary spinors. This takes place in the case of a complex initial vector, and if the initial vector is a real vector, the spinors coincide. A real vector also can be obtained with the direct product of two different real spinors.

SL(2,C) includes rotations by angle α

$$M = \exp\left(\frac{1}{2} i\alpha\sigma_\mu\right)$$

and boosts by the angle β

$$M = \exp\left(\frac{1}{2} \beta\sigma_\mu\right)$$

The vector obtained from the transformed spinor is equal to the vector obtained from the original spinor and subjected to a rotation or boost by the same angle using the Lorentz transformation matrix

$$\Lambda^\mu_\nu = \frac{1}{2}\mathrm{Tr}[\sigma_\mu M \sigma_\nu M^\dagger]$$

The spinor transformation matrix M can be the product of any number of rotations and boosts.

It is possible to consider the spinor as a more fundamental description of the electromagnetic potential; if the coordinate spinor is transformed, the field spinor is transformed by the same law. The corresponding field vectors are automatically transformed by the same law as the coordinate vectors and they are immediately calibrated and have zero length, just like the corresponding coordinate vectors. Now the same transformation of coordinate and field vectors looks more natural,

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since their set of values is equally bounded by zero length in Minkowski space. Thus, it turns out that the electromagnetic field, in particular light, is described by a light-like vector with zero length.

Returning to the above example with a given vector of potential at the origin of coordinates at a zero moment in time, we note that these coordinates correspond to a zero coordinate spinor, which remains zero after the boost, while the corresponding nonzero field spinor changes. If the coordinate spinor is not subjected to boost but to translation, the coordinate vector obtained from it will no longer be zero, that is, it will also undergo translation, although it will have a zero length. We are interested in how the field spinor and the vector obtained from it will change. The translation of a field vector can be described if a four-dimensional momentum vector, which also has zero length in Minkowski space, is given. The effect of translation on the field vector is expressed in the multiplication of the field vector components by the phase multiplier as an exponent of the scalar product of the vector of coordinate translations on the momentum vector. The zero-length momentum vector can be obtained from the corresponding momentum spinor. Thus, each coordinate spinor can be matched with a field spinor and a momentum spinor. The question arises what actions should be performed with the coordinate and momentum spinor to obtain an analogue of the scalar product of the coordinate and momentum vector. The matrix M and its corresponding matrix Λ synchronously transform spinors and coordinate and field vectors; the same matrices synchronously transform spinor and momentum vector. That is, by setting the matrix M, we thereby simultaneously set six synchronous transformations in which the relations between the triplet of spinors and the triplet of vectors remain unchanged. The relations between the spinor $\mathbf{p}$ and the momentum vector $\mathbf{P}$ are

$$P_\mu = \mathbf{p}^\dagger \sigma_\mu \, \mathbf{p}$$

Scalar product of the transformed momentum and coordinate vectors

$$\eta_{\alpha\beta}(\Lambda^\alpha_{\ \mu} P^\mu)(\Lambda^\beta_{\ \nu} X^\nu) = \eta_{\alpha\beta}\Lambda^\alpha_{\ \mu}\Lambda^\beta_{\ \nu} P^\mu X^\nu = \eta_{\ \mu\nu} P^\mu X^\nu$$

coincides with the scalar product of unconverted vectors under the following commutability condition

$$P^\mu \Lambda^\beta_\nu - \Lambda^\beta_\nu P^\mu = 0$$

Obviously, a scalar product of vectors that does not change under the Lorentz transformation must correspond to some relation between the corresponding spinors that does not change under the transformation from the SL(2,C) group. If this is so, then from this scalar product of spinors one can obtain at once the scalar product of vectors, the exponent of which gives the multiplier acting on the field vector during translation. Indeed, the equality is correct

$$[\mathbf{p}^T \sigma_M \mathbf{x}]^* [\mathbf{p}^T \sigma_M \mathbf{x}] = \frac{1}{2}\mathbf{P}^T \eta \mathbf{X}$$

where η is the metric tensor of Minkowski space and σ_M is the metric tensor of spinor space

$$\sigma_M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Another, non-invariant equality also take place

$$[\mathbf{p}^\dagger \mathbf{x}]^* [\mathbf{p}^\dagger \mathbf{x}] = \frac{1}{2}\mathbf{P}^T \mathbf{X} = \mathrm{Tr}[(\mathbf{p}\mathbf{p}^\dagger)^\dagger (\mathbf{x}\mathbf{x}^\dagger)]$$

When the coordinate vector is translated, the field vector is multiplied by the phase multiplier

$$\exp(\mathbf{P}^T \eta \mathbf{X})$$

Similarly, we can define a translation in spinor space, leading to the multiplication of the field spinor by the phase multiplier

$$\exp(\mathbf{p}^T \sigma_M \mathbf{x})$$

If we obtain a vector from the translated spinor, it will differ from the vector obtained by the vector translation by some complex multiplier with a unit module. It is an open question which translations - vector or spinor - more correctly describe the nature.

Since the square of the momentum vector length is zero, the square of the momentum spinor length is also zero, however, this property is characteristic of arbitrary spinors in general (the spinor length is determined by the metric tensor of spinor space)

$$\mathbf{p}^T \sigma_M \mathbf{p} = 0$$

Since, from the physical point of view, the zero length of the momentum vector is associated with the equality to zero mass, we can conclude that the masslessness of the field entails the possibility to describe it using a complex spinor. For a massive field there is no such possibility, it is described by a spintensor or a vector equivalent to it.

The described procedure of transition from an arbitrary complex spinor to a real four-dimensional vector through the direct product of the spinor by the conjugate leads to a vector with boson properties, since when the spinor is rotated or boosted by a certain angle, the vector is rotated or boosted by a doubled angle. It is also possible to form a four-dimensional spinor using the direct sum of spinor spaces. To do this, we form a four-component spinor from the complex spinor $\boldsymbol{\psi}_\mathbf{R}$ and the spinor $\boldsymbol{\psi}_\mathbf{L}$ connected with it in a certain way

$$\boldsymbol{\Psi} = \begin{pmatrix} \boldsymbol{\psi}_\mathbf{R} \\ \boldsymbol{\psi}_\mathbf{L} \end{pmatrix}$$

and using the matrix M from SL(2,C) we define the matrix

$$\hat{\Lambda} = \begin{pmatrix} M & 0 \\ 0 & -\sigma_M M^* \sigma_M \end{pmatrix}$$

The matrix $\hat{\Lambda}$ has the property

$$\hat{\Lambda}^T \begin{pmatrix} \sigma_M & 0 \\ 0 & \sigma_M \end{pmatrix} \hat{\Lambda} = \begin{pmatrix} M^T & 0 \\ 0 & -{\sigma_M}^T {M^*}^T {\sigma_M}^T \end{pmatrix} \begin{pmatrix} \sigma_M & 0 \\ 0 & \sigma_M \end{pmatrix} \begin{pmatrix} M & 0 \\ 0 & -\sigma_M M^* \sigma_M \end{pmatrix}$$
$$= \begin{pmatrix} M^T & 0 \\ 0 & -{\sigma_M}^T {M^*}^T {\sigma_M}^T \end{pmatrix} \begin{pmatrix} \sigma_M M & 0 \\ 0 & -\sigma_M \sigma_M M^* \sigma_M \end{pmatrix}$$
$$= \begin{pmatrix} M^T \sigma_M M & 0 \\ 0 & ({\sigma_M}^T {M^*}^T {\sigma_M}^T) \sigma_M (\sigma_M M^* \sigma_M) \end{pmatrix} = \begin{pmatrix} \sigma_M & 0 \\ 0 & \sigma_M \end{pmatrix}$$

where the property of matrices is used

$$M^T \sigma_M M = \sigma_M \qquad ({\sigma_M}^T {M^*}^T {\sigma_M}^T) \sigma_M (\sigma_M M^* \sigma_M) = \sigma_M$$

The matrix M is a combination of rotations and boosts with arbitrary angles

$$M = exp\left(\frac{1}{2}\beta_1\sigma_1\right) exp\left(\frac{1}{2}i\alpha_2\sigma_2\right) exp\left(\frac{1}{2}\beta_3\sigma_3\right) exp\left(\frac{1}{2}i\alpha_1\sigma_1\right) exp\left(\frac{1}{2}\beta_2\sigma_2\right) exp\left(\frac{1}{2}i\alpha_3\sigma_3\right)$$

When the matrix $\hat{\Lambda}$ acts, the spinor $\boldsymbol{\Psi}$ undergoes the transformation

$$\hat{\Lambda}\,\boldsymbol{\Psi} = \begin{pmatrix} M\boldsymbol{\psi}_\mathbf{R} & 0 \\ 0 & (-\sigma_M M^* \sigma_M)\boldsymbol{\psi}_\mathbf{L} \end{pmatrix}$$

Since the angles do not double, the spinor $\boldsymbol{\Psi}$ behaves as a spinor rather than a vector in the Lorentz transformation. The matrix $\hat{\Lambda}$ is not a Lorentz matrix at finite angles of rotations and boosts, but in the infinitesimal case, i.e. when all angles tend to zero, the Lorentz condition is approximately satisfied

$$\eta_{\alpha\beta} \hat{\Lambda}^\alpha{}_\mu \hat{\Lambda}^\beta{}_\nu = \eta_{\mu\nu}$$

and in this approximation the Dirac equation for the electron, for example, is valid for $\boldsymbol{\Psi}$. This is because at infinitesimal angles the matrix M can be represented as

$$M = \exp\left(\frac{1}{2}\beta_1\sigma_1\right) \exp\left(\frac{1}{2}i\alpha_2\sigma_2\right) \exp\left(\frac{1}{2}\beta_3\sigma_3\right) \exp\left(\frac{1}{2}i\alpha_1\sigma_1\right) \exp\left(\frac{1}{2}\beta_2\sigma_2\right) \exp\left(\frac{1}{2}i\alpha_3\sigma_3\right)$$
$$= 1 + \frac{1}{2}\beta_1\sigma_1 + \frac{1}{2}i\alpha_2\sigma_2 + \frac{1}{2}\beta_3\sigma_3 + \frac{1}{2}i\alpha_1\sigma_1 + \frac{1}{2}\beta_2\sigma_2 + \frac{1}{2}i\alpha_3\sigma_3$$

and the matrix $\hat{\Lambda}$ as

$$\hat{\Lambda} = 1 + \frac{i}{2}\omega_{\mu\nu}\Omega^{\mu\nu}$$

where $\omega_{\mu\nu}$ consists of rotation angles and boosts, and the antisymmetric matrix $\Omega^{\mu\nu}$ is defined through the gamma matrix commutator [1]

$$\Omega^{\mu\nu} = -\frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$$

$$i[\Omega^{\mu\nu}, \Omega^{\rho\sigma}] = \eta^{\mu\nu}\Omega^{\rho\sigma} - \eta^{\nu\rho}\Omega^{\mu\sigma} - \eta^{\mu\rho}\Omega^{\nu\sigma} + \eta^{\sigma\nu}\Omega^{\rho\mu}$$

Let us use the methodology by which the Dirac equation is derived in [2]. There, instead of the product of exponents, the exponent of the sum is used in determining M

$$M = \exp\left(\frac{1}{2}\beta_1\sigma_1 + \frac{1}{2}i\alpha_2\sigma_2 + \frac{1}{2}\beta_3\sigma_3 + \frac{1}{2}i\alpha_1\sigma_1 + \frac{1}{2}\beta_2\sigma_2 + \frac{1}{2}i\alpha_3\sigma_3\right)$$

which is inappropriate in the general case, since not all rotation and boost generators commute, but in the small-angle limit this substitution has the right to be used, due to which the Dirac equation is actually derived. We will use the exact transformation M, which is valid for arbitrary angles. Following [2] we consider the case of all six angles of rotations and boosts being equal to zero, which means that we consider $\mathbf{\Psi}$ in the rest frame at zero boosts, where the left and the right spinors coincide

$$\boldsymbol{\psi}_{\mathbf{R}} = \boldsymbol{\psi}_{\mathbf{L}} = \boldsymbol{\psi}$$

and equal to some spinor, which can be chosen to be real, its components specify two physical degrees of freedom, which a fermion, e.g. an electron, should possess. For arbitrary rotations and boosts, the fermion will be described by a complex vector in Minkowski space

$$\mathbf{\Psi} = \hat{\Lambda}\begin{pmatrix} \boldsymbol{\psi} \\ \boldsymbol{\psi} \end{pmatrix}$$

and the fermion will still be characterized by only two real numbers belonging to the spinor $\boldsymbol{\psi}$.

