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Article

Discovery of New Exact Wave Solutions to the M-Fractional Complex Three Coupled Maccari's System by Sardar Sub-Equation Scheme

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Abstract: In this paper, we succeed to discover the new exact wave solutions to the truncated M-fractional complex three coupled Maccari's system by utilizing the Sardar sub-equation scheme. The obtained solutions are in the form of trigonometric and hyperbolic forms. These solutions have many applications in nonlinear optics, fiber optics, deep water-waves, plasma physics, mathematical physics, fluid mechanics, hydrodynamics and engineering where the propagation of nonlinear waves is important. Achieved solutions are verified with the use of Mathematica software. Some of the achieved solutions are also described graphically by 2-dimensional, 3-dimensional and contour plots with the help of Maple software. The gained solutions are helpful for the further development of concerned model. Finally, this technique is simple, fruitful and reliable to handle the nonlinear FPDEs.

Keywords: time-fractional complex three coupled maccari's system; sardar sub-equation scheme; new exact wave solutions

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1. Introduction

Different types of phenomenon occurring chemically, biologically, economically and others are represented as a non-linear partial differential equations (NLPDEs). Many different methods are developed to gain the analytical wave solutions of these NLPDEs i.e optical soliton solutions of coupled nonlinear Schrödinger equation have been gained with the use of Kudryashov R function technique [1], some new kind of optical soliton solutions of time-fractional perturbed nonlinear Schrödinger equation have been achieved by using the generalized Kudryashov scheme [2], by applying the modified auxiliary equation technique, optical wave solutions of time-fractional resonant non-linear Schrödinger equation have been obtained [3], new optical wave solutions for the time-fractional perturbed non-linear Schrödinger equation have been achieved by utilizing the improved $\tan(\phi(\zeta/2))$-expansion scheme [4].

We utilize the Sardar sub-equation method for our research work. This method have so many applications in the literature. For example; some solitons solutions of perturbed Fokas-lenells model have been obtained with the use of Sardar sub-equation method [5], new kind of soliton wave solutions of (2+1)-dimensional Sawada-Kotera model have been gained by this method [6], the dark, bright and singular optical wave solutions of higher order non-linear Schrödinger have been gained by using Sardar sub-equation scheme [7], The singular, bright, dark, periodic singular, combined dark-bright solitons and other wave solutions of strain wave equation have been achieved with the use of this model [8].

Our study model is complex three coupled Maccari's system along with truncated M-fractional derivative. Different types of exact wave solutions of this model have been obtained with the help of different methods in the literature. Instantly, the dark, bright and complex optical wave solutions have been achieved by using extended rational sine-cosine and rational sinh-cosh techniques [9], trigonometric, rational and hyperbolic solutions have been attained with the use of unified solver

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technique [10], the optical soliton solutions have been gained by utilizing the planar dynamic system scheme [11], some new travelling wave solutions have been obtained by applying the modified trial equation technique [12], the periodic, double periodic, dark, bright, combined soliton solutions etc have been obtained with the help of generalized auxiliary equation mapping technique [13].

Main aim of this research is to discover the new exact wave solutions to the truncated M-fractional complex three coupled Maccari's system with the help of Sardar sub-equation method.

Paper consists of different sections; In Section 2: we describe the main steps of our concerned method Sardar sub-equation.. In Section 3: we explain the our concerned model and it's mathematical analysis. In Section 4: we apply the method to gain the new exact wave solutions of our concerned model. In Section 5: we explain the some obtained solutions through 2-D, 3-D and contour graphs. In Section 6: we give the conclusion of our research work.

