Article

Not peer-reviewed version

Anisotropic universes sourced by modified Chaplygin gas

Saibal Ray *, Sunil Tripathy, Rikpratik Sengupta, Bibhudutta Bal, Sonali Rout

Posted Date: 1 August 2023

doi: 10.20944/preprints202308.0013.v1

Keywords: Anisotropic universe; Modified Chaplygin gas; Cosmological constant; Deceleration parameter

Article

Anisotropic Universes Sourced by Modified Chaplygin Gas

Saibal Ray [1*], Sunil Kumar Tripathy [2], Rikpratik Sengupta [3], Bibhudutta Bal [4] and Sonali Monalisa Rout [5]

1 Centre for Cosmology, Astrophysics and Space Science (CCASS), GLA University, Mathura 281406, India; saibal.ray@gla.ac.in
2 Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Orissa, India; sktripathy@igitsarang.ac.in
3 Department of Physics, Aliah University, Kolkata 700160, West Bengal, India; rikpratik.sengupta@gmail.com
4 Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Orissa, India; balbibhudutta@gmail.com
5 Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Orissa, India; sonalirout514@gmail.com
* Correspondence: saibal.ray@gla.ac.in

Abstract: In this work we perform a comparative study of the Kantowski-Sachs (KS) and Bianchi-I anisotropic universes with Modified Chaplygin gas (MCG) as matter source. We obtain the volume and scale factors as solutions to the Einstein Field Equations (EFEs) for the anisotropic universes, and check whether the initial anisotropy is washed out or not for different values of the MCG parameters present in the solution by obtaining the anisotropy parameters for each solution. The deceleration parameter is also obtained for each solution, the significance of which is discussed in the concluding section. Interestingly there are a number of notable results that appear from our study which help us to compare and contrast the two different anisotropic models along with proper understanding of the role of MCG as matter source in these models.

Keywords: anisotropic universe; Modified Chaplygin gas; cosmological constant; deceleration parameter

1. Introduction

The conditions of homogeneity and isotropy are observed to be satisfied for the present universe on a large scale. The standard cosmological model expects that the observed large scale structure has grown under gravity in the sea of dark matter and with an initial Gaussian random density fluctuation of almost scale-invariant power spectrum. Over a period of time, the standard model and the cosmic isotropy have been tested through the Cosmic Microwave Background (CMB) radiation and the Large Scale Structure (LSS) data.

However, in the very early universe when the energy densities were considerably higher, it is generally believed by cosmologists that the universe was highly anisotropic. Also, certain anomalies such as the lack of correlations on large angular scales, a statistically significant alignment and planarity of the CMB quadrupole and octopole modes and the observed large scale alignment in the quasar polarization vectors imply a possible violation of the statistical isotropy and the break down of the cosmological principle (see [1–5]). Nevertheless, there is no conclusive evidence for the cosmological isotropic hypothesis [4]. Also, there is no consensus on the physical nature and origin of the scale-invariance of the primordial perturbations [6]. However, it is believed that, with the growth of cosmic time, the anisotropy may be washed out to leave the universe isotropic to a greater extent. This imply there must have been some mechanism of washing out this initial anisotropy with time [7]. Also, it may be inferred that the initial conditions of the universe do not influence it's present state.

A statically isotropic universe is usually modelled through an FRW metric. In order to account for the possible anisotropy in the space time, we will be concerned with two such anisotropic frameworks, namely the KS [8] and Bianchi-I. There are a number of Bianchi models in cosmology ,in accordance with the corresponding classification of 3D spaces by Bianchi [9]. Bianchi's classification was first applied to GR by Ellis and others [10–12]. The cosmology of Bianchi-I models have been studied in [5,13–17] whereas KS cosmology has been investigated in [18–21].

In the present work, along with the anisotropic spatial expansion, we consider the MCG as the source of matter field. The MCG is a generalized and modified form of the Chaplygin gas (CG) which was first proposed as an alternative to quintessence for explaining the accelerated expansion of the universe [22] and later on carried out work by several scientists [23–25]. The CG is a perfect fluid having Equation of State (EOS)

$$p = -\frac{A}{\rho}, \tag{1}$$

where A is a positive constant and p and ρ denote the usual pressure and energy density respectively.