The infinitesimality of the Dirac equation indicates that Lorentz invariance in vector space is not a natural property of the fermion. Whereas for the boson the Lorentz symmetry in Minkowski space at any angles follows directly from the field spinor symmetry, for the fermion it takes place only in the limit of small angles of rotations and boosts, and the natural symmetry at arbitrary angles for the fermion is generated by the matrix $\hat{\Lambda}$. At arbitrary finite angles we have exact equalities characterizing the difference between the transformation of bosons and fermions in Minkowski space

$$\Lambda^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Lambda^{\mu}_{\ \nu} = \frac{1}{2}\mathrm{Tr}[\sigma_{\mu} M \sigma_{\nu} M^{\dagger}]$$

$$\hat{\Lambda}^T \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \hat{\Lambda} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\hat{\Lambda} = \begin{pmatrix} M & 0 \\ 0 & -\sigma_M M^* \sigma_M \end{pmatrix}$$

Note that [3] uses another set of gamma matrices, whose anticommutators are equal to the metric tensor of Minkowski space

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

and it is shown that in this case the last two components of the spinor $\boldsymbol{\Psi}$ are equal to zero in the resting frame (at zero boosts)

$$\psi_2 = 0 \quad \psi_3 = 0$$

which agrees with the fact that the electron has two physical degrees of freedom. If we choose the zero gamma matrix somewhat differently

$$\gamma^0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

the necessary anticommutators relations are still satisfied, and zero and the second components of the vector will be equal to zero at zero boosts

$$\psi_0 = 0 \quad \psi_2 = 0$$

In this case one can draw an analogy with the electromagnetic field in which the direct product of identical real spinors leads to a vector of zero length and a zero component in the y axis, while the direct sum of identical real vectors at the specified choice of gamma matrices also leads to a vector with a zero second component and an equal to zero component with a zero index, but with a nonzero length. That is, in this case boson and fermion have field vectors localized in the xz plane.

It is possible to consider coordinate and momentum spinors, forming them from the left and right spinors of the spinor field

$$\mathbf{p} = \begin{pmatrix} \mathbf{p_R} \\ \mathbf{p_L} \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{x_R} \\ \mathbf{x_L} \end{pmatrix}$$

the length of the momentum vector in Minkowski space will not equal to zero, i.e. the particle will possess a mass. If we accept for the momentum vector and spinors the same mechanism of connection between them as for the field vector and spinors, the left and right momentum spinors must be equal to each other at zero momentum value (zero boosts). If we know the current value of momentum, we know velocity and can make a transformation that brings the velocity, and therefore the boost angles, to zero. With this transformation, we will bring both the momentum and field vectors to a state with zero boost, and hence the left and right spinors generating them will become equal to each other. By this sign, we can relate the spinor and the momentum vector with the spinor and the field vector.

Similar to the description of the electron, the most economical way to describe the electromagnetic field is to use two identical real spinors. In this approach, the vector of potential depends only on two real parameters, since not only is its length equal to zero, but its component along the y-axis is always equal to zero

$$A_0 = \mathbf{a}^T \sigma_0 \mathbf{a}$$
$$A_1 = \mathbf{a}^T \sigma_1 \mathbf{a}$$
$$A_2 = 0$$
$$A_3 = \mathbf{a}^T \sigma_3 \mathbf{a}$$
$$A_0^2 = A_1^2 + A_3^2$$

When describing the electromagnetic field quantization procedure in [2], it is noted that by imposing two calibration conditions, the number of independent parameters describing the electromagnetic potential must be reduced to two, there Lorentz calibration and Coulomb calibration are used for this purpose. The approach we have described with the same real spinors is also a calibration that provides the dependence of the potential on only two independent parameters.

If we apply only rotation around the y-axis and boosts on the x- and z-axes to the real spinor

$$\exp\left(\frac{1}{2}\beta_1\sigma_1\right) \qquad \exp\left(\frac{1}{2}i\alpha_2\sigma_2\right) \qquad \exp\left(\frac{1}{2}\beta_3\sigma_3\right)$$

then the spinor remains real, and the components of the transformed vector satisfy the above constraints. These three transformations constitute a group, its elements act on the coordinates of spinor space, and the algebra of its generators has the form

$$[\sigma_1,(i\sigma_2)] = -2\sigma_3 \qquad [(i\sigma_2),\sigma_3] = -2\sigma_1 \qquad [\sigma_1,\sigma_3] = 2(i\sigma_2)$$

if we denote

$$\xi_1 = -\frac{1}{2}\sigma_1 \qquad \xi_2 = -\frac{1}{2}i\sigma_2 \qquad \xi_3 = -\frac{1}{2}\sigma_3$$

then

$$[\xi_1,\xi_2] = \xi_3 \qquad [\xi_2,\xi_3] = \xi_1 \qquad [\xi_3,\xi_1] = -\xi_2$$

Such an algebra does not coincide with either the algebra of the rotation group SO(3) or the algebra of the group SU(2), which should be

$$[\xi_i,\xi_j] = i\mathcal{E}_{ijk}\xi_k$$

The difference from this group is the sign of the commutator $[\xi_3,\xi_1]$.

Let us introduce step-up and step-down operators

$$J_+ = \xi_1 - i\xi_3 \qquad J_- = \xi_1 + i\xi_3 \qquad J_2 = i\xi_2$$

for which

$$[J_2,J_+] = J_+ \qquad [J_2,J_-] = -J_- \qquad [J_+,J_-] = 2J_2$$

The increasing operator increases the eigenvalue of the operator J_2 by one, and the decreasing operator decreases it by one. It is possible to define the operator

$$JJ = \frac{1}{2}(J_+J_- - J_-J_+) + J_2J_2 = \begin{pmatrix} 0.75 & 0 \\ 0 & 0.75 \end{pmatrix} = \frac{1}{2}\left(1+\frac{1}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which is the Casimir operator for the group in question.

Extending this group to an inhomogeneous group by two translations on the coordinates of spinor space x_1 and x_2, and finding an infinitesimal representation of its algebra using infinitesimal operators of translations on the two coordinates

$$\frac{\partial}{\partial x_1} \qquad \frac{\partial}{\partial x_2}$$

one can define its Casimir operators and write differential wave equations for spinor space. It is also possible to find irreducible representations of this five-parameter group. The matrix of the metric tensor in this space is such that the length of any spinor, including the spinor of momentum, is equal to zero

$$\mathbf{p}^T\sigma_M\mathbf{p} = 0$$

Thus, one spinor cannot be compared to any concept analogous to mass in Minkowski space. A nonzero value can be obtained only for the momentum described by the spintensor.

The angles of rotations and boosts in spinor space can depend on two spinor coordinates, which will give an additional contribution to the derivatives on them in the Casimir operators and in the wave equation, similar to that in four-dimensional space, as described further in the part 2 of this paper. Such a spinor-closed approach will probably allow one to obtain all necessary results in terms of spinors, including solving the spinor wave equations and quantization, and to move to ordinary space after obtaining results, calculating the potential vector from the spinor field, and the electric and magnetic fields from it.

Based on this reasoning, we can assume that the electromagnetic field is inherently a real spinor subject to transformations in the form of a combination of one rotation and two boosts. Such a description can simplify the procedure of quantization of the electromagnetic field. The reason for the difference in the properties of the Pauli matrix σ_2 among others is obvious - it coincides with the matrix of the metric tensor of spinor space to within an imaginary unit.

Summarizing the above considerations, we can propose the following concept of describing the electromagnetic field existing in Minkowski space. There is a two-dimensional space of real coordinate spinors, that is, each point in the space is described by two real numbers taking arbitrary values. At each such point, a real spinor of the field, representing two real numbers taking arbitrary values, is defined. From one point of coordinate spinor space one can move to another by translations. A field is also defined at a new point in space, and its values can depend on the value of the field at the first point and on the relation of the spatial points, for example by means of a wave equation. It can also be changed by adding a field from another source. As a result, the coordinate spinor is somehow mapped to two real field values. Having a given field for given coordinates, we postulate that all homogeneous linear coordinate transformations from the group described above lead to field transformations according to exactly the same law. The transformations are the product of two-dimensional square matrices with arbitrary angles

$$\exp\left(\frac{1}{2}\beta_1\sigma_1\right) \qquad \exp\left(\frac{1}{2}i\alpha_2\sigma_2\right) \qquad \exp\left(\frac{1}{2}\beta_3\sigma_3\right)$$

If the coordinates are transformed with their help, the field undergoes the same transformations. From the two spatial coordinates, using three Pauli matrices and the unit matrix, we calculate the coordinates of a point in Minkowski space and the four values of the electromagnetic field potential compared to this point. At different angles, we get coordinate points in the xz plane, and the coordinate and field transformations correspond to two boosts along the x and z axes and rotation around the y axis. But in reality, the field does not exist only in the xz plane, but can have a nonzero y component. This limitation is removed by the consideration that the coordinate and field spinors can also undergo three other homogeneous transformations

$$\exp\left(\frac{1}{2}i\alpha_1\sigma_1\right) \qquad \exp\left(\frac{1}{2}\beta_2\sigma_2\right) \qquad \exp\left(\frac{1}{2}i\alpha_3\sigma_3\right)$$

With this boost and two rotations, the entire four-dimensional field value space and the entire Minkowski space will be filled. Although the spinors will take complex values as well, both the spinors and the corresponding four-dimensional vectors will be determined by only two real numbers in the original coordinate spinor and two numbers in the original field spinor corresponding to them. All further transformations of both spinors and vectors will be determined by the angles of three rotations and three boosts.

It is necessary to recall our basic assumption that the rotation and boost of the field spinor does not occur by itself, but only after the rotation and boost of the coordinate spinor to which the field spinor is mapped. This coordinate spinor in Minkowski space acts as a vector with a double rotation angle. Rotation of the coordinate spinor entails rotation of the field spinor, which, in turn, rotates the boson and fermion vectors. If we want to trace all symmetries, starting from the rotation in Minkowski space, we should not consider boson and fermion at once, we should first return from the transformed coordinate vector to the coordinate spinor, transform it, then transform the field spinor by the same law, and obtain from it the transformed vectors for boson and fermion. The boson will be transformed exactly according to the Lorentz transformation, while the fermion in general case is not, the Lorentz transformation for it is valid only at infinitely small angles of rotations and boosts.

2. Description of accelerated motion

Let us return to Minkowski space and consider the Casimir operator of the Poincaré group, see, for example, [4], which is equal to the square of the length of the translation operator

$$P_\mu = -i\partial_\mu$$

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$$C_1 = P_\mu P^\mu = \eta_{\alpha\beta} P^\alpha P^\beta$$

Instead of this operator, we propose to use an operator of a more general form, composed of the translation operators subjected to the Lorentz transformation

$$C_1 = \eta_{\alpha\beta}(\Lambda^\alpha_\mu P^\mu)(\Lambda^\beta_\nu P^\nu)$$

If the matrix Λ^β_ν does not depend on coordinates, it can be taken out from under the sign of the derivative

$$C_1 = \eta_{\alpha\beta}\Lambda^\alpha_\mu \Lambda^\beta_\nu P^\mu P^\nu = \eta_{\mu\nu} P^\mu P^\nu$$

and get the usual Casimir operator. But if it depends on coordinates, the Casimir operator becomes more complicated

$$C_1 = \eta_{\alpha\beta}(\Lambda^\alpha_\mu P^\mu)(\Lambda^\beta_\nu P^\nu) = \eta_{\alpha\beta}\Lambda^\alpha_\mu [P^\mu(\Lambda^\beta_\nu P^\nu)]$$

$$= \eta_{\alpha\beta}\Lambda^\alpha_\mu [\Lambda^\beta_\nu (P^\mu P^\nu)] + \eta_{\alpha\beta}\Lambda^\alpha_\mu [(P^\mu \Lambda^\beta_\nu) P^\nu]$$

$$= \eta_{\alpha\beta}\Lambda^\alpha_\mu \Lambda^\beta_\nu [(P^\mu P^\nu)] + \eta_{\alpha\beta}\Lambda^\alpha_\mu [(P^\mu \Lambda^\beta_\nu) P^\nu]$$

$$= \eta_{\mu\nu}(P^\mu P^\nu) + \eta_{\alpha\beta}\Lambda^\alpha_\mu [(P^\mu \Lambda^\beta_\nu) P^\nu]$$

Here we add terms with derivatives on coordinates from angles of rotations and boosts.

Similar reasoning can be applied to the second Casimir operator of the Poincaré group formed from the Lubansky-Pauli vector.

$$C_2 = W_\mu W^\mu = \eta_{\alpha\beta} W^\alpha W^\beta$$

The propagation in space and time of fields, realizing the Poincaré group representations, is described by relativistic wave equations [4]. Wave equations are constructed on the basis of the Casimir operators, so when using generalized Casimir operators, the wave equations also become more complicated and acquire new terms. This can also be illustrated directly for specific types of wave equations. For example, consider a wave equation for the electromagnetic field, but use a "natural" calibration in the form of the zero-length requirement for the electromagnetic potential vector.

