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Article

General Quantum Gravity: Quantum Einstein Field Equation from $(3+1)D$ Quantum Mechanics & Extra Eleven Dimensions within $(3+1)D$ Quantum State Spaces

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Abstract: $(3+1)D$ quantum mechanics is a four-dimensional quantum formalism, which is developed neither in a Minkowski spacetime, nor in a purely Hilbert space and not its metric is Lorentzian. This quantum mechanics also yields the quantum Einstein field equation in $(3+1)D$ quantum spacetime. Every observable $(3+1)D$ quantum spacetime can be found to contain internally hidden opposite spin quantum state, that results Supersymmetry. Closed continuous mappings of this hidden quantum state immediately yield more extra hidden dimensions including string. This string is found to be fundamentally eleven-dimensional (rather than lower dimensional) by nature within this quantum spacetime, which is necessarily a natural phenomenon of $(3+1)D$ Quantum Mechanics.

Keywords: quantum gravity; quantum state spaces; quantum geometry; dimension theory

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1. Introduction

Time is not considered as a dimension in $3D$ Quantum Mechanics [1]. That is why gravity is excluded from the quantum spacetime. In this article, we introduce a $(3+1)D$ quantum formalism, which is developed neither in a Minkowski spacetime, nor in a purely Hilbert space and not its metric is Lorentzian. To include gravity in the quantum spacetime, this quantum mechanics yields the (renormalizable) quantum Einstein field equation in $(3+1)D$ quantum spacetime by introducing a purely quantum line element. Analysis of its quantum state spaces shows us that every observable $(3+1)D$ quantum spacetime contains internally hidden opposite spin quantum state. Application of Dimension Theory in this $(3+1)D$ quantum formalism yields that this hidden quantum state immediately acquires more extra hidden dimensions including string. This string, which is found to be fundamentally eleven-dimensional, is inevitably natural and universal but hidden inside every $(3+1)D$ observable quantum spacetime. This finding can make an effective impact on the field of studying and developing String/M-theory in a deeper level.

2. $(3+1)D$ Quantum Mechanics

Let the line element $\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \equiv \left(\frac{\mathrm{d}t}{m}\right)^2 g_{\mu\nu}\,P^\mu P^\nu$ for the 'four-momentum' P^μ, then,

$$\begin{aligned}\mathrm{d}\,\mathcal{S}_P^2 &\equiv m^2\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 = m^2c^2 - m^2\sum\frac{\mathrm{d}x^i}{\mathrm{d}t}\frac{\mathrm{d}x^j}{\mathrm{d}t}\\ &= m\,E - \sum p^i p^j = p^0 p^0 - \sum p^i p^j = g_{\mu\nu}\,P^\mu P^\nu\,, \end{aligned} \tag{1}$$

for $\mu = 0,1,2,3$, $i = 1,2,3$ and $g_{\mu\nu} = \mathrm{diag}[1,-1,-1,-1]$, where, $m^2\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 = m^2\left(1-\frac{(\mathrm{v}^i)^2}{c^2}\right)c^2 = m_0^2\,c^2$ for the rest mass m_0, whereas v^μ is the 'four-velocity', then, the rearrangement of (1) as, $m\,E =$

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$p^i\, p^j + g_{\mu\nu}\, P^\mu\, P^\nu$, may yield the representation of a wave field $\psi(\vec{r},t)$ by superposition of a free particle (de Broglie wave) as,

$$\begin{aligned} \psi(\vec{r},t) &= \frac{1}{(2\pi\hbar)^2}\, \exp\left[\frac{i}{\hbar\, m}\left\{m\left(\vec{p}\cdot\vec{r} + g_{\mu\nu}\,\vec{P}\cdot\vec{R}\right) - mEt\right\}\right] \\ &\equiv \frac{1}{(2\pi\hbar)^2}\, \exp\left[\frac{i}{\hbar\, m}\left\{m\left(\vec{p}\cdot\vec{r} + m\, t\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2\right) - mEt\right\}\right]. \end{aligned} \tag{2}$$

Thus, from (2), we can get the (total) energy operator $\hat{E} \to i\,\hbar\,\partial_t$ (it is analogous with but not exactly the same as the classical Quantum Mechanics, since it is now related to $(3+1)D$ instead of $3D$ spacetime due to the presence of $g_{\mu\nu}$ in its wave field), and the three momentum operator $\hat{\boldsymbol{p}} \to -\,i\,\hbar\,\vec{\nabla}_i$, whereas the 'Four-momentum' operator,

$$\begin{aligned} \hat{\boldsymbol{\mathcal{P}}}^{\,\mu} &\to i\,\hbar\,\vec{\nabla}_\mu = \left\{i\,\hbar\,\frac{\partial}{\partial(ct)},\, -\,i\,\hbar\,\frac{\partial}{\partial x^i}\right\} = \left(\frac{1}{c}\,\hat{E},\,\hat{\boldsymbol{p}}\right), \\ \text{thus,}\quad \hat{\boldsymbol{\mathcal{P}}}_\mu &\to -\,i\,\hbar\,\vec{\nabla}_\mu = \left\{-\,i\,\hbar\,\frac{\partial}{\partial(i\,ct)},\, -\,i\,\hbar\,\frac{\partial}{\partial x^i}\right\} = \left(\frac{i}{c}\,\hat{E},\,\hat{\boldsymbol{p}}\right), \end{aligned} \tag{3}$$

and the mass operator,

$$\hat{m} \to -\,i\,\hbar\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)^2\frac{\partial}{\partial t} \equiv -\,i\,\hbar\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)\frac{\partial}{\partial s} = -\,i\,\hbar\,\frac{\gamma}{c}\,\frac{\partial}{\partial s}, \tag{4}$$

for $c\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right) = 1/\sqrt{1-\frac{\mathrm{v}^2}{c^2}} = \gamma$. Let us prescribe quantum-to-classical metric tensor (i.e., $g_{\mu\nu}$) for the 'Four-momentum' operator $\hat{\boldsymbol{\mathcal{P}}}^{\,\mu} \to i\,\hbar\,\vec{\nabla}_\mu$ as,

$$\begin{aligned} \mathrm{g}_{\mu\nu}\,\psi(\vec{r},t) &\to \mathrm{g}_{\alpha\beta}\begin{pmatrix}\frac{\hat{\boldsymbol{\mathcal{P}}}^{\,\alpha}}{\hat{\boldsymbol{\mathcal{P}}}^{\,\mu}} & \frac{\hat{\boldsymbol{\mathcal{P}}}^{\,\beta}}{\hat{\boldsymbol{\mathcal{P}}}^{\,\nu}}\end{pmatrix}\psi(\vec{r},t) \to \mathrm{g}_{\alpha\beta}\begin{pmatrix}\frac{i\,\hbar\,\vec{\nabla}_\alpha}{i\,\hbar\,\vec{\nabla}_\mu} & \frac{i\,\hbar\,\vec{\nabla}_\beta}{i\,\hbar\,\vec{\nabla}_\nu}\end{pmatrix}\psi(\vec{r},t) \\ &= g^{\alpha\beta}\begin{pmatrix}\frac{\partial x^\mu}{\partial x^\alpha} & \frac{\partial x^\nu}{\partial x^\beta}\end{pmatrix}\psi(\vec{r},t) = g^{\mu\nu}\,\psi(\vec{r},t)\,. \end{aligned} \tag{5}$$

Here, the 'quantum metric tensor' $\mathrm{g}_{\mu\nu}$ is symmetric, i.e., $\mathrm{g}_{\mu\nu} = \mathrm{g}_{\nu\mu}$, and $\det\left(\mathrm{g}_{\mu\nu}\right) \neq 0$. Components of its inverse matrix g^{-1} are themselves the components of matrix g, namely, $\mathrm{g}_{\mu\nu}\,\mathrm{g}^{\mu\gamma} = \mathrm{g}^{\gamma\mu}\,\mathrm{g}_{\mu\nu} = \delta^\gamma_\nu$, where, δ^γ_ν is the Kronecker delta.