The fact that CG is connected to the d-branes appearing in String theory makes it even more interesting. The origin of CG in the universe has been addressed in [26]. The EOS for CG can be generalized to

$$p = -\frac{A}{\rho^{\alpha}}, \tag{2}$$

where α can have any value between 0 and 1.

The above EOS describes an exotic fluid known as Generalized Chaplygin Gas (GCG) which behaves like pressureless dust in the early universe and like a Cosmological constant Λ in the late universe [23]. Even this EOS was further modified to

$$p = B\rho - \frac{A}{\rho^{\alpha}}, \tag{3}$$

where both A and B are positive constants. This is known as the Modified Chaplygin gas [27,28].

It is to note that in the past two decades, the MCG has been used extensively in cosmology. Not only can it be used to describe the accelerated expansion of the universe in recent times [29–31], but also have been used to construct unified models of dark matter and dark energy [32–35] and also to explain early universe phenomena [36,37]. MCG has also been found to be consistent with observational tests like gravitational lensing test [38,39] and gamma ray bursts[40]. The observational constraints on MCG have also been studied in some details [41,42].

We organize the paper as follows. In the next section we consider the solutions for KS universe sourced by MCG followed by the solutions for Bianchi-I universe with MCG in Section 3. We conclude with discussion on the results obtained.

2. KS Model

The line element for the KS universe is given as [8]

$$ds^2 = -dt^2 + a_1(t)^2 dr^2 + a_2(t)^2(d\theta^2 + sin^2\theta d\phi^2). \tag{4}$$

The EFE for the above line element with MCG as source can be written as (following the geometrized unit we take $8\pi G = c = 1$ throughout the paper)

$$\frac{\dot{a_2}^2}{a_2^2} + 2\frac{\dot{a_1}\dot{a_2}}{a_1 a_2} + \frac{1}{a_2^2} = \rho, \tag{5}$$

$$2\frac{\ddot{a_2}}{a_2} + \frac{\dot{a_2}^2}{a_2^2} + \frac{1}{a_2^2} = \frac{A}{\rho^{\alpha}} - B\rho, \tag{6}$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2} = \frac{A}{\rho^{\alpha}} - B\rho. \tag{7}$$

Equation (7) can be rewritten as

$$2\frac{\ddot{a_1}}{a_1}+2\frac{\dot{a_1}\dot{a_2}}{a_1a_2}-\frac{\dot{a_2}^2}{a_2^2}-\frac{1}{a_2^2}=\frac{A}{\rho^\alpha}-B\rho. \tag{8}$$

From Eqs. (5) and (8), we get

$$\frac{d(VH_1)}{dt}=\frac{(1-B)}{2}\rho V+\frac{AV}{2\rho^\alpha}, \tag{9}$$

where V is the volume given by $V=a_1a_2^2$ and H_1 is one of the directional Hubble parameters given as $H_1=\frac{\dot{a_1}}{a_1}$.

The conservation equation has the usual form

$$\dot{\rho}+3H(p+\rho)=0, \tag{10}$$

where the Hubble parameter H is given by $H=\frac{1}{3}(H_1+2H_2)$, H_2 being the second directional derivative $H_2=\frac{\dot{a_2}}{a_2}$.

Equation (10) gives the solution for the energy density as

$$\rho=\rho_0\left[A_s+\frac{1-A_s}{V^{(1+B)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}, \tag{11}$$

where $A_s=\frac{A}{1+B}\frac{1}{\rho_0^{(\alpha+1)}}$ and $B\neq -1$. As we are considering anisotropic space-time, so we must be in the very early epoch when the volume is such that $V\ll 1$. Since, B and α both are positive, the factor in the denominator of the second term of RHS of Eq. (11) is much less than 1 making the second term dominant at early times. So, we get approximate ρ from Eq. (11) of the form

$$\rho\simeq\frac{C}{V^{1+B}}, \tag{12}$$

where $C=\rho_0(1-A_s)^{\frac{1}{1+\alpha}}$.

