Short Note

Not peer-reviewed version

Why No PlaneWaves of Macroscopic Bodies? A Micro-Macro Threshold?

Ladislaus Banyai *

Posted Date: 10 July 2024

doi: 10.20944/preprints202307.0731.v2

Keywords: center of mass; wave-package; plane wave; macroscopic; microscopic

Article

Why No Plane Waves of Macroscopic Bodies? A Micro-Macro Threshold?

Ladislaus Alexander Bányai

Institut für Theoretische Physik, Goethe-Universität, Frankfurt am Main; banyai@itp.uni-frankfurt.de

Abstract: According to Quantum Mechanics a narrow wave-packet of the center of mass of macroscopic objects, due to their heavy mass, remains stable over astronomical times. However Quantum Mechanics allows and energetically even prefers largely smeared out c.m. states that never been seen. Why is this the real state of the world we know? Does it suggest a micro-macro threshold with different theoretical descriptions?

Keywords: center of mass; wave-package; plane wave; macroscopic; microscopic

1. Introduction

One of the strange features of Quantum Mechanics is that for its formulation, one needs the classical physics that actually should emerge as its macroscopic limit. All experiences with quantum objects have to be analyzed through classical "glasses". Naturally, the question arises: where is the threshold (if any) between the microscopic and the macroscopic worlds? In the first half of the twentieth century the big successes of quantum theory were obtained mainly in the field of ("elementary") microscopic particles: electrons, atoms, nuclei. Thereafter, the most spectacular evolution occurred in understanding the properties of condensed matter on the basis of many-body quantum mechanics. This step revolutionized our technology. Now we witness another exceptional development in producing ever smaller pieces of solid matter. We are approaching the microscopic world from above. Therefore, the answer to the above stated question might be very close. Quantum Mechanics is an exceptionally successful theory, nevertheless the relation between macroscopic classical and microscopic quantum mechanical description is still subject of worry. I want to underline some simple but often ignored aspects in this context. The theory of condensed matter is conceived as the quantum mechanical description of bound states of an enormous number of Coulomb interacting electrons and ions. In the absence of an external potential the center of mass (c.m.) motion is fully separable from the relative (internal) one [1] and its wave function satisfies the free Schrödinger equation of a particle with the total mass M of the system. All we are treating is however, only the relative motion, while the center of mass motion is considered irrelevant and left to the classical description. In what follows we shall however discuss only the center of mass motion. In the real world the center of mass states of the macroscopic objects are sharply defined wave packets. According to Quantum Mechanics (Schrödinger equation) these should spread out in time. A somewhat appeasing argument is given in the frame of Ehrenfest's theorem in almost all handbooks about their stability for macroscopic masses. In Section 2 we recapitulate the standard quantum mechanical description of an ideal wave packet (here the free motion of the c.m), while in Section 3 we comment on the not understood absence of broad wave packets of macroscopic objects in the real word.

2. Evolution of a Wave-Packet of The C.M.

Let us consider a solution of the free 1D Schrödinger equation for an object of mass M in the form of a Gaussian wave packet (having the ideal uncertainty $\Delta x \Delta p = \hbar$) traveling with a constant average velocity. It may represent the free motion of the c.m. of a macroscopic object of total mass m. Using the Galilean invariance of the free Schrödinger equation [2] one may always go to its own frame (in our

 doi:10.20944/preprints202307.0731.v2

case the c.m. frame of reference) defined by the vanishing of the average velocity and at some initial time t = 0 one has the wave function

$$\psi(x,0) = \frac{1}{2\pi^{\frac{1}{4}}\sqrt{d}} e^{-\frac{x^2}{4d^2}} \quad .$$

Then after a lapse of time t it decays

$$\psi(x,t) = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{d + \frac{\imath\hbar t}{2dM}}} e^{-\frac{x^2}{4(d^2 \frac{\imath\hbar t}{2dM})}} \quad .$$

This means that the average quadratic width grows as

$$\langle x^2\rangle_t = d^2 + (\frac{\hbar t}{2Md})^2 \quad .$$

One may see, that for $M \to \infty$ the width does not change at all. However, the decisive parameter is not the mass M alone but the product Md. For any finite mass M at $d \to 0$ the wave packet decays instantly.