Then, $i\,\hbar\,(\partial/\partial s)$ in (4) may express as,

$$i\,\hbar\,\frac{\partial}{\partial s} = i\,\hbar\left\{\frac{\partial^2}{\partial(ct)^2} - \frac{\partial^2}{\partial x^i \partial x^j}\right\}^{1/2} = i\,\hbar\,\left\{\mathrm{g}^{\mu\nu}\,\partial_\mu\,\partial_\nu\right\}^{1/2} = i\,\hbar\,\Box^{1/2}. \tag{6}$$

But, (6) immediately tells us that (1) is possible to write as,

$$\begin{aligned} \mathrm{d}\,\mathcal{S}^2_{\mathcal{P}}\,\psi(\vec{r},t) &= \mathrm{g}^{\mu\nu}\,\hat{\boldsymbol{\mathcal{P}}}^{\,\mu}\hat{\boldsymbol{\mathcal{P}}}^{\,\nu}\psi(\vec{r},t) = -\,\hbar^2\,\vec{\nabla}_0\,\vec{\nabla}_0\,\psi(\vec{r},t) + \hbar^2\,\vec{\nabla}_i\,\vec{\nabla}_j\,\psi(\vec{r},t) \\ &= -\,\hbar^2\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^i\,\partial x^j}\right)\psi(\vec{r},t) = -\,\hbar^2\,\frac{\partial^2}{\partial s^2}\,\psi(\vec{r},t) \\ &= -\,\hbar^2\,\mathrm{g}^{\mu\nu}\vec{\nabla}_\mu\,\vec{\nabla}_\nu\,\psi(\vec{r},t)\,. \end{aligned} \tag{7}$$

The quantum line element $\mathrm{d}\,\mathcal{S}^2_{\mathcal{P}}$ has neither a Minkowski spacetime, nor a purely Hilbert space and not its metric is Lorentzian, since $\mathrm{g}_{\mu\nu}$ is satisfying (5).

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Note that, $\left(\frac{\mathrm{ds}}{\mathrm{d}t}\right)^2 = c^2 - (\mathrm{v}^i)^2 \sim c^2$ since $c \gg \mathrm{v}^i$. Hence, for $c \gg \mathrm{v}^i$ and $c = 1$, (4) becomes as, $\hat{m} \rightarrow -i\hbar\,\partial_t$. For constant velocity $\mathrm{v}^i \ll c$ and $c = 1$ in the RHS term of (4) yields an uncertainty principle describing the intrinsic indeterminacy with which $\hat{m}$ and s can be determined by,

$$\Delta\hat{m}\,\Delta s \geq \frac{\hbar}{2}.$$

The mass-energy relation in (1), i.e., $E = m\,c^2$, yields its quantum definition for the mass operator $\hat{m}$ of (4) along with (3) and (6) as,

$$i\hbar\,\frac{\partial}{\partial t}\,\psi(\vec{r},t) = -\,i\hbar\,c\,\gamma\,\frac{\partial}{\partial s}\,\psi(\vec{r},t) = -\,c\,\gamma\,\hat{\boldsymbol{\mathcal{P}}}^{\mu}\,\psi(\vec{r},t)\,. \quad (8)$$

The RHS term of this equation becomes very surprising due to γ, because it is not clear to whom γ acts upon.

Relating the first line of (7) to its last line, the quantum line element yields,

$$-\,\hbar^2\left(1 - \mathrm{g}^{00}\right)\vec{\nabla}_0\,\vec{\nabla}_0\,\psi(\vec{r},t) + \hbar^2\left(1 + \mathrm{g}^{ij}\right)\vec{\nabla}_i\,\vec{\nabla}_j\,\psi(\vec{r},t) = 0\,,$$

or, simply discarding $(1 - \mathrm{g}^{00}) = (1 + \mathrm{g}^{ij}) = 0$, we can get,

$$-\,\hbar^2\,\vec{\nabla}_0\,\vec{\nabla}_0\,\psi(\vec{r},t) + \hbar^2\,\vec{\nabla}_i\,\vec{\nabla}_j\,\psi(\vec{r},t) = 0\,. \quad (9)$$

Then, (9) may give us the Schrödinger equation in $(3+1)D$ spacetime for (8) as,

$$i\hbar\,\frac{\partial}{\partial t}\,\psi(\vec{r},t) + \frac{\hbar^2}{\hat{m}\,c^2}\,\vec{\nabla}_i\,\vec{\nabla}_j\,\psi(\vec{r},t) = 0\,. \quad (10)$$

Again, applying the representation of wave field $\psi(\vec{r},t)$ of (2) into $m\,E = p^i\,p^j + g_{\mu\nu}\,P^{\mu}\,P^{\nu}$ from (1) and replacing $g_{\mu\nu}$ with $\mathrm{g}^{\mu\nu}$ for (5), we can get,

$$\begin{aligned} \hat{E}\,\psi(\vec{r},t) &= \frac{1}{\hat{m}}\left(\hat{\boldsymbol{p}}^i\,\hat{\boldsymbol{p}}^j + \mathrm{g}^{\mu\nu}\,\hat{\boldsymbol{\mathcal{P}}}^{\mu}\hat{\boldsymbol{\mathcal{P}}}^{\nu}\right)\psi(\vec{r},t)\,, \\ \text{i.e.,}\quad i\hbar\,\frac{\partial}{\partial t}\,\psi(\vec{r},t) &= -\,\frac{\hbar^2}{\hat{m}}\left(\vec{\nabla}_i\,\vec{\nabla}_j + \mathrm{g}^{\mu\nu}\,\vec{\nabla}_{\mu}\,\vec{\nabla}_{\nu}\right)\psi(\vec{r},t)\,. \end{aligned} \quad (11)$$

Thus, (11) should be used as a general alternation of (10).

Now, let us check $(3+1)D$ Quantum Mechanics from the perspective of General theory of Relativity [2]. Let us consider a space $\mathcal{V}$. Let $\left((i\hbar)^{-1}x^0, \ldots, (i\hbar)^{-1}x^{n-1}\right)$ is a coordinate system of a point $p \in \mathcal{V}$. Let a line element ($\mathrm{d}\mathfrak{s} \neq \mathrm{d}s$) is,

$$\mathrm{d}\mathfrak{s}^2\,\psi(\vec{r},t) = \mathrm{g}_{\mu\nu}\,\mathrm{d}\left((i\hbar)^{-1}x^{\mu}\right)\,\mathrm{d}\left((i\hbar)^{-1}x^{\nu}\right)\,\psi(\vec{r},t) = (i\hbar)^{-2}\mathrm{g}_{\mu\nu}\,\mathrm{d}\,x^{\mu}\,\mathrm{d}\,x^{\nu}\,\psi(\vec{r},t)\,,$$

hence, this yields (7) as,

$$\frac{\partial^2}{\partial\,\mathfrak{s}^2}\,\psi(\vec{r},t) = (i\hbar)^2\,\mathrm{g}^{\mu\nu}\,\vec{\nabla}_{\mu}\,\vec{\nabla}_{\nu}\,\psi(\vec{r},t) = \mathrm{g}^{\mu\nu}\,\hat{\boldsymbol{\mathcal{P}}}^{\mu}\,\hat{\boldsymbol{\mathcal{P}}}^{\nu}\,\psi(\vec{r},t)\,,$$

where, $\partial^2/(\partial\,\mathfrak{s})^2\,\psi(\vec{r},t) = (i\hbar)^2\,\partial^2/(\partial s)^2\,\psi(\vec{r},t) + \Pi\,\psi(\vec{r},t)$, for (6) and for some value of Π (see (18) below for more details).