Using this form of ρ, Eq. (9) can be written as

$$\frac{d(VH_1)}{dt}=\frac{(1-B)}{2}CV^{-B}+\frac{A}{2C^\alpha}V^{\alpha(1+B)+1}. \tag{13}$$

Similarly, from Eqs. (5), (6) and (12), we get

$$\frac{d(VH_2)}{dt}=\frac{(1-B)}{2}CV^{-B}+\frac{A}{2C^\alpha}V^{\alpha(1+B)+1}-a_1. \tag{14}$$

In the above Eqs. (13) and (14) are the EFE in terms of the volume.

From these equations, we obtain:

$$H_1=H-\frac{2K}{3V},\ H_2=H+\frac{K}{3V}, \tag{15}$$

where $K=\int a_1dt$.

The third EFE in alternative form may be written using Eqs. (5), (6) and (7) as

$$3\dot{H}+H_1^2+2H_2^2=\frac{3AV^{(1+B)\alpha}}{\rho_0^\alpha(1-A_s)^{\frac{\alpha}{1+\alpha}}}-\rho_0(3B+1)(1-A_s)^{\frac{1}{1+\alpha}}V^{-(1+B)}. \tag{16}$$

From the EFE in terms of volume, we get

$$\ddot{V} = \frac{3}{2}(1-B)CV^{-B} + \frac{3}{2}\frac{A}{C^{\alpha}}V^{\alpha(1+B)+1} - 2a_1. \tag{17}$$

This gives $\dot{V}$ of the form

$$\dot{V} = \sqrt{3CV^{1-B} + 3\frac{A}{C^{\alpha}}\frac{V^{\alpha(1+B)+2}}{\alpha(1+B)+2} - 4\int a_1 dV}. \tag{18}$$

From the above expressions, it can be observed that the solutions for the volume and scale factors depend on the MCG parameters α, A and B. However, the dependence on the B-parameter is found to be the strongest, so we choose to vary it for obtaining different solutions to check the anisotropic behaviour and accelerated expansion behaviour of the model. We will now consider three different values of the parameter B to assess its effect on the exact solutions.

Case-1: $B = 1$.

We can write

$$\dot{V}^2 = 3C + 3\frac{A}{C^{\alpha}}\frac{V^{2\alpha+2}}{2\alpha+2} - 4\int a_1 dV. \tag{19}$$

Using small V approximation, i.e., considering the early universe, we obtain the solution as

$$V = C_1 + \sqrt{3C+b}\,t, \tag{20}$$

where b is a constant and C_1 is integration constant.

Using Eq. (15), we now obtain the solutions for the scale factors as

$$a_1 = C_2\left[C_1 + \sqrt{3C+b}\,t\right]^{\frac{1}{3}-\frac{2K}{3\sqrt{3C+b}}}, \tag{21}$$

$$a_2 = \frac{1}{\sqrt{C_2}}\left[C_1 + \sqrt{3C+b}\,t\right]^{\frac{1}{3}+\frac{K}{3\sqrt{3C+b}}}, \tag{22}$$

where C_2 is a constant.

We now consider

$$\Delta H_1 = H_1 - H, \Delta H_2 = H_2 - H. \tag{23}$$

The anisotropy parameter Γ is defined as

$$\Gamma = \frac{1}{3}\left[\left(\frac{\Delta H_1}{H}\right)^2 + 2\left(\frac{\Delta H_2}{H}\right)^2\right]. \tag{24}$$

Using the above obtained scale factors, we find the anisotropy parameter to be

$$\Gamma = \frac{2b}{(3C+b)}, \tag{25}$$

which is a constant.