In [3] the equation for a massive meson field with spin 1 is considered. The corresponding vector field satisfies the field equation

$$(\partial^2 + m^2)A_\mu = 0$$

with the additional Lorentz calibration condition

$$\partial_{\boldsymbol{\mu}} A^{\boldsymbol{\mu}} = 0$$

The equation for the electromagnetic field is obtained in the special case of zero mass. Instead of the calibration condition adopted in [3], we propose to use the "natural" calibration justified above

$$A_{\boldsymbol{\mu}} A^{\boldsymbol{\mu}} = 0 \quad A_{\mathbf{2}} = 0$$

and instead of the equation

$$(\eta^{\mu\nu}\partial_\mu \partial_\nu + m^2)A_\mu = 0$$

consider the equation with transformed vector of derivatives

$$(\eta^{\mu\nu}(\Lambda^\alpha_\mu \partial_\alpha)(\Lambda^\beta_\nu \partial_\beta) + m^2)A_\mu = 0$$

Here the matrix Λ^α_μ depends on angles of rotations and boosts, which generally depend on the four-dimensional coordinates. If the commutation relation is satisfied

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$$\partial_\alpha \Lambda_\nu^\beta - \Lambda_\nu^\beta \partial_\alpha = 0$$

then we can write

$$\eta^{\mu\nu}(\Lambda_\mu^\alpha \partial_\alpha)(\Lambda_\nu^\beta \partial_\beta) = \eta^{\mu\nu}\Lambda_\mu^\alpha \Lambda_\nu^\beta \partial_\alpha \partial_\beta = \eta^{\alpha\beta}\partial_\alpha \partial_\beta$$

and obtain an ordinary wave operator, which, on the other hand, can be obtained from a general view operator simply by putting the angles of rotations and boosts equal to zero. But even at arbitrary constant angles the general operator coincides with the usual one, and therefore it does not seem to change the description of physical phenomena. But the commutative relation

$$\partial_\alpha \Lambda_\nu^\beta - \Lambda_\nu^\beta \partial_\alpha = 0$$

is not always satisfied; it includes, for example, the time and spatial coordinate derivative of the boost angles, and hence the derivative of velocity, which is indirectly included as a parameter in the Lorentz matrix. Both angles and velocity may not be constant. Then new terms appear in the equation where this operator enters, depending, for example, on acceleration, which make an additional contribution to the corresponding propagator and Lagrangian studied in [3].

The usual wave equation, in which all angles of rotations and boosts in the Lorentz matrix are equal to zero, describes behavior of the relativistic field in a stationary observation point, the generalized wave equation describes behavior of the field in a moving observation point, thus, if motion is uniform and speed, and thus angles of boosts, do not depend on four-dimensional coordinates, the generalized equation coincides with the usual one. If the motion is accelerated, the generalized equation contains additional terms.

The motion of a charged particle in an electromagnetic field is known to be accelerated, so the field acting on the moving particle must be calculated by transforming the external field given for the stationary observation point with a boost with an angle depending on coordinates and time. This would make adjustments to the equation of motion of the particle. The difficulty is that the dependence of the boost angle on the coordinates is determined by the resultant field acting on the particle itself. In any case, the equation of motion will differ from the one usually used. In particular, it will take into account the presence of conduction. Maxwell's equations are valid only for free space; in the presence of the right-hand side, the term related to conductivity should appear in them because if there is no conductivity in the point for which the equations are written, then there cannot be any current in it, i.e. the right-hand side of the equations is equal to zero. Maxwell's inhomogeneous equations must be telegraphic equations. The physical expression of conductivity is the accelerated motion of a charged particle under the action of a field, which can be described as the dependence of angles in the Lorentz transformations on the four coordinates of Minkowski space. Note that since the fields in the moving observation point depend on its velocity and acceleration, one force acts on a stationary charged particle and another force acts on the same stationary but not fixed particle, since the particle has a non-zero acceleration.

3. Bosons and fermions in spinor space

Let us consider a universal method for describing bosons and fermions using a four-component complex spinor. Let there be two complex spinors, one of which we will further compare with a boson and the other with a fermion

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_2, b_3)$$
$$\mathbf{f}^{\boldsymbol{T}} = (f_0, f_1, f_2, f_3)$$

Let's define the matrix given by the set of real angles of rotations and boosts

$$M = exp\left(-\frac{1}{2}i\alpha_1\sigma_1\right) exp\left(\frac{1}{2}\beta_1\sigma_1\right) exp\left(-\frac{1}{2}i\alpha_2\sigma_2\right) exp\left(\frac{1}{2}\beta_2\sigma_2\right) exp\left(-\frac{1}{2}i\alpha_3\sigma_3\right) exp\left(\frac{1}{2}\beta_3\sigma_3\right)$$

and the matrix

$$N = -\sigma_M M^* \sigma_M$$

where the two-dimensional spinor space metric is used

$$\sigma_M \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

the matrix N can be written explicitly

$$N = exp\left(-\frac{1}{2} i\alpha_1\sigma_1\right) exp\left(-\frac{1}{2}\beta_1\sigma_1\right) exp\left(-\frac{1}{2} i\alpha_2\sigma_2\right) exp\left(-\frac{1}{2}\beta_2\sigma_2\right) exp\left(-\frac{1}{2} i\alpha_3\sigma_3\right) exp\left(-\frac{1}{2}\beta_3\sigma_3\right)$$

where the sign of all the angles of the boosts has changed. Let us also define the matrix

$$\Lambda^\mu_\nu = \frac{1}{2}\mathrm{Tr}[\sigma_\mu M \sigma_\nu M^\dagger]$$

which can be written down explicitly using the matrices of the rotation and boost generators

$$\Lambda = exp(\alpha_1 L_1) exp(\beta_1 K_1) exp(\alpha_2 L_2) exp(\beta_2 K_2) exp(\alpha_3 L_3) exp(\beta_3 K_3)$$

as well as the matrices

$$MM = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}$$

$$MN = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}$$

$$\Sigma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \quad \Sigma_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \quad \Sigma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \quad \Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\Gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \quad \Gamma_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix} \quad \Gamma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \quad \Gamma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

$$\Gamma 5_0 = \begin{pmatrix} -\sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \quad \Gamma 5_1 = \begin{pmatrix} -\sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix} \quad \Gamma 5_2 = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \quad \Gamma 5_3 = \begin{pmatrix} -\sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

Let us define the real vectors

$$B_\mu = \frac{1}{2}\mathbf{b}^\dagger \Sigma_\mu \mathbf{b}$$

$$F_\mu = \frac{1}{2}\mathbf{f}^\dagger \Gamma_\mu \mathbf{f}$$

which can also be calculated in another way

$$B_\mu = \frac{1}{2}\mathrm{Tr}[\mathbf{b}\mathbf{b}^\dagger \Sigma_\mu]$$

$$F_\mu = \frac{1}{2}\mathrm{Tr}[\mathbf{f}\mathbf{f}^\dagger \Gamma_\mu]$$

Let us subject the spinors to transformations

$$\hat{\mathbf{b}} = MM\mathbf{b}$$

$$\hat{\mathbf{f}} = MN\mathbf{f}$$

then for the transformed vectors

$$\hat{B}_\mu = \frac{1}{2}\hat{\mathbf{b}}^\dagger \Sigma_\mu \hat{\mathbf{b}}$$

$$\hat{F}_\mu = \frac{1}{2}\hat{\mathbf{f}}^\dagger \Gamma_\mu \hat{\mathbf{f}}$$

the equations are valid

$$\hat{\mathbf{B}} = \Lambda \mathbf{B}$$

$$\hat{\mathbf{F}} = \Lambda \mathbf{F}$$

Thus, the spinors of bosons and fermions differ in the transformation matrices and the set of matrices by which real Lorentzian vectors are formed from them.

All the above is also true for the case of different spinors to the left and to the right of the matrices, but in this case the Lorentz invariant vector is complex.

Although a complex spinor has 8 degrees of freedom, in reality both the boson and the fermion have fewer degrees of freedom. If we restrict ourselves to spinors with complex components in the form

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_0, b_1)$$

$$\mathbf{f}^{\boldsymbol{T}} = (f_0, f_1, f_0, f_1)$$

then vector **B** has a zero length (and if all components of **b** are real, then also a zero component along the y-axis), and **F** has only one nonzero component

$$\mathbf{B}^{\boldsymbol{T}} = (B_0, B_1, 0, B_3)$$

$$\mathbf{F}^{\boldsymbol{T}} = (F_0, 0,0,0)$$

and if one chooses

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, {b_1}^*, -{b_0}^*)$$

$$\mathbf{f}^{\boldsymbol{T}} = (f_0, f_1, {f_1}^*, -{f_0}^*)$$

which corresponds to

$$\begin{pmatrix} b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} {b_0}^* \\ {b_1}^* \end{pmatrix}$$

then, on the contrary, vector **F** has already a zero length (and a zero component y if all spinor components are real), and **B** has only one nonzero component

$$\mathbf{B}^{\boldsymbol{T}} = (B_0, 0,0,0)$$

$$\mathbf{F}^{\boldsymbol{T}} = (F_0, F_1, 0, F_3)$$

The rest of the value manifold is obtained from the spinors given by the two complex parameters by means of all possible rotations and boosts. Note that the vector **F** can be interpreted as a current vector [5], and the vector

$$F5_\mu = \frac{1}{2}\mathbf{f}^\dagger \Gamma 5_\mu \mathbf{f}$$

as an axial current vector, the scalar $\eta_{\mu\nu} F_\mu B_\nu$ describes the boson-fermion interaction in the Lagrange function [5].

It is possible not to use gamma matrices at all, because if we use the matrices for the spinor transformation

$$MN = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}$$

then the vector is obtained using the matrices

$$\Gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \Gamma_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix} \Gamma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \Gamma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

but the same result in terms of the behavior of the spinor and the vector is obtained if we transform the spinor by matrices

$$MM = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}$$

and the vector is obtained by matrices

$$\Sigma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix} \Sigma_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \Sigma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

In this connection we can consider only the variant with all matrices M and matrices σ entering only with plus. That is, one can limit oneself to sigma matrices and not use gamma matrices, the differences between bosons and fermions will be determined only by the structure of spinors with the same way of forming vectors from them.

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In fact, it is possible to reduce the number of degrees of freedom by considering only real spinors, which follows from the fact that according to the Dirac equation in the rest frame with zero boosts the fermion spinor has just this form, and the spinor components are real [5]:

$$\mathbf{f}^{T} = (f_0, f_1, f_0, f_1)$$
$$\mathbf{F}^{T} = (F_0, 0,0,0)$$

i.e. in the rest frame (with zero boosts) the three-dimensional current vector is zero, there is only a stationary charge, which is logical, because in the rest frame the momentum vector has exactly the same form

$$\mathbf{P}^{T} = (P_0, 0,0,0)$$

We can thus say that the fulfillment of the Dirac equation leads to the fact that the current vector and the momentum vector are transformed not just by the same Lorentz matrix, but it happens synchronously, that is, the current is created only by a moving charge. From a physical point of view, it cannot be otherwise. We can say that the Dirac equation connects the current vector and the momentum vector. The axial current vector in the considered form of a spinor has the form

$$\mathbf{F}^{T} = (0, F_1, 0, F_3)$$

it is known [5] that the current is always conserved, and the axial current is conserved only in the case of zero mass.

The synchronous transformation of the current vector and the momentum vector allow to suppose that some spinor, which transforms by the same law as the fermion spinor and, moreover, synchronously with it, is also connected with the momentum vector, i.e. in the fermion rest frame this spinor should have the form

$$\mathbf{p}^{T} = (p_0, p_1, p_0, p_1)$$

When the momentum vector is formed from the momentum spinor having this form with the help of Γ_μ matrices, it will have a non-zero length and therefore the fermion's mass will be non-zero. The vector for the boson from a spinor of this format is formed with Σ_μ matrices and has a zero length, assuming that the momentum vector for the boson is also formed with Σ_μ, it will also have a zero length, which explains the zero mass of the boson.

In general, the physical mechanism can be described as follows. There is a spinor of the field with the configuration

$$\mathbf{f}^{T} = (f_0, f_1, f_0, f_1)$$

It is matched with a spinor of coordinates and a spinor of momentum with the same configuration

$$\mathbf{x}^{T} = (x_0, x_1, x_0, x_1)$$
$$\mathbf{p}^{T} = (p_0, p_1, p_0, p_1)$$

Actually, the same configuration combines these spinors in conjunction with the same and simultaneous transformation using the same matrix of rotations and boosts M. Vectors in Minkowski space are formed from all three spinors in two ways - using sigma matrices and gamma matrices. By means of sigma matrices three bosonic vectors are formed - the zero length of the momentum vector provides zero mass, the zero length of the coordinate vector means motion with light speed, and the zero length of the field vector corresponds to the absence of charge. Gamma matrices form three fermionic vectors - non-zero length of the momentum vector means the presence of mass, non-zero length of the coordinate vector means motion with a sub light speed, and non-zero length of the field vector means the presence of charge.

The Dirac equation in the momentum representation is fulfilled here automatically, because at zero boost the field spinor and the momentum vector of the fermion have the form

$$\mathbf{f}^{T} = (f_0, f_1, f_0, f_1)$$
$$\mathbf{P}^{T} = (P_0, 0,0,0)$$

that is, they satisfy the Dirac equation, and at nonzero boost they transform coherently and continue to satisfy the equation. On the other hand, the sigma matrices translate triplet of spinors of the type

$$\mathbf{b}^{T} = (b_0, b_1, b_1, -b_0)$$

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into three vectors of non-zero length, and the gamma matrices translate into three vectors of zero length.