Let (M^n, g) is a smooth, D-dimensional manifold, where M^n is an n-dimensional differentiable manifold and g is a metric, which is either as a positive-definite section of the bundle of symmetric (covariant) 2-tensors $T^*M \otimes_S T^*M$ or as positive-definite bilinear maps, $\mathrm{g}\left((i\hbar)^{-1}x\right) : T_{\left((i\hbar)^{-1}x\right)}M \times$

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$T_{\left((i\hbar)^{-1}x\right)}M \to \mathcal{V}$ for all $(i\hbar)^{-1}x \in M$. Here, $T^*M \otimes_S T^*M$ is the subspace of $T^*M \otimes T^*M$ generated by elements of the form $X \otimes Y + Y \otimes X$. Let $\{(i\hbar)^{-1}x^i\}_{i=1}^{n}$ be local coordinates in a neighborhood U of some point of M. In U the vector fields $\left\{\left(i\hbar\,\vec{\nabla}_i\right)^{-1}\right\}_{i=1}^{n}$ form a local basis for TM and the 1-forms $\left\{i\hbar\,\vec{\nabla}_i\right\}_{i=1}^{n}$ form a dual basis for T^*M, that is, $i\hbar\,\vec{\nabla}_j\left(i\hbar\,\vec{\nabla}_i\right)^{-1} = \delta^i_j$. The metric may then be written in local coordinates as $\mathrm{g} = \mathrm{g}_{ij}\left(i\hbar\,\vec{\nabla}_i\right) \otimes \left(i\hbar\,\vec{\nabla}_j\right)$. Let ∇^{g} denote the Levi-Civita connection of the metric g. The Christoffel symbols are the components of the Levi-Civita connection and are defined in U by $\nabla_{\left(i\hbar\,\vec{\nabla}_i\right)}\left(i\hbar\,\vec{\nabla}_j\right) \doteqdot \Gamma^k_{ij}\left(i\hbar\,\vec{\nabla}_k\right)$, and for $\left[\left(i\hbar\,\vec{\nabla}_i\right), \left(i\hbar\,\vec{\nabla}_j\right)\right] = 0$, we see that they are given by,

$$\Gamma^k_{ij}\,\psi(\vec{r},t) = \frac{1}{2}\,\mathrm{g}^{k\ell}\left[\left(i\hbar\,\vec{\nabla}_i\right)\mathrm{g}_{j\ell} + \left(i\hbar\,\vec{\nabla}_j\right)\mathrm{g}_{i\ell} - \left(i\hbar\,\vec{\nabla}_\ell\right)\mathrm{g}_{ij}\right]\psi(\vec{r},t)\,. \tag{12}$$

Let the curvature $(3,1)$-tensor Rm is defined by,

$$\mathrm{Rm}(X,Y)Z\,\psi(\vec{r},t) \doteqdot \left(\nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z\right)\psi(\vec{r},t)\,.$$

Thus, the curvature tensor, $R^\ell_{ijk}\,\psi(\vec{r},t) = \left(\partial_i\,\Gamma^\ell_{jk} - \partial_j\,\Gamma^\ell_{ik} + \Gamma^p_{jk}\,\Gamma^\ell_{ip} - \Gamma^p_{ik}\,\Gamma^\ell_{jp}\right)\psi(\vec{r},t)$, is purely Quantum Mechanical due to (12).

Let the tensor Rc is the trace of Rm curvature tensor: $\mathrm{Rc}(Y,Z)\,\psi(\vec{r},t) \doteqdot \mathrm{trace}\,(X \mapsto \mathrm{Rm}(X,Y)Z)\,\psi(\vec{r},t)$, defined by $R_{ij}\,\psi(\vec{r},t) \doteqdot \mathrm{Rc}\left(\left(i\hbar\,\vec{\nabla}_i\right), \left(i\hbar\,\vec{\nabla}_j\right)\right)\psi(\vec{r},t)$, and the scalar curvature R is the trace of Rc tensor: $\mathrm{R}\,\psi(\vec{r},t) \doteqdot \sum_{a=1}^{n}\mathrm{Rc}(e_a,e_a)\,\psi(\vec{r},t)$ where $e_a \in T_{\left((i\hbar)^{-1}x\right)}M^n$ is a unit vector spanning $L \subset T_{\left((i\hbar)^{-1}x\right)}M^n$. Then, the Einstein-like tensor $\mathrm{Rc} - \frac{1}{2}\,\mathrm{g}\,\mathrm{R}$ directly acts on a quantum space. Thus, Einstein-like field equation, $\left[\mathrm{Rc} - \frac{1}{2}\,\mathrm{g}\,\mathrm{R}\right]\psi(\vec{r},t) = (i\hbar)^2\,8\pi\,G\,\mathrm{T}\,\psi(\vec{r},t)$, is now 'purely' Quantum Mechanical for (12). But the Ricci tensor $R_{ij}\,\psi(\vec{r},t) \doteqdot \mathrm{Rc}\left(\left(i\hbar\,\vec{\nabla}_i\right), \left(i\hbar\,\vec{\nabla}_j\right)\right)\psi(\vec{r},t) \doteqdot (i\hbar)^2\,\mathrm{Rc}\left(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}\right)\psi(\vec{r},t)$, thus, Einstein field equation (in quantum spacetime) should become as,

$$\begin{aligned}
&\left[\mathrm{Rc} - \frac{1}{2}\,\mathrm{g}\,\mathrm{R}\right]\psi(\vec{r},t) = (i\hbar)^2\,8\pi\,G\,\mathrm{T}\,\psi(\vec{r},t)\,,\\
\therefore\quad (i\hbar)^2&\left[\mathrm{Rc} - \frac{1}{2}\,g\,\mathrm{R}\right]\psi(\vec{r},t) = (i\hbar)^2\,8\pi\,G\,\mathrm{T}\,\psi(\vec{r},t)\,,\\
\Rightarrow\quad &\left[\mathrm{Rc} - \frac{1}{2}\,g\,\mathrm{R}\right] = 8\pi\,G\,\mathrm{T}\,,
\end{aligned} \tag{13}$$

where g is satisfying (5), and where $\left[\mathrm{Rc} - \frac{1}{2}\,g\,\mathrm{R}\right]$ is Einsteinian and not renormalizable, though, in the first line of (13), mass dimension of gravitational constant vanishes due to $\hbar^2\,G$ and if divergences are to be present, they could now be disposed of by the technique of renormalization (though, this will not play a role in our present discussion). Hence, (13) should be used as a renormalizable Einstein field equation in quantum spacetime for general purposes.

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3. Extra Eleven Dimensions from $(3+1)D$ Quantum State Spaces

Both of the above perspectives of $(3+1)D$ Quantum Mechanics yield extra eleven dimensions from its quantum state spaces quite naturally. The wave field $\psi(\vec{r},t)$ in (2) must satisfy the eigenfunctions for a discrete Lorentz transformation as,

$$\begin{aligned}
\Psi &= \frac{1}{\sqrt{2}}\psi_0 - \frac{1}{\sqrt{2}}\sum_{i=1}^{3}\psi_i = \frac{1}{\sqrt{2}}\psi_0 + \frac{1}{\sqrt{2}}\sum_{i=1}^{3}\psi_i^\dagger = -\Psi^\dagger\,, \\
\Psi^\dagger &= \frac{1}{\sqrt{2}}\psi_0 + \frac{1}{\sqrt{2}}\sum_{i=1}^{3}\psi_i = \frac{1}{\sqrt{2}}\psi_0 - \frac{1}{\sqrt{2}}\sum_{i=1}^{3}\psi_i^\dagger = -\Psi\,,
\end{aligned} \tag{14}$$

for $i=(1,2,3)$, when $\psi^\dagger$ is the complex conjugate of ψ. Then, using summation convention and (6), we can write the joint state formalism as,

$$\begin{aligned}
\frac{\partial^2}{\partial s^2}\Psi\bigg|_{s_0} &= \left\{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^i \partial x^j}\right\}\left\{\frac{1}{\sqrt{2}}\psi_0 - \frac{1}{\sqrt{2}}\psi_i\right\}\bigg|_{x_0^\mu} \\
&= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[\psi_0 - \psi_i] - \frac{\partial^2}{\partial x^i \partial x^j}[\psi_0 - \psi_i]\right\}\bigg|_{x_0^\mu} \\
&= \left(\frac{\partial^2}{\partial t^2}\Psi - \frac{\partial^2}{\partial x^i\,\partial x^j}\Psi\right)\bigg|_{x_0^\mu} = \mathrm{g}^{\mu\nu}(s)\,\partial_\mu\,\partial_\nu\Psi\Big|_{x_0^\mu}.
\end{aligned} \tag{15}$$