The average scale factor a_m can be written as

$$a_m = \left(a_1 a_2^2\right)^{\frac{1}{3}}. \tag{26}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 August 2023 doi:10.20944/preprints202308.0013.v1

Hence we can define the deceleration parameter q as

$$q = -\frac{a_m \ddot{a_m}}{\dot{a_m}^2}. \tag{27}$$

Alternatively, it may also be computed as

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{28}$$

Using either of the above relations, we obtain $q = 2$ for the present model. This is the same value of deceleration parameter as obtained for Petrov Type-D anisotropic cosmological solutions by Alvarado [43] and in the case of anisotropic dark energy cosmological model under the framework of generalised Brans-Dicke theory by Tripathy et al. [44].

Case-2: $B = 0$.

In this case, we get

$$\dot{V}^2 = 3CV + 3\frac{A}{C^\alpha}\frac{V^{\alpha+2}}{\alpha+2} - 4\int a_1 dV. \tag{29}$$

In the small V approximation, it is not possible to obtain the solutions in the analytical form, however in the asymptotic limit

$$V \propto (t-t_0)^2, a_1 \propto (t-t_0)^{\frac{2}{3}}, a_2 \propto (t-t_0)^{\frac{2}{3}}, \tag{30}$$

where $V(t_0) = 0$.

In the asymptotic limit, the anisotropy parameter turns out to be zero.

Case-3: $B = \frac{1}{2}$.

In this case, we have

$$\dot{V}^2 = 3CV^{\frac{1}{2}} + 3\frac{A}{C^\alpha}\frac{V^{\frac{3\alpha}{2}+2}}{\frac{3\alpha}{2}+2} - 4\int a_1 dV. \tag{31}$$

We obtain the solutions by using small V approximation as

$$V = \beta^{\frac{4}{3}}(t-t_0)^{\frac{4}{3}}, \tag{32}$$

where $\beta = \frac{3}{4}\sqrt{3C}$.

The scale factors are obtained as

$$a_1 = \frac{\beta^{\frac{4}{3}}}{g^2}(t-t_0)^{\frac{4}{9}}, \tag{33}$$

$$a_2 = g(t-t_0)^{\frac{4}{9}}, \tag{34}$$

where g is integration constant.

The anisotropy parameter has zero value whereas the deceleration parameter has value $q \sim 0.39$.

3. Bianchi-I Model

The line element for the Bianchi-I representing anisotropic flat space is given by

$$ds^2 = -dt^2 + [R_i(t)dx^i]^2, \tag{35}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 August 2023 doi:10.20944/preprints202308.0013.v1

where i=1, 2, 3. We choose $R_1 = a_1$ and $R_2 = R_3 = a_2$.

The EFE for this metric with MCG as source can be written as

$$\frac{\dot{a_2}^2}{a_2^2} + 2\frac{\dot{a_1}\dot{a_2}}{a_1 a_2} = \rho, \tag{36}$$

$$\frac{\dot{a_2}^2}{a_2^2} + 2\frac{\ddot{a_2}}{a_2} = \frac{A}{\rho^\alpha} - B\rho, \tag{37}$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2} = \frac{A}{\rho^\alpha} - B\rho. \tag{38}$$

After some algebraic manipulation, we get from the above equations

$$2\frac{\ddot{a_1}}{a_1} + 2\frac{\dot{a_1}\dot{a_2}}{a_1 a_2} - \frac{\dot{a_2}^2}{a_2^2} = \frac{A}{\rho^\alpha} - B\rho. \tag{39}$$

From Eqs. (30) and (33) we get

$$\frac{d(VH_1)}{dt} = \frac{(1-B)}{2}CV^{-B} + \frac{A}{2C^\alpha}V^{\alpha(1+B)+1}. \tag{40}$$

Again, from Eqs. (30) and (31) we obtain

$$\frac{d(VH_2)}{dt} = \frac{(1-B)}{2}CV^{-B} + \frac{A}{2C^\alpha}V^{\alpha(1+B)+1}. \tag{41}$$

From the above obtained EFEs in terms of volume, we find

$$\ddot{V} = \frac{3}{2}(1-B)CV^{-B} + \frac{3}{2}\frac{A}{C^\alpha}V^{\alpha(1+B)+1}. \tag{42}$$

This gives after integration

$$\dot{V} = \sqrt{3CV^{1-B} + 3\frac{A}{C^\alpha}\frac{V^{\alpha(1+B)+2}}{\alpha(1+B)+2}}. \tag{43}$$

It can be observed that, for the BI universe also, the solutions for the volume and scale factors depend on the MCG parameters α, A and B and as such the dependence on the B-parameter is substantial. Here also, we consider the three different values of the parameter B to assess its effect on the exact solutions.