As a summary, let us formulate the following. The properties of fields are not determined by the Klein-Gordon and Dirac equations, but simply by the structure of spinors at zero boosts. The Klein-Gordon equation for a fermion is just a statement that its momentum is a vector of fixed length

$$P_0^{\,2} - P_1^{\,2} - P_2^{\,2} - P_3^{\,2} = \text{const}$$

The wave equation for a boson is just a statement that the momentum of, for example, a photon is a vector with zero length

$$P_0^{\,2} - P_1^{\,2} - P_2^{\,2} - P_3^{\,2} = 0$$

For the fermion, there is an equation stating that the current is a vector of fixed length

$$F_0^{\,2} - F_1^{\,2} - F_2^{\,2} - F_3^{\,2} = \text{const}$$

this equation is related to the law of conservation of charge. And for the photon, there is a second order equation stating that the electromagnetic potential is also a vector with zero length (taking into account $\mathbf{A} \equiv \mathbf{B}$)

$$A_0^{\,2} - A_1^{\,2} - A_2^{\,2} - A_3^{\,2} = 0$$

Dirac equation states that current is created only when a fermion moves. Dirac equation is written for a spinor, if a spinor satisfies it, then the vector obtained from it satisfies Klein-Gordon equation, the reverse is not true. The vector obtained from the photon spinor satisfies the Klein-Gordon equation, but the photon spinor does not satisfy the Dirac equation, but a first order equation similar to it. The fact that momentum vector of the photon and electromagnetic potential vector are inseparably related and change synchronously, similar to the synchrony of momentum and current transformations in the fermion, can be described by the first order equation similar to the Dirac equation. In the reference frame with zero boosts, analogous to the rest frame of the fermion, it is simply an algebraic relation, and with nonzero boosts the equation undergoes transformations from the Lorentz group. All this is true for complex spinors, so the statement about two physical degrees of freedom of photon and electron is not quite justified, formally the degrees of freedom are four, two for each spin of fermion or helicity of photon.

The question arises what conclusions by analogy with the fermion can be made for the boson in its configuration

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_1^{\,*}, -b_0^{\,*})$$
$$\mathbf{B}^{\boldsymbol{T}} = (B_0, 0,0,0)$$

If the boson had non-zero mass and possessed a charge, it could only have this form of current vector at zero boosts, hence it would have to have momentum in this configuration

$$\mathbf{P}^{\boldsymbol{T}} = (P_0, 0,0,0)$$

and this could be provided by the first-order equation limiting its form, analogous to the Dirac equation for the fermion. The real spinor with format

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_1, -b_0)$$

may be converted to the format

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_0, b_1)$$

by converting $\Sigma_M \mathbf{b}$

$$\Sigma_M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \sigma_M \end{pmatrix}$$

By substituting this spinor into the Dirac equation for the fermion ($\Gamma^{\boldsymbol{\mu}} \equiv \Gamma_\mu$)

$$\left(i\Gamma^{\boldsymbol{\mu}}\partial_\mu - m\right)\mathbf{f} = 0$$

we obtain the first order equation for the boson

$$\left(i\Gamma^{\mu}\partial_{\mu} - m\right)\Sigma_{M}\mathbf{b} = 0$$

$$\left(i(\Gamma^{\mu}\Sigma_{M})\partial_{\mu} - m\Sigma_{M}\right)\mathbf{b} = 0$$

For generality, in all the differential equations considered, one should substitute the boson and fermion spinors transformed by Lorentz with some angles of rotations and boosts, then when taking the derivatives in the equations there will be additional terms with derivatives from angles of rotations and boosts, which depend on the spinor and Minkowski space coordinates, since when a fermion moves in the boson field the speeds and momenta are not constants.

Since vector **B** is a vector of the electromagnetic potential, and the electromagnetic potential may be included in the Dirac equation [6]

$$\left(\Gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) - m\right)\mathbf{f} = 0$$

then by substituting B_{μ} instead of A_{μ}

$$B_{\mu} = \frac{1}{2}\mathbf{b}^{\dagger}\Sigma_{\mu}\mathbf{b}$$

we get

$$\left(\Gamma^{\mu}(i\partial_{\mu} - e\frac{1}{2}\mathbf{b}^{\dagger}\Sigma_{\mu}\mathbf{b}\,) - m\right)\mathbf{f} = 0$$

In momentum space this equation has the form

$$\left(\Gamma^{\mu}\left(P_{\mu} - e\frac{1}{2}\mathbf{b}^{\dagger}\Sigma_{\mu}\mathbf{b}\,\right) - m_{f}\right)\mathbf{f}(\mathbf{P}) = 0$$

The assumption used here is that momentum in the equation acts as a vector. Of course, in the usual treatment of the Dirac equation this is what is meant, but in the light of our consideration it is not at all obvious that it is the momentum vector and not the spinor that enters the equation. After all, **f** is a spinor, and there is no such variable as time in spinor space. Nevertheless, we use this assumption because it is convenient for writing the following form of the equation

$$\left(\Gamma^{\mu}\left(\frac{1}{2}\mathbf{p}_{f}{}^{\dagger}\Sigma_{\mu}\mathbf{p}_{f} - e\frac{1}{2}\mathbf{b}^{\dagger}\Sigma_{\mu}\mathbf{b}\,\right) - m_{f}\right)\mathbf{f}(\mathbf{p}_{f}) = 0$$

One is tempted to write this equation in a more interesting form

$$\left(\Gamma^{\mu}\left(\frac{1}{2}(\mathbf{p}_{f} - \sqrt{e}\mathbf{b})^{\dagger}\Sigma_{\mu}(\mathbf{p}_{f} - \sqrt{e}\mathbf{b})\,\right) - m_{f}\right)\mathbf{f}(\mathbf{p}_{f}) = 0$$

but this transition requires justification.

In what follows we will proceed from the assumption that the quantity $\sqrt{e}\mathbf{b}$ has the dimension of the momentum spinor and is essentially the momentum spinor of the boson

$$\mathbf{p}_{b} = \sqrt{e}\mathbf{b}$$

We can also assume a rigid dependence of the fermion spinor on the spinor of its momentum and the corresponding dependence of vectors

$$\mathbf{f} = \pm\sqrt{\frac{e}{m_{e}}}\mathbf{p}_{e}$$

$$\mathbf{F} = \frac{e}{m_{e}}\mathbf{P}_{e}$$

The squares of the charge and mass of the electron are simply notations for the squares of the length of the electron's field vector and its momentum vector

$$e^{2} \equiv \eta^{\mu\nu}F_{\mu}F_{\nu}$$

$$m_{e}{}^{2} \equiv \eta^{\mu\nu}\mathrm{P}_{e_{\mu}}\mathrm{P}_{e_{\nu}}$$

In turn, based on the ratio

$$\mathbf{p}_e = \pm\sqrt{\frac{m_e}{e}}\mathbf{f}$$

$$\mathbf{P}_e = \frac{m_e}{e}\mathbf{F}$$

we can add the momentum contributed by the fermion vector to the momentum in the equation for the boson (here we have also subtracted an additional contribution, but perhaps it should be added)

$$\left((\Gamma^{\mu}\Sigma_M)\left(P_\mu - \frac{m_e}{e}\frac{1}{2}\mathbf{f}^\dagger\Sigma_\mu\mathbf{f}\right) - m_b\Sigma_M\right)\mathbf{b}(\mathbf{p}_b) = 0$$

or using only spinors

$$\left((\Gamma^{\mu}\Sigma_M)\left(\frac{1}{2}\left(\mathbf{p}_b - \sqrt{\frac{m_e}{e}}\mathbf{f}\right)^\dagger \Sigma_\mu\left(\mathbf{p}_b - \sqrt{\frac{m_e}{e}}\mathbf{f}\right)\right) - m_b\Sigma_M\right)\mathbf{b}(\mathbf{p}_b) = 0$$

As a result, we have two coupled equations for boson and fermion. They can be interpreted in such a way that if in one point of the spinor coordinate space there is a boson and a fermion, then each of them gives an addition to the momentum spinor of the other, proportional to its field spinor, the signs of these additions require specification. Addition of the boson spinor to the spinor of the fermion momentum changes its structure and the fermion ceases to be a fermion, but if we postulate a rigid uniformity of structures of the field and its momentum, the change of the momentum structure must lead to a change in the structure of its field, and it in turn gives an inverse contribution to the momentum of the interacting field. Perhaps such interdependence of the fields will allow us to find out the law of their interaction and evolution.

Actually

$$\mathbf{f} = \pm\sqrt{\frac{e}{m_e}}\mathbf{p}_e$$

$$\mathbf{F} = \frac{e}{m_e}\mathbf{P}_e$$

is a simpler form of Dirac equation, which looks more complicated because it connects spinor of the electron with the of its momentum vector. But in fact it is reduced to a simple relation between spinors of the electron and its momentum.

If for the combined spinor

$$\mathbf{p}_e - \sqrt{e}\mathbf{b}$$

we calculate the vector, then it turns out, for example, that if the electron was at rest and all three components of its momentum vector were equal to zero, then adding to the spinor the boson momentum, that is, imposing an electromagnetic field, leads to a nonzero component of the electron momentum vector, that is, it moves under the action of the field. When field is absent we have

$$\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix} \quad \begin{pmatrix}-2\\5\\-2\\5\end{pmatrix} \qquad \begin{pmatrix}13\\0\\0\\0\end{pmatrix} \quad \begin{pmatrix}29\\-20\\0\\-21\end{pmatrix}$$

after turning on the field (let's put the $\sqrt{e}$ at 0.01) we have

$$\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix} - 0.01 * \begin{pmatrix}-2\\5\\-2\\5\end{pmatrix} \qquad \begin{pmatrix}13.273\\-0.152\\0\\-0.232\end{pmatrix}$$

The energy of the fermion increases. In turn, the electron also affects the field

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$$\begin{pmatrix}-2\\5\\-2\\5\end{pmatrix}-0.01*\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix}\qquad\begin{pmatrix}29.271\\-20.15\\0\\-21.23\end{pmatrix}$$

in the presence of the electron, the field energy increased and the vector potential changed, that is, there was additional radiation. The energy of both particles increased, which is apparently not realistic. If you change the sign

$$\begin{pmatrix}-2\\5\\-2\\5\end{pmatrix}+0.01*\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix}\qquad\begin{pmatrix}28.731\\-19.85\\0\\-20.77\end{pmatrix}$$

then the magnitude of the potential and energy of the field have decreased, and the total energy of the two particles

$$13.273+28.731=42.004$$

slightly increased. Apparently, the field energy was spent to accelerate the massive electron (the mass of the electron in our example is 13). The mass of the electron increased to 13.27 and the mass of the photon also became non-zero 0.269, the total mass increased to 13.539.

We used the same coefficient $\sqrt{e}$ to account for the effect of the electron on the field, which is apparently incorrect; in fact, the modified boson momentum has the form

$$\mathbf{p}_b-\sqrt{\frac{m_e}{e}}\mathbf{f}$$

but for our illustrative computational examples this is not fundamental.

Here we can clarify the meaning of the Lorentz calibration

$$\partial_{\boldsymbol{\mu}}A^{\boldsymbol{\mu}}=0$$

which in the momentum vector space has the form

$$(\mathbf{P}_b)_{\boldsymbol{\mu}}A^{\boldsymbol{\mu}}=0$$

and taking into account the ratio

$$\mathbf{p}_b=\sqrt{e}\mathbf{b}$$
$$\mathbf{P}_b=e\mathbf{B}=e\mathbf{A}$$

we receive

$$eA_{\boldsymbol{\mu}}A^{\boldsymbol{\mu}}=0$$

that is, the Lorentz calibration means zero length of the potential vector. And instead of the Coulomb calibration

$$A_{\mathbf{0}}=0$$

we use a similar condition

$$A_{\mathbf{2}}=0$$

But in our case calibration is not an artificially imposed external condition, but a natural consequence of the particular structure of the photon spinor.

Let us apply our approach to the electromagnetic field tensor of a single photon

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu=(\mathbf{P}_b)_{\boldsymbol{\mu}}A_\nu-(\mathbf{P}_b)_\nu A_\mu=A_{\boldsymbol{\mu}}A_\nu-A_\nu A_\mu$$
$$=\mathbf{a}^\dagger\Sigma_\mu\mathbf{a}\mathbf{a}^\dagger\Sigma_\nu\mathbf{a}-\mathbf{a}^\dagger\Sigma_\nu\mathbf{a}\mathbf{a}^\dagger\Sigma_\mu\mathbf{a}=\mathbf{0}$$

Replacing the photon's momentum by its field $\mathbf{p}_b=\sqrt{e}\mathbf{b}=\sqrt{e}\mathbf{a}$, we got a result in the form of zero electric and magnetic fields, which is a consequence of direct proportionality of momentum components to potential components. The zero fields of a free photon are not something absurd. One can suppose that the direct proportionality of the momentum spinor to the field spinor takes place only for a free photon, and if it interacts with another field, there is an addition to the momentum spinor, so that the direct proportionality of momentum and field no longer takes place. Therefore, the components of the electric and magnetic fields do not become equal to zero. We can say that the effect of the electromagnetic field is manifested only if there is an object for that effect.