But, (15) may also intend to,

$$\begin{aligned}
\frac{\partial^2}{\partial s^2}\Psi\bigg|_{s_0} &= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[\psi_0 - \psi_i] - \frac{\partial^2}{\partial x^i \partial x^j}[\psi_0 - \psi_i]\right\}\bigg|_{x_0^\mu} \\
&= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[\psi_0] - \frac{\partial^2}{\partial x^i\,\partial x^j}[-\psi_i]\right\}\bigg|_{x_0^\mu} + \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_i] - \frac{\partial^2}{\partial x^i\,\partial x^j}[\psi_0]\right\}\bigg|_{x_0^\mu} \\
&= \frac{1}{\sqrt{2}}\left\{\mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\partial_\nu\,[\psi_0 - \psi_i]\right\}\Big|_{x_0^\mu} + \frac{1}{\sqrt{2}}\left\{\widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\,[\psi_0 - \psi_i]\right\}\Big|_{x_0^\mu} \\
&= \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\partial_\nu\,\Psi\Big|_{x_0^\mu} + \widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\,\Psi\Big|_{x_0^\mu} = \rho\,\Psi\Big|_{x_0^\mu} + \Gamma\,\Psi\Big|_{x_0^\mu},
\end{aligned} \tag{16}$$

for $\partial_\mu^2 = \widehat{\partial}_\mu^2 = \left[\frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial x^i\,\partial x^j}\right]$ and $\mathrm{g}_{\mu\nu} = \widehat{\mathrm{g}}_{\mu\nu} = \mathrm{diag}[1,-1,-1,-1]$, whereas $\widehat{\partial}$ implies either ∂_t is in company with ψ_i or ∂_i is in company with ψ_0, exclusively, i.e., the 'wrong' distribution of indices for ∂ and ψ. Here, ρ and Γ have been chosen arbitrarily. Hence, the complex conjugate of (16) is,

$$\begin{aligned}
\frac{\partial^2}{\partial s^2}\Psi^\dagger\bigg|_{s_0} &= \left\{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^i \partial x^j}\right\}\left\{\frac{1}{\sqrt{2}}\psi_0 + \frac{1}{\sqrt{2}}\psi_i\right\}\bigg|_{x_0^\mu} \\
&= \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\partial_\nu\,\Psi^\dagger\Big|_{x_0^\mu} + \widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\,\Psi^\dagger\Big|_{x_0^\mu} = \rho\,\Psi^\dagger\Big|_{x_0^\mu} + \Gamma\,\Psi^\dagger\Big|_{x_0^\mu}.
\end{aligned} \tag{17}$$

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If we take the line element as, $\mathrm{d}s^{-2}\varphi = (\mathrm{d}t^2 - \sum \mathrm{d}x^i\,\mathrm{d}x^j)^{-1}\varphi$, where φ is an operator, then we can say that, $(\mathrm{d}t^2 - \sum \mathrm{d}x^i\,\mathrm{d}x^j)^{-1} \neq \mathrm{d}t^{-2} - \sum(\mathrm{d}x^i\,\mathrm{d}x^j)^{-1}$, though, we may assume without any objection that, $(\mathrm{d}t^2 - \sum \mathrm{d}x^i\,\mathrm{d}x^j)^{-1} \doteqdot \frac{\partial^2}{\partial t^2} - \sum \frac{\partial^2}{\partial x^i\,\partial x^j} - \Pi$, for some value of Π. Thus,

$$\begin{aligned}\mathrm{d}s^{-2}\varphi &= (\mathrm{d}t^2 - \mathrm{d}x^i\,\mathrm{d}x^j)^{-1}\varphi \doteqdot \left\{\frac{\partial^2\varphi}{\partial t^2} - \left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}\right)\right\} - \Pi\varphi \\ &= \frac{\partial^2}{\partial s^2}\varphi - \Pi\varphi = \mathrm{g}^{\mu\nu}\,\partial_\mu\,\partial_\nu\varphi - \Pi\varphi\,. \end{aligned} \tag{18}$$

Then, comparing with (16), if $\mathrm{g}^{\mu\nu}\,\partial_\mu\partial_\nu \equiv \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\partial_\nu$, we can write that,

$$\frac{\partial^2}{\partial s^2}\Psi\Big|_{s_0} - \Gamma\,\Psi\Big|_{x_0^\mu} \quad\Longleftrightarrow\quad \mathrm{d}s^{-2}\varphi + \Pi\varphi\,.$$

This yields, $\frac{\partial^2}{\partial s^2}\Psi\Big|_{s_0} - \Gamma\,\Psi\Big|_{x_0^\mu} \doteqdot \mathrm{d}s^{-2}\varphi + \Pi\varphi$, if $\Gamma \to -\Pi$, which gives the equivalency of (18) and (16).

No more combinations are possible from (16) apart from ρ, Γ and (15) itself. The arrangement of ρ and Γ implies that $\mathrm{g}_{\mu\nu}(s) = \frac{1}{2}\big(\mathrm{g}_{\mu\nu}(\rho) + \widehat{\mathrm{g}}_{\mu\nu}(\Gamma)\big)$. Thus, (16) should be rewritten by considering (15) and additionally replacing $\mathrm{g}_{\mu\nu}(s)$ with $\mathrm{g}_{\mu\nu}(s) = \frac{1}{2}\big(\mathrm{g}_{\mu\nu}(\rho) + \widehat{\mathrm{g}}_{\mu\nu}(\Gamma)\big)$ as follows,

$$\frac{1}{2}\big(\mathrm{g}^{\mu\nu}(\rho) + \widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\big)\,\partial_\mu\,\partial_\nu\,\Psi\Big|_{x_0^\mu} = \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\,\partial_\nu\Psi\Big|_{x_0^\mu} + \widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\,\widehat{\partial}_\mu\,\widehat{\partial}_\nu\Psi\Big|_{x_0^\mu}\,. \tag{19}$$

Note that, Π exclusively has to depend upon spacetime. Comparing (18) with (16), let us say that, when $\Gamma \to -\Pi$,

$$\frac{\partial^2}{\partial s^2}\Psi\Big|_{s_0} = \rho\Psi\Big|_{x_0^\mu} - \Pi\Psi\Big|_{x_0^\mu}\,. \tag{20}$$

Since $\mathrm{g}_{\mu\nu}(s) \neq \frac{1}{2}\big(\mathrm{g}_{\mu\nu}(\rho) - \widehat{\mathrm{g}}_{\mu\nu}(\Pi)\big) \equiv 0$ as long as $\mathrm{g}_{\mu\nu} = \widehat{\mathrm{g}}_{\mu\nu} = \mathrm{diag}[1,-1,-1,-1]$, then, for $\mathrm{g}_{\mu\nu}(s) \neq 0$, the '$-$'-sign of (20) must have to switch its position in such a way that,

$$\frac{1}{2}\big(\mathrm{g}^{\mu\nu}(\rho) + \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\big)\,\partial_\mu\,\partial_\nu\,\Psi\Big|_{x_0^\mu} = \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\,\partial_\nu\Psi\Big|_{x_0^\mu} + \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\,\widehat{\partial}_\mu\,\widehat{\partial}_\nu\,[-\Psi]\Big|_{x_0^\mu}\,. \tag{21}$$

Reduction of (19) or (21) is impossible since both of their LHS are only depended upon ∂, thus,

1. The spacetimes of ρ and Γ (so as Π) are not easily dissociative even up to a very high energy scale.
2. Since $\mathrm{g}_{\mu\nu}(s)$ is independent of $\widehat{\partial}$, the spacetime of Γ (so as Π) must be an internally hidden property of the overall system (in other words, inside the observable spacetime of ρ). Thus, the observable spacetime is always ∂-dependent.