2. Description of Sardar Sub-Equation METHOD

Here, we explain the fundamental points of Sardar sub-equation method [14]. Let's consider the non-linear fractional partial differential equation:

$$F(g, g_t, g_{xx}, g_{xt}, gg_{tt}, g_{xy}, ...) = 0. \tag{1}$$

here $g = g(x, y, t)$ represents a wave function.
Applying the wave transformations given as follow:

$$g(x, y, t) = G(\zeta), \zeta = \lambda x + \kappa y + \mu t \tag{2}$$

We obtain a non-linear ODE shown as:

$$Y(G, G'', GG'', G'G^2, ...) = 0. \tag{3}$$

Consider Equation (3) posses the results in the given shape:

$$G(\zeta) = \sum_{i=0}^{m} b_i \psi^i(\zeta). \tag{4}$$

here $\psi(\zeta)$ fulfill the ODE shown as:

$$\psi'(\zeta) = \sqrt{\sigma + \kappa\psi^2(\zeta) + \psi^4(\zeta)}. \tag{5}$$

here σ and κ are constants.
Using Equation (4) into Equation (3) with Equation (5) and collecting the coefficients of each power of ψ^i. Putting co-efficient of each power equal to 0, we gain a sets of algebraic equations in the term b_i, λ, μ. By solving the obtained system of equation, we get the values of parameters..
Case 1: If $\kappa > 0$ and $\sigma = 0$, then

$$\psi_1^{\pm} = \pm\sqrt{-\kappa ab}\, sech_{ab}(\sqrt{\kappa}\,\zeta), \tag{6}$$

$$\psi_2^{\pm} = \pm\sqrt{\kappa ab}\, csch_{ab}(\sqrt{\kappa}\,\zeta), \tag{7}$$

where, $sech_{ab}(\zeta) = \frac{2}{ae^{\zeta}+be^{-\zeta}}$, $csch_{ab}(\zeta) = \frac{2}{ae^{\zeta}-be^{-\zeta}}$
Case 2: If $\kappa < 0$ and $\sigma = 0$, then

$$\psi_3^{\pm} = \pm\sqrt{-\kappa ab}\ \sec_{ab}(\sqrt{-\kappa}\,\zeta), \tag{8}$$

$$\psi_4^{\pm} = \pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta), \tag{9}$$

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where, $\sec_{ab}(\zeta) = \frac{2}{ae^{\iota\zeta}+be^{-\iota\zeta}}$, $\csc_{ab}(\zeta) = \frac{2\iota}{ae^{\iota\zeta}-be^{-\iota\zeta}}$
Case 3: If $\kappa < 0$ and $\sigma = \frac{\kappa^2}{4}$, then

$$\psi_5^{\pm} = \pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta), \tag{10}$$

$$\psi_6^{\pm} = \pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta), \tag{11}$$

$$\psi_7^{\pm} = \pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\ sech_{ab}(\sqrt{-2\kappa}\,\zeta)), \tag{12}$$

$$\psi_8^{\pm} = \pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\ csch_{ab}(\sqrt{-2\kappa}\,\zeta)), \tag{13}$$

$$\psi_9^{\pm} = \pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta)), \tag{14}$$

where, $\tanh_{ab}(\zeta) = \frac{ae^{\zeta}-be^{-\zeta}}{ae^{\zeta}+be^{-\zeta}}$, $\coth_{ab}(\zeta) = \frac{ae^{\zeta}+be^{-\zeta}}{ae^{\zeta}-be^{-\zeta}}$
Case 4: If $\kappa > 0$ and $\sigma = \frac{\kappa^2}{4}$, then

$$\psi_{10}^{\pm} = \pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta), \tag{15}$$

$$\psi_{11}^{\pm} = \pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta), \tag{16}$$

$$\psi_{12}^{\pm} = \pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta) \pm \sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta)), \tag{17}$$

$$\psi_{13}^{\pm} = \pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta) \pm \sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta)), \tag{18}$$

$$\psi_{14}^{\pm} = \pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta) + \cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta)), \tag{19}$$

where, $\tan_{ab}(\zeta) = -\iota\frac{ae^{\iota\zeta}-be^{-\iota\zeta}}{ae^{\iota\zeta}+be^{-\iota\zeta}}$, $\cot_{ab}(\zeta) = \iota\frac{ae^{\iota\zeta}+be^{-\iota\zeta}}{ae^{\iota\zeta}-be^{-\iota\zeta}}$