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As noted above, the spinor of the photon momentum can be added to the spinor of the electron momentum with a plus or minus sign

$\mathbf{p}_b = \sqrt{e}\mathbf{b} = \sqrt{e}\mathbf{a}$, we got a result in the form of zero electric and magnetic fields, which is a consequence of direct proportionality of momentum components to potential components. The zero fields of a free photon are not something absurd. One can suppose that the direct proportionality of the momentum spinor to the field spinor takes place only for a free photon, and if it interacts with another field, there is an addition to the momentum spinor, so that the direct proportionality of momentum and field no longer takes place. Therefore, the components of the electric and magnetic fields do not become equal to zero. We can say that the effect of the electromagnetic field is manifested only if there is an object for that effect. As noted above, the spinor of the photon momentum can be added to the spinor of the electron momentum with a plus or minus sign

$$\mathbf{p}_b + \sqrt{\frac{m_e}{e}}\mathbf{f}$$

In this case, the components of the electromagnetic field tensor have the form $\left(\sqrt{\frac{m_e}{e}} \equiv \lambda\right)$

$$\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu = (\mathbf{P}_b)_{\boldsymbol{\mu}} A_\nu - (\mathbf{P}_b)_\nu A_\mu \\
&= \frac{1}{4}(\mathbf{p}_b + \lambda\mathbf{f})^\dagger \Sigma_\mu (\mathbf{p}_b + \lambda\mathbf{f})\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} - \frac{1}{4}(\mathbf{p}_b + \lambda\mathbf{f})^\dagger \Sigma_\nu (\mathbf{p}_b + \lambda\mathbf{f})\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} \\
&= \frac{1}{4}\left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)^\dagger \Sigma_\mu \left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} - \frac{1}{4}\left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)^\dagger \Sigma_\nu \left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} \\
&= \frac{1}{4}\left(e\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} + \sqrt{e}\lambda\mathbf{a}^\dagger \Sigma_\mu \mathbf{f} + \lambda\sqrt{e}\mathbf{f}^\dagger \Sigma_\mu \mathbf{a} + \lambda^2 \mathbf{f}^\dagger \Sigma_\mu \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} \\
&- \frac{1}{4}\left(e\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} + \sqrt{e}\lambda\mathbf{a}^\dagger \Sigma_\nu \mathbf{f} + \lambda\sqrt{e}\mathbf{f}^\dagger \Sigma_\nu \mathbf{a} + \lambda^2 \mathbf{f}^\dagger \Sigma_\nu \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} \\
&= \frac{1}{4}\left(\sqrt{e}\lambda\mathbf{a}^\dagger \Sigma_\mu \mathbf{f} + \lambda\sqrt{e}\mathbf{f}^\dagger \Sigma_\mu \mathbf{a} + \lambda^2 \mathbf{f}^\dagger \Sigma_\mu \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} \\
&- \frac{1}{4}\left(\sqrt{e}\lambda\mathbf{a}^\dagger \Sigma_\nu \mathbf{f} + \lambda\sqrt{e}\mathbf{f}^\dagger \Sigma_\nu \mathbf{a} + \lambda^2 \mathbf{f}^\dagger \Sigma_\nu \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\mu \mathbf{a}
\end{aligned}$$

$$\begin{aligned}
E_y &= \frac{1}{4}\left(\sqrt{m_e}\mathbf{a}^\dagger \Sigma_0 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_0 \mathbf{a} + \frac{m_e}{e}\mathbf{f}^\dagger \Sigma_0 \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_2 \mathbf{a} \\
&\quad - \frac{1}{4}\left(\sqrt{m_e}\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_2 \mathbf{a} + \frac{m_e}{e}\mathbf{f}^\dagger \Sigma_2 \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_0 \mathbf{a} \\
B_x &= \frac{1}{4}\left(\sqrt{m_e}\mathbf{a}^\dagger \Sigma_3 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_3 \mathbf{a} + \frac{m_e}{e}\mathbf{f}^\dagger \Sigma_3 \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_2 \mathbf{a} \\
&\quad - \frac{1}{4}\left(\sqrt{m_e}\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_2 \mathbf{a} + \frac{m_e}{e}\mathbf{f}^\dagger \Sigma_2 \mathbf{f}\right)\mathbf{a}^\dagger \Sigma_3 \mathbf{a}
\end{aligned}$$

The field depends on the charge on which it acts, which is not surprising either, since the charge has its own field that distorts the external one.

Thus, when finding the momentum vector, we sum the momentum spinors of the boson and fermion, and when finding the field vector, we do not sum the field spinors. This approach can be tried to apply to Maxwell's equations. In this case, we replace the derivative of the field vector component on the coordinate vector component with the product of the corresponding momentum vector component and the field vector component.

$$\partial_\mu A_\nu = (\mathbf{P}_b)_{\boldsymbol{\mu}} A_\nu = \frac{1}{4}(\mathbf{p}_b + \lambda\mathbf{f})^\dagger \Sigma_\mu (\mathbf{p}_b + \lambda\mathbf{f})\mathbf{a}^\dagger \Sigma_\nu \mathbf{a} = \frac{1}{4}\left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)^\dagger \Sigma_\mu \left(\sqrt{e}\mathbf{a} + \lambda\mathbf{f}\right)\mathbf{a}^\dagger \Sigma_\nu \mathbf{a}$$

In the left parts of Maxwell's equations there are derivatives on the coordinate vector component of the electric or magnetic field component, for example

$$\begin{aligned}\partial_\mu E_y = \partial_\mu F_{02} &= \partial_\mu((\mathbf{P}_b)_0 A_2) - \partial_\mu((\mathbf{P}_b)_2 A_0)\\ &= (\mathbf{P}_b)_0 \partial_\mu A_2 + A_2 \partial_\mu (\mathbf{P}_b)_0 - (\mathbf{P}_b)_2 \partial_\mu A_0 - A_0 \partial_\mu (\mathbf{P}_b)_2\\ &= (\mathbf{P}_b)_0 (\mathbf{P}_b)_\mu A_2 + A_2 \partial_\mu (\mathbf{P}_b)_0 - (\mathbf{P}_b)_2 (\mathbf{P}_b)_\mu A_0 - A_0 \partial_\mu (\mathbf{P}_b)_2\\ &= (\mathbf{P}_b)_\mu ((\mathbf{P}_b)_0 A_2 - (\mathbf{P}_b)_2 A_0) + A_2 \partial_\mu (\mathbf{P}_b)_0 - A_0 \partial_\mu (\mathbf{P}_b)_2\\ &= (\mathbf{P}_b)_\mu E_y + A_2 \partial_\mu (\mathbf{P}_b)_0 - A_0 \partial_\mu (\mathbf{P}_b)_2\\ &= (\mathbf{P}_b)_\mu E_y + A_2 \left(\frac{1}{2} e^2 A_\mu A_0 + \frac{1}{2}\frac{{m_e}^2}{e^2} F_\mu F_0 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_0 \mathbf{f} + \mathbf{f}^\dagger \Sigma_0 \mathbf{a})\right)\\ &\quad - A_0 \left(\frac{1}{2} e^2 A_\mu A_2 + \frac{1}{2}\frac{{m_e}^2}{e^2} F_\mu F_2 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \mathbf{f}^\dagger \Sigma_2 \mathbf{a})\right)\end{aligned}$$

where taken into account

$$\begin{aligned}\partial_\mu (\mathbf{P}_b)_2 &= \frac{1}{2}\partial_\mu \left((\mathbf{p}_b + \lambda \mathbf{f})^\dagger \Sigma_\mu (\mathbf{p}_b + \lambda \mathbf{f})\right)\\ &= \frac{1}{2}\partial_\mu \left({\mathbf{p}_b}^\dagger \Sigma_2 \mathbf{p}_b + \lambda {\mathbf{p}_b}^\dagger \Sigma_2 \mathbf{f} + \lambda \mathbf{f}^\dagger \Sigma_2 \mathbf{p}_b + \lambda^2 \mathbf{f}^\dagger \Sigma_2 \mathbf{f}\right) =\\ &= \frac{1}{2}\partial_\mu \left(e\mathbf{a}^\dagger \Sigma_2 \mathbf{a} + \sqrt{m_e}\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_2 \mathbf{a} + \frac{m_e}{e}\mathbf{f}^\dagger \Sigma_2 \mathbf{f}\right)\\ &= \frac{1}{2}\partial_\mu \left(eA_2 + \sqrt{m_e}\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \sqrt{m_e}\mathbf{f}^\dagger \Sigma_2 \mathbf{a} + \frac{m_e}{e} F_2\right)\\ &= \frac{1}{2} e \partial_\mu A_2 + \frac{1}{2}\frac{m_e}{e}\partial_\mu F_2 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \mathbf{f}^\dagger \Sigma_2 \mathbf{a})\\ &= \frac{1}{2} e (\mathbf{P}_b)_\mu A_2 + \frac{1}{2}\frac{m_e}{e}(\mathbf{P}_e)_\mu F_2 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \mathbf{f}^\dagger \Sigma_2 \mathbf{a})\\ &= \frac{1}{2}(\mathbf{P}_b)_\mu (\mathbf{P}_b)_2 + \frac{1}{2}(\mathbf{P}_e)_\mu (\mathbf{P}_e)_2 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \mathbf{f}^\dagger \Sigma_2 \mathbf{a})\\ &= \frac{1}{2} e^2 A_\mu A_2 + \frac{1}{2}\frac{{m_e}^2}{e^2} F_\mu F_2 + \frac{1}{2}\sqrt{m_e}\partial_\mu (\mathbf{a}^\dagger \Sigma_2 \mathbf{f} + \mathbf{f}^\dagger \Sigma_2 \mathbf{a})\end{aligned}$$

In the process of this calculations there is a real vector

$$\frac{1}{2}\left(\mathbf{a}^\dagger \Sigma_\mu \mathbf{f} + \mathbf{f}^\dagger \Sigma_\mu \mathbf{a}\right)$$

about which we can say that it characterizes the interaction of fields.

Thus, in the left parts of the Maxwell's equations we can get rid of all derivatives, except for derivatives from the real vector of interaction $\frac{1}{2}(\mathbf{a}^\dagger \Sigma_\mu \mathbf{f} + \mathbf{f}^\dagger \Sigma_\mu \mathbf{a})$, and in the right parts there are only the components of the field vector of currents $\mathbf{F}$. But the derivative of the interaction vector remains a problem. Something similar can be applied to the equation of motion of the electron in the electromagnetic field to also take into account the mutual influence of the fields.

In quantum electrodynamics, to account for the interaction of the electron with the electromagnetic field, the Lagrangian includes the value

$$e\eta_{\mu\nu} F_\mu B_\nu = e\eta_{\mu\nu}\left(\mathbf{f}^\dagger \Sigma_\mu \mathbf{f}\right)\left(\mathbf{b}^\dagger \Sigma_\nu \mathbf{b}\right)$$

Alternatively, we can consider another scalar quantity derived from the interaction vector just given

$$\frac{1}{2} e\eta_{\mu\nu}\left(\mathbf{f}^\dagger \Sigma_\mu \mathbf{f} + \mathbf{b}^\dagger \Sigma_\mu \mathbf{b}\right)\left(\mathbf{f}^\dagger \Sigma_\nu \mathbf{f} + \mathbf{b}^\dagger \Sigma_\nu \mathbf{b}\right)$$

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or

$$\frac{1}{2}e\eta_{\mu\nu}\big((\mathbf{f}+\mathbf{b})^{\dagger}\Sigma_{\mu}(\mathbf{f}+\mathbf{b})\big)\big((\mathbf{f}+\mathbf{b})^{\dagger}\Sigma_{\nu}(\mathbf{f}+\mathbf{b})\big)$$

It is clear from general considerations that when convolving the interaction vector into a scalar, we lose some information about this interaction. Interestingly, if both spinors $\mathbf{f}$ and $\mathbf{b}$ were bosons, then an additional equality would also be true

$$e\eta_{\mu\nu}F_{\mu}B_{\nu} = e\eta_{\mu\nu}\big(\mathbf{f}^{\dagger}\Sigma_{\mu}\mathbf{f}\big)\big(\mathbf{b}^{\dagger}\Sigma_{\nu}\mathbf{b}\big) = \frac{1}{2}e[\mathbf{f}^{T}\Sigma_{MM}\mathbf{b}]^{*}[\mathbf{f}^{T}\Sigma_{MM}\mathbf{b}]$$

$$\Sigma_{MM} = \begin{pmatrix}\sigma_M & 0\\ 0 & \sigma_M\end{pmatrix}$$

Above we considered an example for zero angles of rotations and boosts. Let us see what changes if one of the fields is transformed by an *MM* matrix with arbitrary angles of rotations and boosts, e.g.

$$\alpha = \begin{pmatrix}2\\5\\1\end{pmatrix} \qquad \beta = \begin{pmatrix}-0.7\\1\\0.3\end{pmatrix}$$

First we transform the electron without changing the field

$$\begin{pmatrix}-2.786-2.282i\\2.578+1.685i\\5.493-0.335i\\-1.847-1.5i\end{pmatrix}\quad\begin{pmatrix}-2\\5\\-2\\5\end{pmatrix}\qquad\begin{pmatrix}29.2\\-20.672\\-7.667\\14.054\end{pmatrix}\quad\begin{pmatrix}29\\-20\\0\\-21\end{pmatrix}$$

let's turn on the field

$$\begin{pmatrix}-2.786-2.282i\\2.578+1.685i\\5.493-0.335i\\-1.847-1.5i\end{pmatrix}-0.01*\begin{pmatrix}-2\\5\\-2\\5\end{pmatrix}\qquad\begin{pmatrix}29.221\\-20.795\\-7.794\\14.143\end{pmatrix}$$

$$\begin{pmatrix}-2\\5\\-2\\5\end{pmatrix}+0.01*\begin{pmatrix}-2.786-2.282i\\2.578+1.685i\\5.493-0.335i\\-1.847-1.5i\end{pmatrix}\qquad\begin{pmatrix}28.985\\-19.881\\0.126\\-21.089\end{pmatrix}$$

energy of the electron increased, the field energy decreased, and the total energy increased from 58.2 to 58.206. The mass of the electron decreased from 13 to 12.675 and the mass of the photon became 0.328, the total mass increased to 13.003.