Case-1: $B = 1$.

In small V limit, the above differential equation reduces to

$$\dot{V} = \sqrt{3C}. \tag{44}$$

This gives the solution

$$V = ae^{\sqrt{3C}t},\ a_1 = \frac{a}{b^2}e^{\sqrt{3C}t},\ a_2 = be^{\sqrt{3C}t}, \tag{45}$$

where both a and b are constants.

Here the anisotropy parameter is evaluated to be zero. On the other hand, the deceleration parameter has value $q = -1$.

Case-2: $B = 0$.

In small V limit, Eq. (37) reduces to

$$\dot{V}^2 = 3CV. \tag{46}$$

The solution is of the form

$$V = \frac{3C}{4}(t - t_0)^2. \tag{47}$$

The scale factors can be found to be

$$a_1 = \frac{3C}{4c^2}(t - t_0)^{\frac{2}{3}}, a_2 = c(t - t_0)^{\frac{2}{3}}, \tag{48}$$

where c is constant of integration. The anisotropy and the deceleration parameters are respectively turn out to be zero and $q = 0.5$.

Case-3: $B = \frac{1}{2}$.

In this case, we have

$$\dot{V}^2 = 3CV^{\frac{1}{2}} + 3\frac{A}{C^\alpha}\frac{V^{\frac{3\alpha}{2}+2}}{\frac{3\alpha}{2}+2}. \tag{49}$$

We obtain the solutions by using small V approximation as

$$V = \beta^{\frac{4}{3}}(t - t_0)^{\frac{4}{3}}, \tag{50}$$

where $\beta = \frac{3}{4}\sqrt{3C}$.

The scale factors are obtained as

$$a_1 = \frac{\beta^{\frac{4}{3}}}{g^2}(t - t_0)^{\frac{4}{9}}, \tag{51}$$

$$a_2 = g(t - t_0)^{\frac{4}{9}}, \tag{52}$$

where g is integration constant.

The anisotropy parameter is found to have zero value whereas the deceleration parameter turns out to be $q \sim 0.39$.

4. Discussion and Conclusion

In this paper, we have studied two anisotropic universes with MCG as matter source. We see that the solutions for the volume and scale factors depend on the MCG parameters α, A and B. The dependence on the B-parameter is found to be the strongest, so we choose to vary it for obtaining different solutions.

The solutions are identical for $B = \frac{1}{2}$ for both the universes. However, for the rest of the two cases, the solutions differ significantly. For both the models there is a large initial anisotropy which washes out over a small period of time as isotropization starts quickly to produce the homogeneity and thus an isotropic universe observed on large scales.

For $B = 0$ we find that the KS universe does not give an analytical solution while for Bianchi-I universe we get an analytical solution. However, we can obtain an approximate behaviour of the volume and scale factors in the asymptotic limit which is similar to that for the Bianchi-I universe.

The initial anisotropy decays out with time and rate of isotropization for Bianchi-I model is almost identical to the rate for KS model in asymptotic limit. It may, however, be noted that after elapsing of some time this isotropy has been disappeared but again regained at later times to give the observed isotropic universe. This may be physically interesting as the losing and regaining of isotropy at the intermediate and later times, respectively, may correspond to the process of structure formation.