But, the RHS of (21) gives us,

$$\begin{aligned}\{|\Psi\rangle \in V\otimes V : F|\Psi\rangle = |\Psi\rangle\} &= \mathrm{Sym}^2\,V\,,\\ \{|\Psi\rangle \in V\otimes V : F|\Psi\rangle = -|\Psi\rangle\} &= \mathrm{Anti}^2\,V\,,\end{aligned}$$

where, the swap operator $F|\Psi\rangle = \exp[i\,\theta]$ for some phase $\exp[i\,\theta]$, whereas V is a vector space. Then the corresponding eigenspaces are called the symmetric and antisymmetric subspaces and are denoted by the state spaces $\mathrm{Sym}^2\,V$ and $\mathrm{Anti}^2\,V$, respectively. Note that, we have not intended here that ρ and Π individually are two distinguishable particles for the state space $\mathrm{Sym}^2\,V$ or $\mathrm{Anti}^2\,V$; the above equations are just the generalization forms of their kinds, because ρ and Π do not have distinguished

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(opposite) spins until otherwise they are dissociated as free particles at very high energy scale; so, the observable spin is always the spin of ρ, since Π is an internally hidden property of the overall system and the observable spacetime is always ∂-dependent. Thus, (21) tells us that, if we allow Π to be dissociated as a free particle at very high energy, the internally hidden spacetime of Π then must be transformed into a fermionic particle, whereas, the overall ∂-dependent system remains bosonic, since, the observable spacetime is always ∂-dependent.

Similarly, (17) yields,

$$\begin{aligned} \frac{\partial^2}{\partial s^2}\Psi^\dagger\Big|_{s_0} &= \rho\,\Psi^\dagger\Big|_{x_0^\mu} - \Pi\,\Psi^\dagger\Big|_{x_0^\mu}, \\ \therefore\quad \frac{1}{2}\left(\mathrm{g}^{\mu\nu}(\rho) + \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\right)\partial_\mu\partial_\nu\left[-\Psi\right]\Big|_{x_0^\mu} &= \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\partial_\nu\left[-\Psi\right]\Big|_{x_0^\mu} + \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\Psi\Big|_{x_0^\mu}, \end{aligned} \tag{22}$$

which tells us that the internally hidden spacetime of Π must be now bosonic, whereas, the overall system is fermionic, since, the observable spacetime is always ∂-dependent. So, whatever (21) and (22) want to tell us is that the ∂-dependent overall system has Supersymmetry and since the spacetimes of ρ and its supersymmetric partner Π are not easily dissociative even upto a very high energy scale; thus, Π must require extremely high energy to dissociate itself from the overall system as a free particle. Instead of being a free supersymmetric partner, Π actually works quite differently inside of the observable spacetime ρ, though, at the same time, Π is still satisfying the properties of Supersymmetry. We will show you Π's actual purpose very soon in the below. But before that, Supersymmetry needs extra dimensions and we have to discuss it now.

Remark 1 (Dark Matter). *By the way, before we proceed with anything, we can develop a $\widehat{\partial}$-dependent scenario as follows,*

$$\begin{aligned} \frac{\widehat{\partial}^2}{\widehat{\partial} s^2}\Psi\Big|_{s_0} &= \Pi\Psi\Big|_{x_0^\mu} - \rho\Psi\Big|_{x_0^\mu}, \\ \therefore\quad \frac{1}{2}\left(\widehat{\mathrm{g}}^{\mu\nu}(\Pi) + \mathrm{g}^{\mu\nu}(\rho)\right)\widehat{\partial}_\mu\widehat{\partial}_\nu\,\Psi\Big|_{x_0^\mu} &= \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\Psi\Big|_{x_0^\mu} + \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\,\partial_\nu\left[-\Psi\right]\Big|_{x_0^\mu}, \end{aligned} \tag{23}$$

and the complex conjugate of Ψ is,

$$\begin{aligned} \frac{\widehat{\partial}^2}{\widehat{\partial} s^2}\Psi^\dagger\Big|_{s_0} &= \Pi\,\Psi^\dagger\Big|_{x_0^\mu} - \rho\,\Psi^\dagger\Big|_{x_0^\mu}, \\ \therefore\quad \frac{1}{2}\left(\widehat{\mathrm{g}}^{\mu\nu}(\Pi) + \mathrm{g}^{\mu\nu}(\rho)\right)\widehat{\partial}_\mu\widehat{\partial}_\nu\left[-\Psi\right]\Big|_{x_0^\mu} &= \widehat{\mathrm{g}}^{\mu\nu}(\Pi)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\left[-\Psi\right]\Big|_{x_0^\mu} + \mathrm{g}^{\mu\nu}(\rho)\,\partial_\mu\,\partial_\nu\Psi\Big|_{x_0^\mu}. \end{aligned} \tag{24}$$

It is not important which state spaces satisfy such bosonic or fermionic representations of (23) and (24), here, the most important thing is that the overall system as a free observable particle is must not be baryonic matter because now only the internally hidden spacetime of ρ has 'proper' distribution of indices for ∂ and ψ resulting its ∂-dependency, whereas, the overall (observable) system is $\widehat{\partial}$-dependent, i.e., it has 'wrong' distribution of indices for ∂ and ψ. Despite of ρ's ∂-dependency, here, being a supersymmetric partner, if it is allowed to be free at very high energy, it must not be baryonic either and we should not be confused with it. The only candidate to have such properties like (23) and (24) is definitely non-baryonic, hence, it is Dark Matter.

The internally hidden spacetime of Π in (21) and (22) also provides us some additional geometries for its $\widehat{\mathrm{g}}^{\mu\nu}(\Pi)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\left[\pm\Psi\right]$ structures. Suppose, for $\widehat{\mathrm{g}}^{\mu\nu}(\Gamma)\,\widehat{\partial}_\mu\widehat{\partial}_\nu\left[\Psi\right]$, we have, $\frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}\left[-\psi_i\right] - \frac{\partial^2}{\partial x^i\,\partial x^j}\left[\psi_0\right]\right\}$, where, both spacetimes have the 'wrong' distribution of indices for ∂ and ψ within the curvey brackets. These 'wrong' distributions must have a noticeable effect on the acceptable spacetime, i.e., its temporal part must influence over the spatially depended ψ_i, or its spatial part must influence over the temporally depended ψ_0, or vice versa. In other words, the acceptable spacetime may not be four-dimensional in this case. Let us check it.

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Let, $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) \equiv \left(\widehat{\partial}^2_{[0]} \Psi_{[i]} \mp \widehat{\partial}^2_{[i]} \Psi_{[0]}\right)$, when $\widehat{\partial}^2_{[0]} \Psi_{[i]} = \frac{\partial^2}{\partial t^2}[\mp\psi_i]$ and $\widehat{\partial}^2_{[i]} \Psi_{[0]} = \frac{\partial^2}{\partial x^i \partial x^j}[\pm\psi_0]$. Suppose, for $\frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_i] - \frac{\partial^2}{\partial x^i \partial x^j}[\psi_0]\right\}$, we should consider a dimension function,

$$\dim\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) \equiv \dim\left(\widehat{\partial}^2_{[0]} \Psi_{[i]} - \widehat{\partial}^2_{[i]} \Psi_{[0]}\right) \;\forall\, i \in \{1,2,3\}\,. \tag{25}$$

Let the space $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ satisfy a normal T_1-space. Let $\mathcal{U}$ be a collection in an initially $(3+1)D$ topological spacetime $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$, i.e., $\dim\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) = (3+1)$, which is actually hidden inside an observable $(3+1)D$ spacetime, $\left(\partial^2_{[0,i]} \otimes \Psi_{[0,i]}\right)$ (be careful about the subscript indices $[0,i]$ here – do not confuse observable spacetime with the hidden spacetime, which always carries subscript indices $[0,i]$ and $[i,0]$ both at the same time).