Now let us transform by means of the *MM* matrix the field without changing the state of the electron

$$\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix}\quad\begin{pmatrix}-1.22+3.092i\\-1.609-0.805i\\-1.22+3.092i\\-1.609-0.805i\end{pmatrix}\qquad\begin{pmatrix}13\\0\\0\\0\end{pmatrix}\quad\begin{pmatrix}14.285\\-1.051\\11.915\\7.808\end{pmatrix}$$

Let's see how the fields mutually influence each other

$$\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix}-0.01*\begin{pmatrix}-1.22+3.092i\\-1.609-0.805i\\-1.22+3.092i\\-1.609-0.805i\end{pmatrix}\qquad\begin{pmatrix}12.933\\-0.045\\-0.145\\0.093\end{pmatrix}$$

$$\begin{pmatrix}-1.22+3.092i\\-1.609-0.805i\\-1.22+3.092i\\-1.609-0.805i\end{pmatrix}+0.01*\begin{pmatrix}3\\-2\\-2\\-3\end{pmatrix}\qquad\begin{pmatrix}14.354\\-1.006\\12.062\\7.7160\end{pmatrix}$$

the total energy increased again from 27.285 to 27.287, the field energy increased this time, but the energy of the electron decreased, which is somewhat strange. The mass of the electron again decreased to 12.932 and the mass of the photon became 0.1, the total mass increased to 13.031.

Note that in these calculations we were not interested in the coordinates of the particles at all, we assumed only that they coincide. That is, our calculations are valid for any frame of reference, and at all transformations of fields the frame of reference did not change. This suggests that the mention of the coordinate transformation at definition of relativistic fields is superfluous, the only thing we postulate is the synchronicity of the transformation of the field and its momentum.

We can switch to another reference frame by acting on the coordinate, field and momentum spinors with the same matrix *MM* with some angles of rotations and boosts. Then all energy values will change simply because of changes in kinetic energy, but the qualitative relations between field energies in the presence of interaction and without it will remain the same. What really does not change at all when changing the coordinate system is the mass of particles. Without interaction the photon mass is always zero, in the presence of interaction it becomes non-zero, and the electron mass changes, but these values of masses in the presence of interaction in any frame of reference are the same both in total and separately.

Let us return to the question of choosing a sign to account for the interaction

$$\mathbf{p}_f - 0.01 * \mathbf{p}_b$$
$$\mathbf{p}_b + 0.01 * \mathbf{p}_f$$

which we used in the above examples. With this choice, the total energy in the interaction always increases. If you choose both signs minus, then in the interaction of particles, one of which has the momentum transformed, in the same examples it turns out that the total energy decreases, although the total mass increases. In this regard, we can assume that the choice of the plus sign is correct.

It is possible to increase the dimensionality of the spinor and to consider, for example, a spinor with six arbitrary complex components

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_2, b_3, b_4, b_5)$$

and the corresponding matrices

$$MMM = \begin{pmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}$$

$$\Sigma\Sigma\Sigma_0 = \begin{pmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \Sigma\Sigma\Sigma_1 = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{pmatrix} \Sigma\Sigma\Sigma_2 = \begin{pmatrix} \sigma_2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_2 \end{pmatrix} \Sigma\Sigma\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 & 0 \\ 0 & \sigma_3 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

Then still the vector

$$B_\mu = \frac{1}{2}\mathbf{b}^\dagger \Sigma\Sigma\Sigma_\mu \mathbf{b}$$

will be Lorentzian and transformed by the matrix

$$\Lambda^\mu_\nu = \frac{1}{2}\mathrm{Tr}[\sigma_\mu M \sigma_\nu M^\dagger]$$

One can change any matrix *M* in *MMM* to a matrix *N* by changing the σ sign in the correspondingq position of the ΣΣΣ matrices. For example, if

$$MNM = \begin{pmatrix} M & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & M \end{pmatrix}$$

$$\Sigma\Sigma\Sigma_0 = \begin{pmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \Sigma\Sigma\Sigma_1 = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -\sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{pmatrix} \Sigma\Sigma\Sigma_2 = \begin{pmatrix} \sigma_2 & 0 & 0 \\ 0 & -\sigma_2 & 0 \\ 0 & 0 & \sigma_2 \end{pmatrix} \Sigma\Sigma\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 & 0 \\ 0 & -\sigma_3 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

then the behavior of the spinor and the corresponding vector will not change. In this connection, we can consider only the variant with all matrices M and matrices σ entering only with plus. That is, one can limit oneself to sigma matrices and not to use gamma matrices, the differences between bosons and fermions will be determined only by the structure of spinors with the same way of forming vectors from them.

Let us leave the components of the spinor still complex, but not arbitrary. Let the spinor have the form

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, b_0, b_1, b_0, b_1)$$

then the vector **B** obtained from it will have a zero length, and if the spinor is real, it will also have a zero component in the y-axis. One can add as many b_0, b_1 pairs to the spinor as one wants, simultaneously increasing the dimensionality of all matrices, and the vector will always have these properties. This behavior can explain the ability of any number of bosons to be in the same quantum state.

If the complex spinor has the form

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, {b_1}^*, -{b_0}^*, 0,0)$$

then the vector obtained from it has only one nonzero component

$$\mathbf{B}^{\boldsymbol{T}} = (B_0, 0,0,0)$$

If we try to add a pair of components

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, {b_1}^*, -{b_0}^*, b_0, b_1)$$

or

$$\mathbf{b}^{\boldsymbol{T}} = (b_0, b_1, {b_1}^*, -{b_0}^*, {b_1}^*, -{b_0}^*)$$

then all components of the vector are non-zero, and we cannot get a vector with one zero component. That is, we cannot get a particle with zero momentum, that is, stationary in some frame of reference. In other words, a spinor corresponding to a fermion can have only two pairs of nonzero components connected by the relation

$$\begin{pmatrix} b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} {b_0}^* \\ {b_1}^* \end{pmatrix}$$

Only such a spinor can possess a rest system, that is, exist as a particle of matter. These conclusions are valid for complex spinors, but we must consider the more particular case of real spinors, since real bosons and fermions are considered to have only two degrees of freedom.

If one considers a complex spinor with even number of components, one can already obtain a vector with zero momentum at zero boosts, for example, this is true for a spinor of the form

$$\mathbf{f}^{\boldsymbol{T}} = (f_0, f_1, {f_1}^*, -{f_0}^*, f_0, f_1, {f_1}^*, -{f_0}^*)$$

and also for a spinor of the form

$$\mathbf{f}^{\boldsymbol{T}} = (f_0, f_1, {f_1}^*, -{f_0}^*, d_0, d_1, {d_1}^*, -{d_0}^*)$$

Let us consider in detail the structure of boson and fermion spinors in a zero-bust frame of reference. Let us represent the complex boson spinor as a sum of spinors of special structure, about which it is customary to say that one of them has a helicity of one and the other minus one

$$\begin{pmatrix} b_0 \\ b_1 \\ b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} b_0 \\ 0 \\ b_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b_1 \\ 0 \\ b_1 \end{pmatrix}$$

Then from these three spinors we get three real vectors of zero length

$$\begin{pmatrix} B1_0 + B2_0 \\ B_1 \\ B_2 \\ B2_0 - B1_0 \end{pmatrix} \quad \begin{pmatrix} B1_0 \\ 0 \\ 0 \\ -B1_0 \end{pmatrix} \quad \begin{pmatrix} B2_0 \\ 0 \\ 0 \\ B2_0 \end{pmatrix}$$

As we see the vector of the boson, for example, the vector of electromagnetic potential is not equal to the sum of vectors corresponding to photons of different helicity. Let us represent the complex spinor of a fermion as a sum of spinors, one of which is commonly referred to as having spin unity and the other minus unity

$$\begin{pmatrix} f_0 \\ f_1 \\ {f_1}^* \\ -{f_0}^* \end{pmatrix} = \begin{pmatrix} 0 \\ f_1 \\ {f_1}^* \\ 0 \end{pmatrix} + \begin{pmatrix} f_0 \\ 0 \\ 0 \\ -{f_0}^* \end{pmatrix}$$

Then from these three spinors we get three real vectors

$$\begin{pmatrix} F1_0 + F2_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F1_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} F2_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The vector for a fermion, that is, the current vector, unlike the potential vector, is equal to the sum of vectors corresponding to fermions with different spins, in particular, the masses of fermions are summed up.

Since all electrons are the same and have the same mass and charge at rest, we can express the field and momentum spinors of electrons with different spins at zero boosts through the mass and charge of the electron.

$$\begin{pmatrix} \sqrt{m_e} \\ 0 \\ 0 \\ -\sqrt{m_e} \end{pmatrix} \quad \begin{pmatrix} 0 \\ \sqrt{m_e} \\ \sqrt{m_e} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{e} \\ 0 \\ 0 \\ -\sqrt{e} \end{pmatrix} \quad \begin{pmatrix} 0 \\ \sqrt{e} \\ \sqrt{e} \\ 0 \end{pmatrix}$$

A third kind of field can also be imagined, which is neither a boson nor a fermion

$$\begin{pmatrix} d_0 \\ d_1 \\ d_0 \\ -d_1 \end{pmatrix} = \begin{pmatrix} d_0 \\ 0 \\ d_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ d_1 \\ 0 \\ -d_1 \end{pmatrix}$$

$$\begin{pmatrix} B1_0 + B2_0 \\ 0 \\ 0 \\ B2_0 - B1_0 \end{pmatrix} = \begin{pmatrix} B1_0 \\ 0 \\ 0 \\ -B1_0 \end{pmatrix} + \begin{pmatrix} B2_0 \\ 0 \\ 0 \\ B2_0 \end{pmatrix}$$

this field, like the fermion, has non-zero mass and non-zero charge, but, unlike the fermion, creates a non-zero current even at zero boosts. It, like boson, can be represented as a sum of fields with two different spins, each of them has zero mass, but unlike boson its vector is equal to the sum of vectors obtained from spinors with different spins.

In [6] the solution of the Dirac equation for a free particle is sought as

$$\begin{pmatrix} \psi_0(\mathbf{X}) \\ \psi_1(\mathbf{X}) \\ \psi_2(\mathbf{X}) \\ \psi_3(\mathbf{X}) \end{pmatrix} = \begin{pmatrix} u_0(\mathbf{P}) \\ u_1(\mathbf{P}) \\ u_2(\mathbf{P}) \\ u_3(\mathbf{P}) \end{pmatrix} \exp\left(-i\eta_{\mu\nu} P_\mu X_\nu\right)$$

that is, the scalar product of vectors is used to describe the spinor translation, even though these vectors are obtained from the corresponding spinors by the formulas

$$P_\mu = \frac{1}{2}\mathbf{p}^\dagger \Sigma_\mu \mathbf{p}$$

$$X_\mu = \frac{1}{2}\mathbf{x}^\dagger \Sigma_\mu \mathbf{x}$$

and for the particular case of the boson the equality takes place

$$\eta_{\mu\nu} P_\mu X_\nu = \frac{1}{2}[\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x}]^* [\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x}]$$

$$\Sigma_{MM} = \begin{pmatrix} \sigma_M & 0 \\ 0 & \sigma_M \end{pmatrix}$$

When considering the translation of a field described by a spinor, it is logical, at least in the case of a boson, to stay within the spinor representations and use the phase multiplier calculated directly from the spinors of coordinates and momentum. It is possible to represent the spinor field in the form

$$\begin{pmatrix} \psi_0(\mathbf{x}) \\ \psi_1(\mathbf{x}) \\ \psi_2(\mathbf{x}) \\ \psi_3(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} u_0(\mathbf{p}) \\ u_1(\mathbf{p}) \\ u_2(\mathbf{p}) \\ u_3(\mathbf{p}) \end{pmatrix} \exp(-i\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x})$$

Note that this phase multiplier is a scalar quantity and does not change in Lorentz transformations. For the case of a fermion and in general for arbitrary four-component spinors, the relation between the scalar products of vectors and their generating spinors does not hold, although they are both invariant under the Lorentz transformations. Then it is possible to put a question which way of calculating the phase multiplier at translations is more adequately describing the nature. At infinitesimal translations these phase multipliers are close, and at finite translations the scalar product of spinors may be more adequate description.