3. The governing model and it's mathematical treatment

Assume the M-fractional three coupled non-linear Maccari's system shown in [15]:

$$\begin{aligned}\iota D_{M,t}^{\alpha,\gamma}U + U_{xx} + ZU = 0,\\ \iota D_{M,t}^{\alpha,\gamma}V + V_{xx} + ZV = 0,\\ \iota D_{M,t}^{\alpha,\gamma}W + W_{xx} + ZW = 0,\\ D_{M,t}^{\alpha,\gamma}Z + Z_y + (|U+V+W|^2)_x = 0.\end{aligned} \tag{20}$$

where

$$D_{M,t}^{\alpha,\Upsilon}U(t) = \lim_{\tau\to 0}\frac{U\left(t\,E_{\Upsilon}(\tau t^{1-\alpha})\right) - U(t)}{\tau}, \quad 0 < \alpha \le 1, \ \ \Upsilon > 0, \tag{21}$$

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here $E_Y(.)$ represents truncated Mittag-Leffler function of one parameter given in [16,17].
Let's consider the following transformations:

$$U(x,y,t) = U(\zeta) \times \exp(\iota(\theta_1 x + \mu y + \tau\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha)) + \vartheta)),$$
$$V(x,y,t) = V(\zeta) \times \exp(\iota(\theta_1 x + \mu y + \tau\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha)) + \vartheta)),$$
$$W(x,y,t) = W(\zeta) \times \exp(\iota(\theta_1 x + \mu y + \tau\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha)) + \vartheta)),$$
$$Z(x,y,t) = Z(\zeta) \qquad where\ \ \zeta = \lambda(x + y - 2\beta\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha) \tag{22}$$

here θ_1, σ,τ,μ and β are the unknowns and ϑ is arbitrary constant.
Substituting Equation (22) into Equation (20), yields the real and imaginary parts:
Real parts:

$$\lambda^2 U'' - (\tau + \theta_1^2)U + UZ = 0,$$
$$\lambda^2 V'' - (\tau + \theta_1^2)V + VZ = 0,$$
$$\lambda^2 W'' - (\tau + \theta_1^2)W + WZ = 0,$$
$$\lambda(1 - 2\beta)Z' + \lambda((U + V + W)^2)' = 0 \tag{23}$$

Imaginary parts:

$$(-2\beta + 2\theta_1)U' = 0, (-2\beta + 2\theta_1)V' = 0, (-2\beta + 2\theta_1)W' = 0. \tag{24}$$

By Equation (24), implies $\beta = \theta_1$.
Integrating the fourth equation of system (23), yields

$$Z = -\frac{(U + V + W)^2}{1 - 2\theta_1} \tag{25}$$

Putting Equation (25) into first three terms of Equation (33), we get

$$\lambda^2 U'' - (\tau + \theta_1^2)U - \frac{(U + V + W)^2}{1 - 2\theta_1}U = 0,$$
$$\lambda^2 V'' - (\tau + \theta_1^2)V - \frac{(U + V + W)^2}{1 - 2\theta_1}V = 0,$$
$$\lambda^2 W'' - (\tau + \theta_1^2)W - \frac{(U + V + W)^2}{1 - 2\theta_1}W = 0, \tag{26}$$

Taking $V = \theta_2$ U and Taking $W = c$ U, we obtain

$$\lambda^2 U'' - (\tau + \theta_1^2)U - \frac{(1 + \theta_2 + c)^2}{1 - 2\theta_1}U^3 = 0 \tag{27}$$

4. Solutions through Sardar Sub-Equation Method

Equation (4) changes into given form for m=1.

$$G(\zeta) = b_0 + b_1\psi(\zeta) \tag{28}$$

Putting Equation (28) into Equation (27) with Equation (5). By collecting co-efficients of every order of $\psi(\zeta)$ and taking them equal to 0, we get a set of algebraic equations. By solving the obtained set of

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equations by Mathematica software, we gain the following sets.
Set 1:

$$\left\{b_0 = 0, b_1 = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}, \tau = \kappa\lambda^2 - \theta_1^2\right\} \tag{29}$$