See [3–7] for the required background of Dimension Theory to construct a mapping f of the spacetime $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ into a spacetime S, which is a closed (open) mapping, if the image of every closed (open) set of $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ is closed (open) in S. Then the continuous mappings which lower dimensions of the spacetime $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ should be defined as follows,

Theorem 1. *Let f be a closed continuous mapping of the $(3+1)D$ spacetime $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ onto the spacetime S such that $\dim f^{-1}(q) \le k$ for every $q \in S$. Then,*

$$\dim\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) \le \dim S + \dim K\,, \tag{26}$$

where $\dim K \le k$ for the space K, when $0 < \dim S \le 2$, since i should not be zero in (25).

Note that, here and hereafter, the sign '$\le$' always intends to give meaning to Dimension Theory [3], rather than its traditional algebraic meaning.

Proof. Using Theorem III.6 of [3], we can easily prove this theorem. Other good references are [4–7]. theorem □

Since, the temporal axis is unaltered in Lorentz transformation, as we have already seen it in (14), we can express the maximal continuous mapping of the $(3+1)D$ spacetime $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ onto the spacetime S of (26) as,

$$\begin{aligned} X^\mu(\tau,\sigma) \;&\le\; \left\{\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) \mapsto S \;\middle|\; S \le \left(\widehat{\partial}^2_{[0,1]} \otimes \Psi_{[1,0]}\right)\right\}\,, \\ \text{thus,}\quad \mathfrak{X}^\mu \;&\le\; S \cup K \text{ inside the } (3+1)D \text{ observable spacetime } \left(\partial^2_{[0,i]} \otimes \Psi_{[0,i]}\right)\,, \end{aligned}$$

since i should not be zero in (25), then the spacetime S definitely intends the basic structure of a 2-dimensional worldsheet $X^\mu(\tau,\sigma)$ with the joint states, $\frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial \tau^2}[-\psi_\sigma] - \frac{\partial^2}{\partial \sigma^2}[\psi_\tau]\right\}$ for the spacetime $\mathfrak{X}^\mu$, where S is a $(1+1)D$ spacetime, i.e., string. Obviously, a string can sweep out the 2-dimensional worldsheet $X^\mu(\tau,\sigma)$ for the spacetime $\mathfrak{X}^\mu$. But for the space K, we need to discuss it in more details, what we are going to do below in Theorem 2.

Before that, since the spacetime of Π is hidden inside the overall system of (21), i.e., in other words, inside the observable spacetime of ρ, then the increment of spacetime $\mathfrak{X}^\mu$ should not be observable by any means, i.e., the extra dimensions of $\mathfrak{X}^\mu$ remain hidden forever inside the observable spacetime of ρ. As these internally hidden extra dimensions inside the observable spacetime of ρ are considered as the representation of the spacetime S and the space K, thus, we can conclude,

1. Strings (i.e., the spacetime S for the hidden spacetime $\mathfrak{X}^\mu$) are natural and universal but forever hidden inside every $(3+1)D$ observable system, here, it is spacetime $\left(\partial^2_{[0,i]} \otimes \Psi_{[0,i]}\right)$, in Quantum Mechanics.

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2. Every $(3+1)D$ observable system in Quantum Mechanics must contain forever hidden extra dimensions (i.e., the space K for the hidden spacetime $\mathfrak{X}^{\mu}$) independent of any external observer whether she/he considers any string in this system or not (for more details, see (28) below and its following text therein).

But beyond $\dim K \leq k$, the space K can raise more extra hidden dimensions by a closed continuous mapping by adopting the following,

Theorem 2. *Let f be a closed continuous mapping of a space R onto a space K such that for each point q of K, $B(f^{-1}(q))$ contains at most $m+1$ points $(m \geq 0)$; then $\dim K \leq \dim R + \dim M$, when $\dim R \leq \left[\dim\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) - \dim S\right]$ and $\dim M \leq m$, where $\dim\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) \leq (3+1)$.*

Proof. Using Theorem III.7 of [3], we can easily prove this theorem. Other good references are [4–7]. □

Then, we can say for the overall spacetime $\mathfrak{X}^{\mu}_{\text{ov}}$ that,

$$\mathfrak{X}^{\mu}_{\text{ov}} \leq \Big\{ \left(\partial^{2}_{[0,i]} \otimes \Psi_{[0,i]}\right) \cup S \cup K \;\Big|\; S \leq \left(\widehat{\partial}^{2}_{[0,1]} \otimes \Psi_{[1,0]}\right) \text{ and } K \leq \left[\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) - S\right] \cup M \;\forall\; \dim M \leq m \;\exists\; m \geq 0 \Big\}, \tag{27}$$

hence, for the value of K,

$$\mathfrak{X}^{\mu} \leq S \cup K \leq \left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) \cup M \;\forall\; \dim M \leq m \;\exists\; m \geq 0 \text{ inside the } (3+1)D \text{ observable spacetime } \left(\partial^{2}_{[0,i]} \otimes \Psi_{[0,i]}\right). \tag{28}$$

Note here that stringy spacetime S vanishes in the overall spacetime $\mathfrak{X}^{\mu}_{\text{ov}}$ of (27) for the space K, leaving behind the forever hidden extra dimensions m in $\mathfrak{X}^{\mu}_{\text{ov}}$. Thus, in other words, strings are experimentally unobservable forever, whereas, their actions should be mandatory in the purpose of overall spacetime $\mathfrak{X}^{\mu}_{\text{ov}}$ (see end of this section for more details). Also notice that Supersymmetry (now having extra dimensions m for $\mathfrak{X}^{\mu}$ due to (28)) remains unchanged in $\mathfrak{X}^{\mu}_{\text{ov}}$ of (27). Thus, with these extra dimensions, the above scenario is now perfect for Supersymmetry and String Theory without any further objections and/or adjustments.

Along with Theorem 1, what (28) actually wants to say us is,

$$\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) \leq \mathfrak{X}^{\mu} \leq \left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) \cup M,$$

when $\mathfrak{X}^{\mu} \leq S \cup K$, which yields,

$$\left[\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) - S\right] \leq K \leq \left[\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) - S\right] \cup M. \tag{29}$$

Since $S \leq \left(\widehat{\partial}^{2}_{[0,1]} \otimes \Psi_{[1,0]}\right)$ in (27), let the LHS of (29) gives,

$$\left[\left(\widehat{\partial}^{2}_{[0,i]} \otimes \Psi_{[i,0]}\right) - \left(\widehat{\partial}^{2}_{[0,1]} \otimes \Psi_{[1,0]}\right)\right] \leq \left\{ \left(\widehat{\partial}^{2}_{[0,2]} \otimes \Psi_{[2,0]}\right) \cup \left(\widehat{\partial}^{2}_{[0,3]} \otimes \Psi_{[3,0]}\right) \right\}. \tag{30}$$

The most disturbing thing here is that the temporal axis is a part of S spacetime but not the part of K space, but both $\left(\widehat{\partial}^{2}_{[0,2]} \otimes \Psi_{[2,0]}\right)$ and $\left(\widehat{\partial}^{2}_{[0,3]} \otimes \Psi_{[3,0]}\right)$ spaces are influenced by the (mutual) temporal axis, despite the fact that neither of them have contained any temporal axis within themselves. On the other hand, it is evidence that only an influence should not be sufficient to emerge a temporal axis within M (or K) space. Moreover, Theorem 2 yields no temporal axis for M (or K) space either. But the influenced of the temporal axis should not ease to avoid in (30).