Let us compare the translations generated by a finite spinor of coordinates **x** in two ways

$$X_\mu = \frac{1}{2}\mathbf{x}^\dagger \Sigma_\mu \mathbf{x}$$

$$\Psi_\mu = \frac{1}{2}\boldsymbol{\psi}^\dagger \Sigma_\mu \boldsymbol{\psi}$$

$$P_\mu = \frac{1}{2}\mathbf{p}^\dagger \Sigma_\mu \mathbf{p}$$

$$\boldsymbol{\Psi 1} = \boldsymbol{\Psi} * \exp(-i\eta_{\rho\sigma} P_\rho X_\sigma) = \boldsymbol{\Psi} * \exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger \Sigma_\rho \mathbf{p})(\mathbf{x}^\dagger \Sigma_\sigma \mathbf{x})\right)$$

$$\boldsymbol{\psi 2} = \boldsymbol{\psi} * \exp(-i\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x})$$

$$\Psi 2_\mu = \frac{1}{2}\boldsymbol{\psi 2}^\dagger \Sigma_\mu \boldsymbol{\psi 2} = \frac{1}{2}(\boldsymbol{\psi} * \exp(-i\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x}))^\dagger \Sigma_\mu (\boldsymbol{\psi} * \exp(-i\mathbf{p}^{\boldsymbol{T}} \Sigma_{MM} \mathbf{x}))$$

$$\Psi 1_\mu = \Psi_\mu * \exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger \Sigma_\rho \mathbf{p})(\mathbf{x}^\dagger \Sigma_\sigma \mathbf{x})\right)$$

$$= \frac{1}{2}\boldsymbol{\psi}^\dagger \Sigma_\mu \boldsymbol{\psi} * \exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger \Sigma_\rho \mathbf{p})(\mathbf{x}^\dagger \Sigma_\sigma \mathbf{x})\right)$$

$$\frac{\Psi 2_\mu}{\Psi 1_\mu}=\frac{(\boldsymbol{\psi}*\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}))^\dagger\Sigma_\mu(\boldsymbol{\psi}*\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}))}{\boldsymbol{\psi}^\dagger\Sigma_\mu\boldsymbol{\psi}\ *\exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger\Sigma_\rho\mathbf{p})(\mathbf{x}^\dagger\Sigma_\sigma\mathbf{x})\right)}$$

$$=\frac{(\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}))^\dagger\boldsymbol{\psi}^\dagger\Sigma_\mu\boldsymbol{\psi}\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})}{\boldsymbol{\psi}^\dagger\Sigma_\mu\boldsymbol{\psi}\ *\exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger\Sigma_\rho\mathbf{p})(\mathbf{x}^\dagger\Sigma_\sigma\mathbf{x})\right)}$$

$$=\frac{(\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}))^*\exp(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})}{\exp\left(-i\frac{1}{2}\eta_{\rho\sigma}(\mathbf{p}^\dagger\Sigma_\rho\mathbf{p})(\mathbf{x}^\dagger\Sigma_\sigma\mathbf{x})\right)}$$

$$=\exp\left((-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})^*+(-i\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})-\left(-i\frac{1}{2}\eta_{\rho\sigma}\left(\mathbf{p}^\dagger\Sigma_\rho\mathbf{p}\right)(\mathbf{x}^\dagger\Sigma_\sigma\mathbf{x})\right)\right)$$

$$=\exp\left(-i\left((\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})^*+\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}-\frac{1}{2}\eta_{\rho\sigma}\left(\mathbf{p}^\dagger\Sigma_\rho\mathbf{p}\right)(\mathbf{x}^\dagger\Sigma_\sigma\mathbf{x})\right)\right)$$

This ratio is a complex number with unit module, invariant to Lorentz transformations. If we treat the square of the field vector modulus as a probability density, nothing will change for a free particle, but the interference pattern will be different for interfered particles.

For boson the formula is simplified

$$\frac{\Psi 2_\mu}{\Psi 1_\mu}=\exp\left(-i\left((\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x})^*+\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}-\frac{1}{4}[\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}]^*[\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}]\right)\right)$$

What is the fundamental difference between spinor and vector translation? It consists in the fact that phase $\eta_{\rho\sigma}P_\rho X_\sigma$ is a real number and phase $\mathbf{p}^{\boldsymbol{T}}\Sigma_{MM}\mathbf{x}$ is a complex number. Consequently, the phase in spinor translation carries more information that is important in the superposition of field propagation paths. In particular, in vector translation the exponent is always a real value multiplied by the imaginary unit, so the modulus of the exponent is unity, the plane wave does not attenuate, and the attenuation is taken into account by dividing by some degree of the distance between the starting point and the end point. In spinor translation, the phase is complex and the modulus of the exponent can be less than unity, which will cause attenuation without additional division by distance. As an example, the experiment with the passage of an electron through two slits may be considered. The process can be described by the following logical sequence of events

$$\boldsymbol{\psi}\mathbf{1}=\boldsymbol{\psi}\mathbf{0}*exp(-i\mathbf{p0}^{\boldsymbol{T}}\Sigma_{MM}(\mathbf{x1}-\mathbf{x0}))$$

$$\boldsymbol{\psi}\mathbf{2}=\boldsymbol{\psi}\mathbf{0}*exp(-i\mathbf{p0}^{\boldsymbol{T}}\Sigma_{MM}(\mathbf{x2}-\mathbf{x0}))$$

$$\mathbf{p1}=\sqrt{\frac{m_e}{e}}\,\boldsymbol{\psi}\mathbf{1}$$

$$\mathbf{p2}=\sqrt{\frac{m_e}{e}}\,\boldsymbol{\psi}\mathbf{2}$$

$$\boldsymbol{\psi}\mathbf{31}=\boldsymbol{\psi}\mathbf{1}*exp(-i\mathbf{p1}^{\boldsymbol{T}}\Sigma_{MM}(\mathbf{x3}-\mathbf{x1}))$$

$$\boldsymbol{\psi}\mathbf{32}=\boldsymbol{\psi}\mathbf{2}*exp(-i\mathbf{p2}^{\boldsymbol{T}}\Sigma_{MM}(\mathbf{x3}-\mathbf{x2}))$$

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$$\boldsymbol{\psi}\mathbf{3} = \boldsymbol{\psi}\mathbf{31} + \boldsymbol{\psi}\mathbf{32}$$
$$= \boldsymbol{\psi}\mathbf{1} * exp\left(-i\mathbf{p1}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x1})\right) + \boldsymbol{\psi}\mathbf{2} * exp\left(-i\mathbf{p2}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x2})\right)$$
$$= \boldsymbol{\psi}\mathbf{0} * exp(-i\mathbf{p0}^{T}\Sigma_{MM}(\mathbf{x1}-\mathbf{x0})) * exp\left(-i\mathbf{p1}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x1})\right) + \boldsymbol{\psi}\mathbf{0}$$
$$* exp(-i\mathbf{p0}^{T}\Sigma_{MM}(\mathbf{x2}-\mathbf{x0})) * exp\left(-i\mathbf{p2}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x2})\right)$$
$$= \boldsymbol{\psi}\mathbf{0} * exp\left(-i\mathbf{p0}^{T}\Sigma_{MM}(\mathbf{x1}-\mathbf{x0}) - i\mathbf{p1}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x1})\right) + \boldsymbol{\psi}\mathbf{0}$$
$$* exp\left(-i\mathbf{p0}^{T}\Sigma_{MM}(\mathbf{x2}-\mathbf{x0}) - i\mathbf{p2}^{T}\Sigma_{MM}(\mathbf{x3}-\mathbf{x2})\right)$$

Note another difference in the properties of boson and fermion spinors. Their corresponding vectors can also be found by the formulas

$$B_{\mu} = \frac{1}{2}\mathrm{Tr}[(\mathbf{bb}^{\dagger})\Sigma_{\mu}]$$
$$F_{\mu} = \frac{1}{2}\mathrm{Tr}[(\mathbf{ff}^{\dagger})\Sigma_{\mu}]$$

and we can form spintensors from the vectors

$$S_f = \sum_{\mu} B_{\mu}\Sigma_{\mu}$$

$$S_f = \sum_{\mu} F_{\mu}\Sigma_{\mu}$$

Though the determinant of the direct product of both spinors is zero

$$det\,(\mathbf{bb}^{\dagger}) = 0 \qquad det\,(\mathbf{ff}^{\dagger}) = 0$$

this is not the case for the determinants of spintensors; for the boson it is also zero, and for the fermion it is equal to the square of the vector length

$$det\,S_b = 0 \qquad det\,S_f = (\mathbf{F}^{T}\eta\mathbf{F})^{2}$$

Let's try to find an explanation of why the boson obeys the Bose statistic and the fermion the Fermi statistic. Bosons with one or another definite helicity, as well as their sum, have the form of a four-component spinor

$$\begin{pmatrix} b_0 \\ b_1 \\ b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} b_0 \\ 0 \\ b_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b_1 \\ 0 \\ b_1 \end{pmatrix}$$

You can join another spinor of the same kind with half the overlap and get a six-component spinor

$$\begin{pmatrix} b_0 \\ b_1 \\ b_0 \\ b_1 \\ b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} b_0 \\ 0 \\ b_0 \\ 0 \\ b_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b_1 \\ 0 \\ b_1 \\ 0 \\ b_1 \end{pmatrix}$$

This combination can be thought of as two bosons in the same quantum state. Since the bosons are coupled, they are not independent. Another boson in the same state can be added to them with the same overlap and so on to infinity. If *n* photons are coupled, the chain length is *n*+1 identical pairs, i.e. if a photon is born from a vacuum state, it contains two pairs at once, and each next born photon adds only one pair.

It is impossible to join two fermions with half overlap; the only possibility is to join two fermions with different spins

$$\begin{pmatrix} f_0 \\ f_1 \\ f_1{}^* \\ -f_0{}^* \end{pmatrix} = \begin{pmatrix} 0 \\ f_1 \\ f_1{}^* \\ 0 \end{pmatrix} + \begin{pmatrix} f_0 \\ 0 \\ 0 \\ -f_0{}^* \end{pmatrix}$$

that is why fermions cannot be in the same state.

How to explain the presence of antiparticles in the framework of the considered concept? The simplest way is to use the opposite sign of the matrix sigma in the transition from spinors to vectors. Then we get negative energy, opposite sign charge and even negative time. In this case there is no difference between spinors of the particle and antiparticle and it is not clear what controls the choice of a sign sigma when creating a vector from them. It would be more logical to assume the difference between particles and antiparticles already at the level of spinors, for example, the spinor of the antiparticle is equal to the spinor of the particle multiplied by the imaginary unit, but the complex conjugation in the formula for a vector leads to the fact that the energy is still positive. If we remove the conjugation from the formula, the vector is complex and then its physical interpretation is incomprehensible.

Suppose that each fermion and boson has a label - plus or minus. The sign determines the sign of the sigma matrices, which form the vector from the spinor. If the fermion and antifermion meet, that is, have the same coordinate, then the components of all spinors - field, coordinate and momentum are transformed by the law

$$\begin{pmatrix} f_2{}' \\ f_3{}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} f_2{}^* \\ f_3{}^* \end{pmatrix}$$

The fermions become bosons, but retain the label sign. So, they lose charge and mass, their coordinates remain the same, but their vectors now have zero length; in addition, the momentum vectors change immediately. If, for example, they had zero components, i.e. particles were motionless, they acquire non-zero components of opposite sign because of different signs of sigma, bosons fly in opposite directions with light speed, energy of each particle with its sign remains the same, and the law of conservation of momentum vector for each separate component is also satisfied. Let us illustrate the interaction process by a numerical example for spinors and vectors of a fermion and an antifermion with the same spinor, but different labels before

$$\begin{pmatrix} + \\ 3 \\ -2 \\ -2 \\ -3 \end{pmatrix} \begin{pmatrix} - \\ 3 \\ -2 \\ -2 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} 13 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -13 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and after the interaction

$$\begin{pmatrix} + \\ 3 \\ -2 \\ 3 \\ -2 \end{pmatrix} \begin{pmatrix} - \\ 3 \\ -2 \\ 3 \\ -2 \end{pmatrix} \qquad \begin{pmatrix} 13 \\ -12 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} -13 \\ 12 \\ 0 \\ -5 \end{pmatrix}$$

Using a different label sign for the coordinate spinor causes the corresponding vectors to be different when the coordinate spinors coincide and the particles cannot meet in vector space. Or it is necessary to accept the artificial assumption that the label signs act on the sign of the sigma matrices only in the formation of the momentum vector and the field, i.e. the assumption about the possibility of changing the sign of time is rejected. We are forced to assume that the coordinate spinor always has a plus sign, otherwise it would not make sense for the momentum spinor to have a minus sign, because if both the coordinate and momentum spinors have a minus sign, their scalar product is still positive, the phase multiplier remains the same and both particles fly in the same direction. That is, we distinguish the coordinate spinor space, which is always transformed to a vector with a positive label, and the field and momentum spinors can have both plus and minus labels. But there are still spinors of different types - bosonic and fermionic spinors in coordinate spinor space.

If one proceeds from the assumption that the coordinate spinor of some field repeats the structure of the spinor of that field, the question arises whether one can obtain from the coordinate spinor of a boson a vector which can also be obtained from some coordinate spinor of a fermion, that is, whether a boson and a fermion can meet at one point of Minkowski space. If not, we will have to reject the hypothesis that a coordinate spinor can have a definite structure and admit that the structure of the coordinate spinor corresponding to the field spinor can be arbitrary. One can also consider the intermediate assumption that the coordinate spinor of the boson necessarily has a bosonic structure, because the boson must move at light speed, while the coordinate spinor of the fermion can have an arbitrary structure, other than bosonic one, since the fermion can be at any point in vector space.