Case 1

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\, sech_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{30}$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\, sech_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{31}$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\, sech_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{32}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\, sech_{ab}(\sqrt{\kappa}\,\zeta)))^2 \tag{33}$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\, csch_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{34}$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\, csch_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{35}$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\, csch_{ab}(\sqrt{\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{36}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\, csch_{ab}(\sqrt{\kappa}\,\zeta)))^2 \tag{37}$$

Case 2:

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \sec_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{38}$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \sec_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{39}$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \sec_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{40}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \sec_{ab}(\sqrt{-\kappa}\,\zeta)))^2 \tag{41}$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{42}$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{43}$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{44}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta)))^2 \tag{45}$$

Case 3:

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{46}$$

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$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (47)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (48)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)))^2 \quad (49)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (50)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (51)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (52)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)))^2 \quad (53)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\ sech_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (54)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\ sech_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (55)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\ sech_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (56)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\, sech_{ab}(\sqrt{-2\kappa}\,\zeta))))^2 \quad (57)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\ csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (58)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\ csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (59)$$

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$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (60)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta))))^2 \quad (61)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (62)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (63)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (64)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))))^2 \quad (65)$$

Case 4:

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (66)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (67)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (68)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)))^2 \quad (69)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (70)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (71)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (72)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)))^2 \quad (73)$$

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$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (74)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (75)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (76)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))))^2 \quad (77)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (78)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (79)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (80)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))))^2 \quad (81)$$

$$U(x,y,t) = -\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta)+\cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (82)$$

$$V(x,y,t) = -\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta)+\cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (83)$$

$$W(x,y,t) = -\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta)+\cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (84)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(-\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\;\zeta)+\cot_{ab}(\sqrt{\frac{\kappa}{8}}\;\zeta))))^2 \quad (85)$$

Set 2:

$$\left\{b_0=0,b_1=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}},\tau=\kappa\lambda^2-\theta_1^2\right\} \quad (86)$$

Case 1

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;sech_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (87)$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;sech_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (88)$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;sech_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (89)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;sech_{ab}(\sqrt{\kappa}\,\zeta)))^2 \quad (90)$$

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\;csch_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (91)$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\;csch_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (92)$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\;csch_{ab}(\sqrt{\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (93)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\kappa ab}\;csch_{ab}(\sqrt{\kappa}\,\zeta)))^2 \quad (94)$$

Case 2:

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;\sec_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (95)$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;\sec_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (96)$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;\sec_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1 x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}))+\vartheta)) \quad (97)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\;\sec_{ab}(\sqrt{-\kappa}\,\zeta)))^2 \quad (98)$$

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$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{99}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{100}$$

$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{101}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\kappa ab}\ \csc_{ab}(\sqrt{-\kappa}\,\zeta)))^2 \tag{102}$$

Case 3:

$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{103}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{104}$$

$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{105}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \tanh_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)))^2 \tag{106}$$

$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{107}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{108}$$

$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{109}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}\ \coth_{ab}(\sqrt{-\frac{\kappa}{2}}\,\zeta)))^2 \tag{110}$$

$$\begin{aligned}U(x,y,t)=&\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta)\pm\iota\sqrt{ab}\,sech_{ab}(\sqrt{-2\kappa}\,\zeta)))\\&\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta))\end{aligned} \tag{111}$$

$$\begin{aligned}V(x,y,t)=&\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta)\pm\iota\sqrt{ab}\,sech_{ab}(\sqrt{-2\kappa}\,\zeta)))\\&\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta))\end{aligned} \tag{112}$$

$$\begin{aligned}W(x,y,t)=&\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta)\pm\iota\sqrt{ab}\,sech_{ab}(\sqrt{-2\kappa}\,\zeta)))\\&\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta))\end{aligned} \tag{113}$$

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$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}\left(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\tanh_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \iota\sqrt{ab}\,sech_{ab}(\sqrt{-2\kappa}\,\zeta)))\right)^2 \tag{114}$$