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From Theorem 2, if we think that the dimension of M space depends only on $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right) = \left\{\left(\widehat{\partial}^2_{[0,1]} \otimes \Psi_{[1,0]}\right), \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right), \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\right\}$, then we should be mistaken, M is not independent from either elements of the set $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$. Thinking otherwise, let $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$ are related to new quantities Q_i and T_i, differently, which are the curvilinear coordinates of $\left(\widehat{\partial}^2_{[0,i]} \otimes \Psi_{[i,0]}\right)$. Let the corresponding members $\{T_1, T_2, T_3\} \subset T$ are determining Δ, then if each pair of members from the either sides of these curvilinear coordinates joining the pairs of points Q_i and T_i $(i = 1, 2, 3)$ meet in points $\mathfrak{m}_i \in M$ separately, then the three points of intersection U_i of the pairs of coordinates q_i and t_i $(i = 1, 2, 3)$ lie on a line. Let each of the pairs of coordinates Q_i, T_i $(i = 1, 2, 3)$ consists of two distinct coordinates and in which $q_i \neq t_i \; \forall \; i$. Let the coordinate vectors of $\mathfrak{m}_i$ be denoted by $\mathfrak{z}_i$, that of Q_i by $\mathfrak{r}_i$ $(i = 1, 2, 3)$ and that of T_i by $\mathfrak{y}_i$ $(i = 1, 2, 3)$. Then $\mathfrak{z}_i$ can be represented by a linear combination of the $\mathfrak{r}_i$ and $\mathfrak{y}_i$ for each $i = 1, 2, 3$, say, $\mathfrak{z}_i = \mathfrak{r}_1 + \mathfrak{y}_1 = \mathfrak{r}_2 + \mathfrak{y}_2 = \mathfrak{r}_3 + \mathfrak{y}_3$. Hence, $\mathfrak{r}_1 - \mathfrak{r}_2 = \mathfrak{y}_2 - \mathfrak{y}_1$; $\mathfrak{r}_2 - \mathfrak{r}_3 = \mathfrak{y}_3 - \mathfrak{y}_2$ and $\mathfrak{r}_3 - \mathfrak{r}_1 = \mathfrak{y}_1 - \mathfrak{y}_3$. Let us choose two set of coordinates,

$$
\begin{aligned}
a_i = (a_1, a_2, a_3) &= \frac{1}{\sqrt{2}} \left\{ \frac{\partial^2}{\partial t^2} [-\varphi_i] - \frac{\partial^2}{\partial x^i \, \partial x^j} [\varphi_0] \right\} = \left(\widehat{\partial}^2_{[0,i]} \otimes \boldsymbol{\varphi}_{[i,0]}\right) \in \Lambda \,, \\
b_i = (b_1, b_2, b_3) &= \frac{1}{\sqrt{2}} \left\{ \frac{\partial^2}{\partial t^2} [-\varrho_i] - \frac{\partial^2}{\partial x^i \, \partial x^j} [\varrho_0] \right\} = \left(\widehat{\partial}^2_{[0,i]} \otimes \boldsymbol{\varrho}_{[i,0]}\right) \in \Lambda \,,
\end{aligned} \tag{31}
$$

for $i, j = 1, 2, 3$, such that $\{a_i\} \in \mathfrak{r}_i$, $\{b_i\} \in \mathfrak{y}_i$ and $\{a, b\}$ is a basis of Λ, whereas $\Lambda \cap Q_i = \{O\}$, where Q_i is the interior of $\boldsymbol{Q}$ and O is the origin, i.e., Λ is admissible for $\boldsymbol{Q}$. Let the quadratic form,

$$
\begin{aligned}
&\mathcal{Q}\left(\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right), \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\right) \\
&= \sum_{1 \le i, j \le 3} \frac{1}{2} \left[(a_i - a_j) \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) + (b_i - b_j) \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) \right] \\
&= \frac{1}{2} \left[A \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)^2 + 2B \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) + C \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)^2 \right] ,
\end{aligned}
$$

say, is reduced. The last fact means that $2\,|B| \le A \le C$, so that $3A^2 \le 4\left(A^2 - B^2\right) \le 4\left(AC - B^2\right)$. Since Λ is admissible for $\boldsymbol{Q}$, the coordinates $a + mc$ (m an integer) do not belong to int $\boldsymbol{Q}$. Thus,

$$
\{ |(m + a_1)\,(m + a_2)\,(m + a_3)| \ge 1 \;\forall \text{ integers } m \} \,,
$$

this implies that,

$$
\begin{aligned}
A &= (a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 \\
&= 2\left(a_1^2 + a_2^2 + a_3^2\right) - 2\,(a_1\,a_2 + a_2\,a_3 + a_3\,a_1) \,.
\end{aligned}
$$

So as,

$$
\begin{aligned}
C &= (b_1 - b_2)^2 + (b_2 - b_3)^2 + (b_3 - b_1)^2 \\
&= 2\left(b_1^2 + b_2^2 + b_3^2\right) - 2\,(b_1\,b_2 + b_2\,b_3 + b_3\,b_1) \,.
\end{aligned}
$$

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and we can easily find that $B = 0$. Here,

$$\begin{aligned}
\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)^2 &= \left[\frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right\}\right]^2 \\
&= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right\} \times \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right\} \\
&= \frac{1}{2}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] \times \frac{\partial^2}{\partial t^2}[-\psi_2] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] \times \frac{\partial^2}{\partial t^2}[-\psi_2] - \right. \\
&\qquad \left. - \frac{\partial^2}{\partial t^2}[-\psi_2] \times \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] + \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] \times \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right\} \\
&= \frac{1}{2}\left\{\left(\frac{\partial^2}{\partial t^2}[-\psi_2]\right)^2 + \left(\frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right)^2\right\} - \frac{\partial^2}{\partial t^2}[\psi_0]\,\frac{\partial^2}{\partial x^2\,\partial x^2}[-\psi_2] \\
&= \frac{1}{2}\left(\widehat{\partial}^4_0\,\Psi^2_2 + \widehat{\partial}^4_2\,\Psi^2_0\right) - \partial^2_0\,\Psi_0\,\partial^2_2\,\Psi_2\,. \qquad (32)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)^2 &= \left[\frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_3] - \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\}\right]^2 \\
&= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_3] - \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\} \times \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_3] - \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\} \\
&= \frac{1}{2}\left(\widehat{\partial}^4_0\,\Psi^2_3 + \widehat{\partial}^4_3\,\Psi^2_0\right) - \partial^2_0\,\Psi_0\,\partial^2_3\,\Psi_3\,. \qquad (33)
\end{aligned}$$

In the same way,

$$\begin{aligned}
&\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) \\
&= \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0]\right\} \times \frac{1}{\sqrt{2}}\left\{\frac{\partial^2}{\partial t^2}[-\psi_3] - \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\} \\
&= \frac{1}{2}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] \times \frac{\partial^2}{\partial t^2}[-\psi_3] - \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] \times \frac{\partial^2}{\partial t^2}[-\psi_3] - \right. \\
&\qquad \left. - \frac{\partial^2}{\partial t^2}[-\psi_2] \times \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0] + \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] \times \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\} \\
&= \frac{1}{2}\left\{\frac{\partial^2}{\partial t^2}[-\psi_2] \times \frac{\partial^2}{\partial t^2}[-\psi_3] + \frac{\partial^2}{\partial x^2\,\partial x^2}[\psi_0] \times \frac{\partial^2}{\partial x^3\,\partial x^3}[\psi_0]\right\} - \\
&\qquad - \frac{1}{2}\,\frac{\partial^2}{\partial t^2}[\psi_0]\left\{\frac{\partial^2}{\partial x^2\,\partial x^2}[-\psi_3] + \frac{\partial^2}{\partial x^3\,\partial x^3}[-\psi_2]\right\} \\
&= \frac{1}{2}\left(\widehat{\partial}^4_0\,\Psi^2_{\{2,3\}} + \widehat{\partial}^4_{\{2,3\}}\,\Psi^2_0\right) - \frac{1}{2}\,\partial^2_0\,\Psi_0\left(\partial^2_{\{2,3\}} \otimes \Psi_{\{3,2\}}\right)\,. \qquad (34)
\end{aligned}$$