Leaving aside these ideal limits, let us consider a numerical example for sake of illustration. Let us consider a piece of crystal of mass $M = 1g$ with a lattice constant of $5\AA$. To observe its lattice structure by a diffraction experiment the imprecision of its c.m. must be then less than $1\AA$. That implies however that after 20.000 years its crystal structure may not be seen any more. This is worth to think about. Of course, the discussion may be extended to the motion in the presence of a slowly varying external potential on the scale of the macroscopic object as in Ehrenfest's Theorem. All these aspects are well-known. My own short comments follow in the next sections.

3. What About the "Initial" State?

The provoking question is:**Why all macroscopic objects we know have very precise center of mass position?** Ultimately, why are there no plane waves of macroscopic objects, although energetically more favorable. In our example of the Gaussian packet the average kinetic energy is

$$\frac{\hbar^2}{2M}\langle k^2\rangle = \frac{\hbar^2}{8Md^2} \quad .$$

For non-ideal wave-packets $\Delta x \Delta p > \hbar$ the average kinetic energy is even greater. Therefore the more smeared is the wave packet, the smaller its conserved average energy is. Although energetically preferred, no smeared out (on macroscopic scale) macroscopic objects have ever been seen. It looks like a "super-selection" rule for macroscopic objects. "God allowed them only in this state of precise c.m.?" On the other hand, a world with smeared out center of masses is hard to imagine. It won't function. The macroscopic world, as we know it, is the only possible world!?

Actually there is another striking old problem about the application of quantum-mechanical concepts to macroscopic objects. There are no known superpositions of states of macroscopic systems. It was illustrated by the famous "Schrödinger's cat".

On our current way toward micro-miniaturization we might perhaps get an answer to this question. Where is the threshold between micro and macro? One cannot exclude the possibility, that the macroscopic physics is not just the simple limit of the quantum theory. (In a mathematical sense it is surely not the $\hbar \to 0$ limit.) This should solve also the "bootstrap" aspect of Quantum Mechanics that needs for its formulation the existence of macroscopic laws. In other words: Quantum Mechanics has its limited field of application as any other physical theory.

 doi:10.20944/preprints202307.0731.v2

4. Comments

I am aware that just touching this point is a heresy. In our human-centered world-image motivated by the exceptional scientific progresses of the last centuries we cheer to construct a unified picture of the world without loopholes and crevices. One looks even for "God's equations" forgetting that mathematics has to be connected to experiment by some interpretation and this is the most difficult one. Just think about the strange idea of an assembly of elementary particles (we) formulating the theory of elementary particles.

Actually, all theories of physics of the past were just fragments of knowledge with loose connections between them. We construct beautiful models that work surprisingly well in many domains. Of course, thermodynamics has to do with statistical mechanics, but this is a rather vague connection. Whether one may derive every-day quantum theory of condensed matter from the modern quantum field theory of elementary particles remains a question of belief. Actually the former had to be formulated independently (see more comments in Ref. [3]). The unification of gravitation and quantum theory looks merely a dream.

I am afraid, that we are misled by our need for harmony and exaggerated confidence in our brains. We understand nothing, however explain everything.

Acknowledgments: The author thanks Mircea Bundaru† and Paul Gartner for fruitful discussions on this topic and for reading the manuscript.

1. Albert Messiah, Quantum Mechanics, Volume 1, North Holland Publishing Company, Amsterdam. pp 361-366 (1961)
2. J. Schwinger, Quantum Mechanics. Springer. p 183 (2001)
3. Ladislaus Alexander Bányai and Mircea Bundaru, About Non-relativistic Quantum Mechanics and Electromagnetism, Recent Progress in Materials, **4**, issue 4, (2022), doi:10.21926/rpm.2204027