The coordinate vector of the photon moving with the speed of light cannot coincide with the coordinate vector of the electron having a sub light speed, because then the vectors would have the same length, but the boson has zero length and the fermion does not. But at their interaction the photon acquires mass and its speed becomes less than the speed of light and the argument about the impossibility of coincidence does not work. Not coordinate vectors in pure form should coincide, but coordinates modified taking into account the interaction

Both boson and fermion consist of a pair of coupled two-component spinors and have a common plus or minus label, but we can assume that under certain conditions these two-component spinors can exist independently without being coupled. Each has its own plus or minus label, if two spinors with the same label are coupled you get a boson with either two minuses or two pluses, i.e. their common sign may be different. If two spinors with different signs are coupled, you get a fermion, and their common sign may also be different, because the first spinor in the pair may be minus and the second plus, and vice versa. Suppose once the total energy of the world was high, and all spinors existed separately, then they bonded together into bosons, fermions, antifermions, and antibosons. When they interact, they decouple and recouple, sometimes in a different order. For this interaction matter fermions must be accelerated to provide the energy of uncoupling, and matter and antimatter fermions interact spontaneously.

Let us consider the mechanism that leads to a change in the sign of the wave function during the interchange of coordinates of two electrons. In Minkowski space, electrons are represented by vectors whose spatial parts represent a three-dimensional vector with a certain direction. Interchange can be performed by rotating the entire three-dimensional space so that the coordinate points swap places. In this case, the direction of the electron field vector will change and to bring the picture to its original form it is necessary to rotate each field vector by a certain angle. To rotate the vectors by this angle, it is necessary to rotate the corresponding spinor by half the angle. If we sum up the angles of rotation of the two electron vectors, we should get 360 degrees and the total angle of rotation of the spinors respectively 180 degrees. If the wave function of the system includes among others two complex multipliers in the form of field spinors, their rotation by a total angle of 180 degrees (the angles are included in the exponents of the factors, so they sum up) will change the sign of the whole wave function of the system. This can be demonstrated most clearly in the case where both field vectors lie in the same plane and this plane, together with the entire space, rotates flat around the middle of the segment connecting the electron locations. After rotation, the field vectors turn out to be rotated and to bring the whole picture to its original state it is necessary to rotate them in this plane by 180 degrees each, for which the spinors must be rotated by 90 degrees each.

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For realization of the given mechanism it is necessary that the state of the system of particles is described by some product of exponents from quantities, in general case being matrices. Since the change of a sign of the state of the system at the exchange of particles is considered proved, this is an indirect confirmation that the state of the system should be described by such product of exponents.

The transition to consideration of the electron as a complex spinor, as opposed to a real vector, allowed taking into account the interference of particles, but this transition was not complete, the coordinates and momentum continue to be considered as vectors, while they also have a spinor nature. In addition, the photon field also continues to be treated as a vector, although it too is a complex spinor and therefore also subject to interference. Including a complex spinor rather than a real vector in the wave function is justified because it allows one not to lose the information that determines interference in the interaction of fields. In particular, if one ignores the spinor complex essence of the photon, one cannot account for its interference properties. But after all the calculations are done, it is necessary to go to the real values. The transition from a complex spinor to a real vector using Pauli matrices performs the same task as the transition in quantum mechanics from the complex wave function to the square of its modulus. Quantum mechanics, in a sense, is a special case of quantum field theory, in which there are not four dimensions, but only one, and in which of four Pauli matrices only the zero index, i.e. unit matrix, is used. While quantum mechanics treats probability as a real number, quantum field theory deals with probability as a four-dimensional real vector. The place of the probability amplitude, which in quantum mechanics is a complex number, in quantum field theory is taken by a complex spinor.

Let's try to connect the theory with experiment. Let's take the usual formula for the interaction of the electron with the electromagnetic field in the presence of only one photon

$$P_\mu - eA_\mu = P_\mu - e\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a}$$

The field of one photon in the experiment is difficult to measure, so let us assume that the field is formed by n photons in the same state, which corresponds to a chain of n+1 identical two-component spinors, one photon consisting of two such spinors. Hence the potential of n photons is $\frac{n+1}{2}$ times the field of one photon. We will consider an electron as a single one.

$$P_\mu - e\frac{n+1}{2}A_\mu = \frac{1}{2}\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{p}_e - e\frac{n+1}{2}\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a}$$

Let's find the correction for the interaction of one electron with one photon

$$\left(\frac{1}{2}(\mathbf{p}_e - \sqrt{e}\mathbf{a})^\dagger \Sigma_\mu (\mathbf{p}_e - \sqrt{e}\mathbf{a})\right) = \frac{1}{2}\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{p}_e + e\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} - \frac{1}{2}\sqrt{e}\left(\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{a} + \mathbf{a}^\dagger \Sigma_\mu \mathbf{p}_e\right)$$

$$= P_\mu + eA_\mu - \frac{1}{2}\sqrt{e}\left(\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{a} + \mathbf{a}^\dagger \Sigma_\mu \mathbf{p}_e\right)$$

Here the field action appears with a plus sign, while in the conventional formula there is a minus sign. Let's try to assume for a while that the sign in the interaction is really plus

$$P_\mu + eA_\mu = P_\mu + e\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a}$$

Let's subtract the correction and set the task to check in the experiment which formula is more correct

$$P_\mu + e\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a} - \frac{1}{2}\sqrt{e}\left(\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{a} + \mathbf{a}^\dagger \Sigma_\mu \mathbf{p}_e\right)$$

For a real experiment with a strong enough field it is necessary to compare two formulas

$$P_\mu + e\frac{n+1}{2}A_\mu = \frac{1}{2}\mathbf{p}_e{}^\dagger \Sigma_\mu \mathbf{p}_e + e\frac{n+1}{2}\frac{1}{2}\mathbf{a}^\dagger \Sigma_\mu \mathbf{a}$$

and

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$$P_\mu + e\frac{n+1}{2}A_\mu - n\frac{1}{2}\sqrt{e}\left(\mathbf{p}_e{}^\dagger\Sigma_\mu\mathbf{a} + \mathbf{a}^\dagger\Sigma_\mu\mathbf{p}_e\right)$$
$$= \frac{1}{2}\mathbf{p}_e{}^\dagger\Sigma_\mu\mathbf{p}_e + e\frac{n+1}{2}\frac{1}{2}\mathbf{a}^\dagger\Sigma_\mu\mathbf{a} - n\frac{1}{2}\sqrt{e}\left(\mathbf{p}_e{}^\dagger\Sigma_\mu\mathbf{a} + \mathbf{a}^\dagger\Sigma_\mu\mathbf{p}_e\right)$$

Here we have assumed that the correction for n photons for interaction with one electron is n times greater than for the electron with one photon, although it may be that the correct ratio is $\frac{n+1}{2}$ times greater, this question needs clarification. In the experiment we could measure the magnitude of the field $e\frac{n+1}{2}A_\mu$ and the momentum P_μ, use them to calculate the spinors $\mathbf{a}$ and $\mathbf{p}$, then apply them to the corrected formula; we do not know the number of photons n. To avoid the need to calculate their number, we may consider n to be large, and neglect the difference with $n+1$. Then the question remains whether to use the correction for one photon with multiplier n or with multiplier $\frac{n+1}{2}$, it depends on whether the electron interacts with each photon separately - then the coefficient will be n, or it interacts with a chain of overlapping photons as a whole - then the coefficient will be $\frac{n+1}{2}$. There also remains the question of the sign of the interaction of the electron with the field $P_\mu \pm eA_\mu$, since we are looking for a correction contribution with a plus sign, while the standard formula includes a minus.

4. Spintensor model of quantum system

Let us consider a representation of a quantum system of several particles having the same coordinates. The question about the possibility of coincidence of coordinates of a boson and a fermion will be left without consideration for the moment. For each four-component spinor $\mathbf{s_i}$ we will correspond to the spintensor

$$S_i = \mathbf{s_i}\mathbf{s_i}^\dagger$$

from which we can obtain the vector

$$(V_i)_\mu = \frac{1}{2}\mathrm{Tr}[(S_i)\Sigma_\mu]$$

Then a system of n field spinors with the same coordinates is matched to a spintensor in the form of a product of the spintensors, obtained from these spinors, as well as the corresponding vector

$$S = \mathbf{\Pi} S_i$$

$$V_\mu = \frac{1}{2}\mathrm{Tr}[S\Sigma_\mu]$$

Since all rotations and boosts are realized by multiplying the spinor by square matrices which are exponents of other square matrices, it seems logical to represent the spintensor in the same form, namely, as an exponent of its logarithm. During the transformation, the spintensor is multiplied by the transformation matrix twice, on the left and on the right; in general, this will look like the product of the exponent of the spintensor logarithm by the product of the transformation matrices in the form of exponents on the left and by the product of these transformation matrices on the right in the opposite order. If there are several particles, their set can be represented as the product of

$$S = \mathbf{\Pi} S_i = \mathbf{\Pi} exp(ln(S_i)) = exp(ln(\mathbf{\Pi} S_i))$$

If the matrices $ln(S_i)$ commutate with each other, which is wrong in the general case, then the exponent of their sum would be equal to the product of the exponents from the logarithms, that is, simply the product of the spintensors

$$S = exp(\mathbf{\Sigma} ln(S_i)) = \mathbf{\Pi} exp(ln(S_i)) = \mathbf{\Pi} S_i$$

At finite angles of turns and boosts there is no coincidence, but perhaps the exponent from the sum of logarithms of spintensors reflects reality more adequately than the product of spintensors, and both representations have Lorentz invariance, and at infinitesimal angles of turns and boosts both descriptions coincide in the limit.

Let us also touch upon the question of the interaction of particles. When studying the interaction of electrons, a complex value is used

$$\mathbf{f}_1{}^{\dagger}\Sigma_\mu\mathbf{f}_2$$

which describes the interaction of two fermions. Unified spinor description allows constructing similar Lorentz invariant complex vectors for any combination of bosons and fermions

$$\mathbf{b}_1{}^{\dagger}\Sigma_\mu\mathbf{b}_2 \qquad \mathbf{f}_1{}^{\dagger}\Sigma_\mu\mathbf{b}_2$$

or use a more convenient real vector

$$\frac{1}{2}\mathbf{f}_1{}^{\dagger}\Sigma_\mu\mathbf{b}_2+\frac{1}{2}\mathbf{b}_2{}^{\dagger}\Sigma_\mu\mathbf{f}_1$$

As an alternative way to describe the interaction, we can consider a vector of symmetrized sum, generalizing it to three or more particles

$$\frac{1}{6}\mathbf{s}_1{}^{\dagger}\Sigma_\mu\mathbf{s}_2+\frac{1}{6}\mathbf{s}_2{}^{\dagger}\Sigma_\mu\mathbf{s}_1+\frac{1}{6}\mathbf{s}_2{}^{\dagger}\Sigma_\mu\mathbf{s}_3+\frac{1}{6}\mathbf{s}_3{}^{\dagger}\Sigma_\mu\mathbf{s}_2+\frac{1}{6}\mathbf{s}_3{}^{\dagger}\Sigma_\mu\mathbf{s}_1+\frac{1}{6}\mathbf{s}_1{}^{\dagger}\Sigma_\mu\mathbf{s}_3$$

this vector is real for any number of particles.

There is a right to exist also a variant of a simple summation of field spinors for particles having the same coordinates

$$(\mathbf{s}_1+\mathbf{s}_2+\mathbf{s}_3)^{\dagger}\Sigma_\mu(\mathbf{s}_1+\mathbf{s}_2+\mathbf{s}_3)$$

5. Conclusion

The description of the electromagnetic field in the form of a four-component complex spinor, from which a vector of electromagnetic potential with two degrees of freedom, calibrated by two conditions - zero length and zero component in the y-axis - is obtained with the help of Pauli matrices.

A similar approach is applied to the field of a fermion, such as an electron. A unified way to describe bosons and fermions in spinor space is proposed. It is shown that bosons and fermions, of which the electron and the photon are examples, can be described by complex spinors whose structure determines whether they belong to a boson or a fermion. Each spinor by means of the universal formula corresponds to a real vector, in the case of a fermion it is a current vector, in the case of a boson it is a vector, for example, of the electromagnetic potential. Each spinor of a field is matched with a spinor of coordinates and a spinor of momentum, which are transformed by the same Lorentz transformations and which have the same structure as the corresponding field spinor, that is, momentum and coordinates of boson have a bosonic spinor structure, while momentum and coordinates of fermion have a fermionic spinor structure. The field, coordinate and momentum vectors formed from spinors by the universal formula automatically have a zero length for the boson, while for the fermion they all have a nonzero length, so the fermion, in contrast to the boson, has a nonzero mass, a nonzero charge and moves with a sub light speed.

If we assume that all particles, as well as their impulses and coordinates, are spinors, then their interaction, as well as their evolution in time (which does not exist in spinor space in an explicit form) and propagation in space should be studied and described in two-dimensional complex spinor space, and then the obtained results should be expressed in terms of real vector space.

References

1. Steven Weinberg, The Quantum Theory of Fields, Volume 1, Cambridge University Press, 1999.
2. L. H. Ryder, Quantum Field Theory, 2nd Ed., Cambridge University Press, New York, 1996.
3. A Zee, Quantum Field Theory in a Nutshell, 2nd Edition. – Princeton University Press, 2003.
4. Buchbinder I.L., Samsonov I.B., Relativistic wave equations: tutorial. - Tomsk: TPU Publishing House, 2007. - 97 p., p. 55].
5. Peskin M.E., Shroeder D.V., An Introduction to Quantum Field Theory. – Addison-Wesley Publishing Company, 1995.
6. R. P. Feynman, Quantum Electrodynamics, W. A. Benjamin, Inc, New York, 1961.

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