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{115}$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{116}$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{117}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}\left(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{2}}(\coth_{ab}(\sqrt{-2\kappa}\,\zeta) \pm \sqrt{ab}\,csch_{ab}(\sqrt{-2\kappa}\,\zeta)))\right)^2 \tag{118}$$

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{119}$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{120}$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{121}$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}\left(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{-\frac{\kappa}{8}}(\tanh_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta) + \coth_{ab}(\sqrt{-\frac{\kappa}{8}}\,\zeta)))\right)^2 \tag{122}$$

Case 4:

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\frac{\kappa}{2}}\,\tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{123}$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^{\,2}}}(\pm\sqrt{\frac{\kappa}{2}}\,\tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \tag{124}$$

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$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{125}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \tan_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)))^2 \tag{126}$$

$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{127}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{128}$$

$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta))\times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{129}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}\ \cot_{ab}(\sqrt{\frac{\kappa}{2}}\,\zeta)))^2 \tag{130}$$

$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{131}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{132}$$

$$W(x,y,t)=\frac{c\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))) \times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{133}$$

$$Z(x,y,t)=-\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\tan_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \sec_{ab}(\sqrt{2\kappa}\,\zeta))))^2 \tag{134}$$

$$U(x,y,t)=\frac{\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{135}$$

$$V(x,y,t)=\frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta)\pm\sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times\exp(\iota(\theta_1x+\mu y+(\kappa\lambda^2-\theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^\alpha))+\vartheta)) \tag{136}$$

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$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta) \pm \sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (137)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{2}}(\cot_{ab}(\sqrt{2\kappa}\,\zeta) \pm \sqrt{ab}\ \csc_{ab}(\sqrt{2\kappa}\,\zeta))))^2 \quad (138)$$

$$U(x,y,t) = \frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta) + \cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (139)$$

$$V(x,y,t) = \frac{\theta_2\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta) + \cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (140)$$

$$W(x,y,t) = \frac{c\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta) + \cot_{ab}(\sqrt{\frac{\kappa}{8}}\,\zeta))) \times \exp(\iota(\theta_1 x + \mu y + (\kappa\lambda^2 - \theta_1^2)\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})) + \vartheta)) \quad (141)$$

$$Z(x,y,t) = -\frac{(1+\theta_2+c)^2}{1-2\theta_1}(\frac{\sqrt{2}\sqrt{(1-2\theta_1)\,\lambda^2}}{\sqrt{(c+\theta_2+1)^2}}(\pm\sqrt{\frac{\kappa}{8}}(\tan_{ab}(\sqrt{\frac{\kappa}{8}}\ \zeta) + \cot_{ab}(\sqrt{\frac{\kappa}{8}}\ \zeta))))^2 \quad (142)$$

where $\zeta = \lambda(x + y - 2\theta_1\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha})$

5. Graphical Representation of Solutions

In this section we present the graphically behavior of solutions in the 3D, contour and 2D surfaces.

We gave graphs for solution (30) in Figure 1a,b in Contour and Figure 1c 2D with different values of t=-1 is represent the red line, t=0 is represent the black line and t=1 is represent the yellow with $\theta_1 = 0.003, \theta_2 = 0.01, \lambda = 0.5, \beta = 0.5, c = 1, \Upsilon = 1, \mu = -1, \vartheta = 0, \kappa = 0.1, y = 0, \alpha = 0.9$, $-10 < t < 10$ and $-10 < x < 10$.

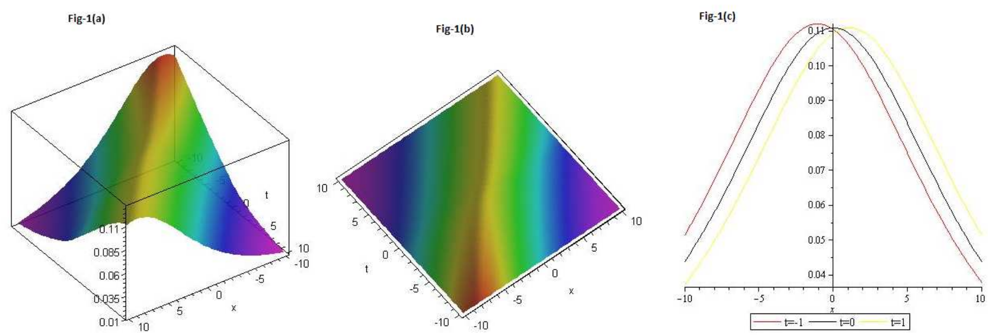

Figure 1. Graphical representation of (30).