In the last line we have used subscripts $\{\,,\,\}$, which are quite different from the subscripts $[\,,\,]$ we have been using yet, however, their purposes are quite obvious here. Since, the temporal axis is a part of S spacetime but not the part of K space, so both $\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)$ and $\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)$ spaces, as well as a_i and b_i spaces of (31), are influenced by the (mutual) temporal axis, though, neither of them have contained any temporal axis within themselves. Then we can say that all axes of $a_i\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)$ and $b_i\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)$ (for $i = 1,2,3$) in K space are interrelated with the (mutual) temporal axis coming from string spacetime S, since the temporal axis is a part of $S = \left(\widehat{\partial}^2_{[0,1]} \otimes \Psi_{[1,0]}\right)$ but not the part of K space, thus, $a_i\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)$ and $b_i\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)$ (for $i = 1,2,3$) in K space have

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individual existences as independent axes $x^{(1+i)}$ and $x^{(1+(i+\ell))}$ (for $\ell = \max i$) influenced by the (mutual) temporal axis x^0. Let us assume that each member of $a_i \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)$ or $b_i \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)$ (for $i = 1, 2, 3$) in K space have maximal weight as 1 for a dimension which is an independent axis for $x^{(1+i)}$ or $x^{(1+(i+\ell))}$, respectively, which yields,

$$\begin{aligned} \dim\left[(a_1 + a_2 + a_3)\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)\right] &\leq 3\,, \\ \text{and} \quad \dim\left[(b_1 + b_2 + b_3)\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\right] &\leq 3\,. \end{aligned} \tag{35}$$

Hence, they have 'proper' dimensions. But, comparing the last line of (34) with (32) and (33), we can determine that if (32) and (33) give us some 'proper' dimensions, then (34) definitely gives us some 'improper' dimensions, since both $\left(\widehat{\partial}^4_0 \Psi^2_{\{2,3\}} + \widehat{\partial}^4_{\{2,3\}} \Psi^2_0\right)$ and $\left(\partial^2_{\{2,3\}} \otimes \Psi_{\{3,2\}}\right)$ are depending on x^2 and x^3 axes, simultaneously. Since a and b are satisfying (31), then $a_i a_j$ and $b_i b_j$ (for $i, j = 1, 2, 3, i \neq j$) must give us 'improper' dimensions, too. If we consider these 'improper' dimensions $(a_i\, a_j)^{1/2}\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)$ or $(b_i\, b_j)^{1/2}\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)$ (for $i, j = 1, 2, 3, i \neq j$) in K space have individual existences as independent axes $x_a^{(1+(i+\ell)+\frac{1}{2}j)}$ or $x_b^{(1+(i+\ell)+\frac{1}{2}j)}$, respectively, (since they are depending on x^2 and x^3 axes, simultaneously) influenced by the (mutual) temporal axis x^0, then, on the contrary of (35), let us assume that they have maximal weight as 0.5 of each dimension for $x_a^{(1+(i+\ell)+\frac{1}{2}j)}$ or $x_b^{(1+(i+\ell)+\frac{1}{2}j)}$, so as they can give $x^{(1+(i+\ell)+j)} = x_a^{(1+(i+\ell)+\frac{1}{2}j)} + x_b^{(1+(i+\ell)+\frac{1}{2}j)}$ to yield the maximal weight as 1 for a 'proper' dimension. Thus, we can say that,

$$\begin{aligned} \dim\left[\left\{(a_1\, a_2)^{1/2} + (a_2\, a_3)^{1/2} + (a_3\, a_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)\right] &\leq 1.5\,, \\ \dim\left[\left\{(b_1\, b_2)^{1/2} + (b_2\, b_3)^{1/2} + (b_3\, b_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\right] &\leq 1.5\,. \end{aligned}$$

Hence, altogether they have,

$$\begin{aligned} \dim\Big[\left\{(a_1\, a_2)^{1/2} + (a_2\, a_3)^{1/2} + (a_3\, a_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) & \\ \bigcup\, (a_1 + a_2 + a_3)\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right)\Big] &\leq 4.5\,, \\ \dim\Big[\left\{(b_1\, b_2)^{1/2} + (b_2\, b_3)^{1/2} + (b_3\, b_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) & \\ \bigcup\, (b_1 + b_2 + b_3)\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\Big] &\leq 4.5\,. \end{aligned}$$

Since $B = 0$, the K space yields,

$$\begin{aligned} K = \Big[& (a_1 + a_2 + a_3)\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) \\ & \bigcup \left\{(a_1\, a_2)^{1/2} + (a_2\, a_3)^{1/2} + (a_3\, a_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) \\ & \bigcup\, (b_1 + b_2 + b_3)\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) \\ & \bigcup \left\{(b_1\, b_2)^{1/2} + (b_2\, b_3)^{1/2} + (b_3\, b_1)^{1/2}\right\}\left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right)\Big]\,, \end{aligned}$$

i.e.,

$$\dim K \leq (4.5 + 4.5) = 9\,.$$

Thus, $\mathfrak{X}^{\mu} \leq S \cup K$ has the spacetime axes as (using summation convention),

$$\begin{aligned} \mathfrak{X}^{\mu} &= \Big[\left(\widehat{\partial}^2_{[0,1]} \otimes \Psi_{[1,0]}\right), a_i \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right), b_i \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right), \\ & \qquad \left\{ \left(a_i a_j\right)^{1/2} \left(\widehat{\partial}^2_{[0,2]} \otimes \Psi_{[2,0]}\right) + \left(b_i b_j\right)^{1/2} \left(\widehat{\partial}^2_{[0,3]} \otimes \Psi_{[3,0]}\right) \right\} \Big] \\ &\mapsto \left(x^0, x^1, x^{(1+i)}, x^{(1+(i+\ell))}, x^{(1+(i+\ell)+j)} \right), \end{aligned} \tag{36}$$

where $\left(\widehat{\partial}^2_{[0,1]} \otimes \Psi_{[1,0]}\right) \mapsto (x^0, x^1)$, whereas, other maps are obvious, for $i, j = 1, 2, 3$, $i \neq j$ and $\ell = \max i$. So, (36) yields,

$$\dim \mathfrak{X}^{\mu} \leq \dim (S \cup K) \leq (2 + 9) = 11,$$

where, the dimensions of K is strongly dependent upon the dimensions of S, that is why string has eleven-dimensions by nature – as a result, eleven is the maxinium spacetime dimension in which one can formulate a consistent Supergravity theory.

4. Conclusion

Present physics is unable to provide us a more acceptable scenario of Einstein field equation which is developed in a quantum spacetime, but in $(3+1)D$ quantum mechanics, we have quantized the classical theory of General Relativity by contributing a very natural geometric way so as we can write a fundamental theory of renormalizable quantum Einstein field equation in $(3+1)D$ quantum spacetime.

We did not know much about the origin of Supersymmetry/string, neither we knew about the properties of eleven dimensions in the higher dimensional spacetime. This work is the first time answer of the natural and universal origin of Supersymmetry/string, as well as the properties and/or purposes of eleven dimensions, within a $(3+1)D$ quantum spacetime.

In our present work, we have not chosen string intentionally, but it has come up quite naturally from the $(3+1)D$ quantum geometry, and surprisingly, which actually is a quite natural phenomenon (as a result, eleven-dimensional Supergravity becomes necessarily a natural phenomenon within the quantum spacetime of $(3+1)D$ Quantum Mechanics). This finding can make an effective impact on the field of studying and developing String/M-theory in a deeper level.

This $(3+1)D$ Quantum Mechanics is a new kind of quantum formalism, which is truly inspiring us for a new beginning of both theoretical and experimental fields related to contemporary Quantum Mechanics, for example, Quantum Dots, Quantum Gates, Quantum Computation, Quantum Coding, Quantum Cryptography, etc.

Application of this extra dimensional quantum state spaces in Quantum Cryptography may result an infinitely secure networking. Similarly, this finding may help us to develop a new kind of Quantum Coding for building up a superfastest Quantum Computation.

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