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We gave graphs for solution (38) in Figure 2a,b in Contour and Figure 2c 2D with different values of t=-1 is represent the red line, t=0 is represent the black line and t=1 is represent the yellow with $\theta_1 = 0.003, \theta_2 = 0.01, \lambda = 0.5, \beta = 0.5, c = 1, \Upsilon = 1, \mu = -1, \vartheta = 0, \kappa = -0.1, y = 0, \alpha = 0.9$, $-10 < t < 10$ and $-10 < x < 10$.

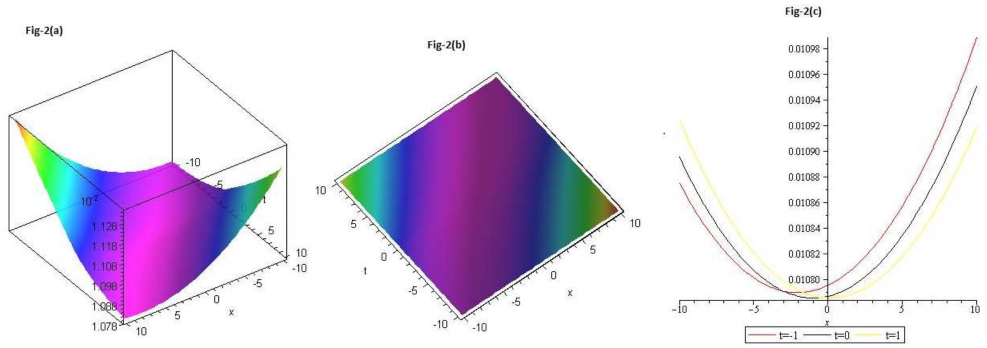

Figure 2. Graphical representation of (38).

We gave graphs for solution (46) in Figure 3a,b in Contour and Figure 3c 2D with different values of t=-1 is represent the red line, t=0 is represent the black line and t=1 is represent the green with $\theta_1 = 0.3, \theta_2 = 0.5, \lambda = 0.5, \beta = 0.05, c = 1, \Upsilon = 1, \mu = 0.1, \vartheta = 0, \kappa = -0.1, y = 0, \alpha = 0.9$, $-10 < t < 10$ and $-10 < x < 10$.

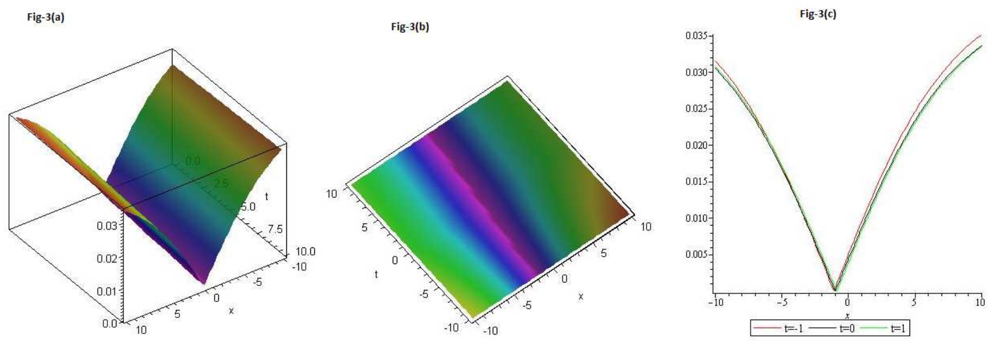

Figure 3. Graphical representation of (46).

We gave graphs for solution (66) in Figure 4a,b in Contour and Figure 4c 2D with different values of t=-1 is represent the red line, t=0 is represent the black line and t=1 is represent the green with $\theta_1 = 0.03, \theta_2 = 0.001, \lambda = 0.5, \beta = 0.05, c = 1, \Upsilon = 1, \mu = -0.1, \vartheta = 0, \kappa = 0.1, y = 0, \alpha = 0.9$, $-10 < t < 10$ and $-10 < x < 10$.

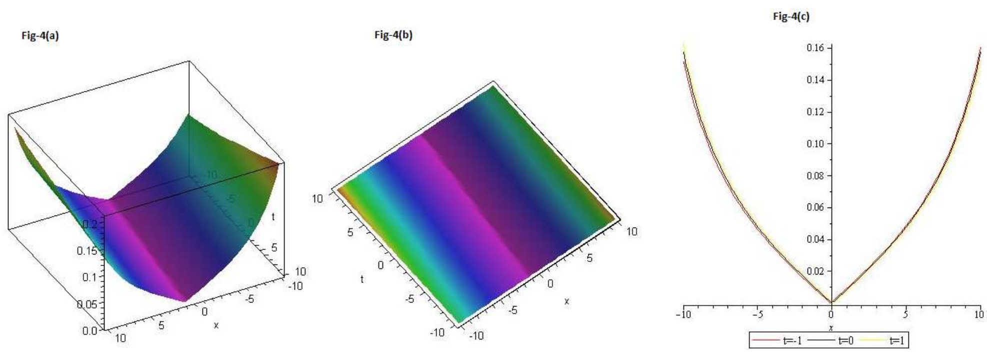

Figure 4. Graphical representation of (66).

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We gave graphs for solution (106) in Figure 5a,b in Contour and Figure 5c 2D with different values of t=-1 is represent the red line, t=0 is represent the black line and t=1 is represent the green with $\theta_1 = 0.03, \theta_2 = 0.002\lambda = 0.5, \beta = 0.05, c = 1, \Upsilon = 1, \mu = -0.1, \vartheta = 0, \kappa = -0.1, y = 0, \alpha = 0.9,$ $-10 < t < 10$ and $-10 < x < 10$.

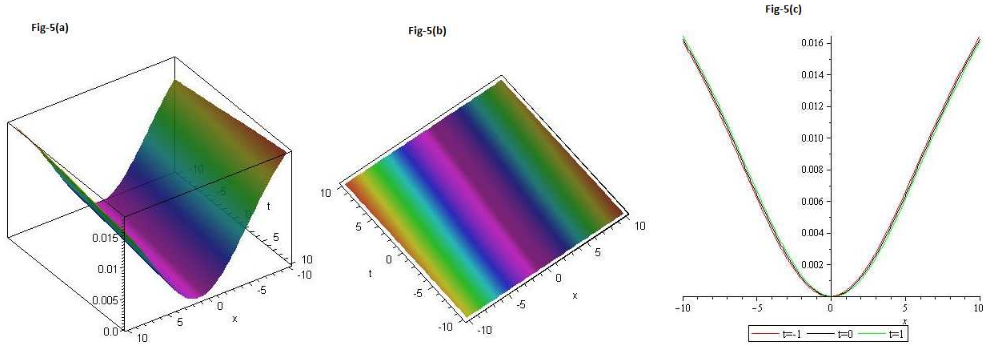

Figure 5. Graphical representation of (106).

6. Conclusions

Overall, this paper contributes to our understanding of the truncated M-fractional complex three coupled Maccari's system and provides a useful technique for handling nonlinear partial differential equations. This paper describes the successful application of the Sardar sub-equation method to obtain new optical wave solutions for the truncated M-fractional complex three coupled Maccari's system. The obtained solutions are useful for future studies of the concerned model and provide insights into the behavior of optical waves in complex media. The Sardar sub-equation method is shown to be a simple, fruitful, and reliable technique for handling nonlinear partial differential equations. The solutions were verified using Mathematica software and were also described graphically using 2-dimensional, 3-dimensional, and contour plots through Maple software.

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