The most significant difference in behaviour between the two models is for $B = 1$ case. In this case, for the KS universe we find that unlike all other cases, the anisotropy parameter is a non-zero constant indicating that it does not vanish at any later time. The large initial anisotropy is retained significantly to prevent isotropization. On the contrary, for the Bianchi-I model, not only do we get a vanishing anisotropic parameter indicating isotropic universe, but also the solutions with de Sitter like behaviour. Interestingly, although we have considered presence of no Cosmological Constant Λ in our study, the constant C involving all the three MCG parameters behaves effectively as $\frac{\Lambda}{9} \sim \Lambda_{effective}$.

For $B = 1$, in the case of the KS universe, besides remaining anisotropy, we get $q = 2$ which gives a closed universe decelerating fast enough to collapse whereas for the Bianchi-l universe we get $q = -1$ providing an accelerating early universe in confirmity with the effective Λ behaviour. On the other hand, for $B = 0$, we get a flat matter dominated Bianchi-l universe with $q = +0.5$, while for $B = 1/2$ we get both the KS and Bianchi-l solutions with q value 0.39 giving open, forever expanding universes. It is to be noted that theoretically Mishra et al. [45] in their model have obtained the lowest values of q at -0.495 and -0.558 whereas highest values as 0.4 and 0.2. The other theoretical reported value of the deceleration parameter lies nearly $q = -1$ [46,47] to $q > -1$ [48]. However, the observational constraints as set upon the parameter in the present epoch from type Ia supernova and X-ray cluster gas mass fraction measurements is $q = -0.81 \pm 0.14$ [49] whereas that from $H(z)$ and SN-Ia data to be $q = -0.34 \pm 0.05$ [50]. Therefore, our obtained value seems acceptable one as far as the accelerating early universe is concerned.

Thus, there are a number of very interesting results that appear from our study which help us to compare and contrast the two different anisotropic models along with proper understanding of the role of MCG as matter source in these models.

Author Contributions: Conceptualization, S.R. and R.S.; writing—original draft preparation, S.R. and R.S.; writing—review and editing, S.K.T., B.B. and S.M.R. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Acknowledgments: SR and SKT are thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing the Visiting Associateship under which a part of this work was carried out. SR also acknowledged the facilities under ICARD at CCASS, GLA University, Mathura, India.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Antoniou, I.; Perivolaropoulos, L. Searching for a Cosmological Preferred Axis: Union2 data analysis and comparison with other probes. *J. Cosmol. Astropart. Phys.* **2010**, *12*, 012. [CrossRef]
2. Colin, J.; Mohayaee, R.; Rameez, M.; Sarkar, S. Evidence for anisotropy of cosmic acceleration. *Astron. Astrophys.* **2019**, *631*, L13. [CrossRef]
3. Fosalba, P.; Gaztanaga, E. Explaining cosmological anisotropy: Evidence for causal horizons from CMB data. *Mon. Not. R. Astron. Soc.* **2021**, *504*, 5840-5862. [CrossRef]
4. Planck Collaboration et al., Planck 2018 results: VII. Isotropy and statistics of the CMB. *Astron. Astrophys.* **2020**, *641*, A7. [CrossRef]
5. Tripathy, S.K. Late time acceleration and role of skewness in anisotropic models. *Astrophys. Space Sci.* **2014**, *350*, 367–374. [CrossRef]
6. Schwarz, D.J.; Copi, C.J.; Huterer, D.; Starkman, G.D. CMB anomalies after Planck, *Class. Quant. Grav.* **2016**, *33*, 184001. [CrossRef]

7. Misner, C.W. The Isotropy of the Universe, *Astrophys. J.* **1968**, *151*, 431–457. [CrossRef]
8. Kanotowski, R.; Sachs, R.K. Some Spatially Homogeneous Anisotropic Relativistic Cosmological Models, *J. Math. Phys.* **1966**, *7*, 443–446. [CrossRef]
9. Bianchi, L. *Mem. Mat. Fis. Soc. Ita. Sci., Ser. Ter., Tom.* **1898** *XI*, 267–352; [On three dimensional spaces which admit a group of motions, English translation: Gen. Relativ. Gravit. **2001**, *33*, 2157-2253]. [CrossRef]
10. Taub, A.H. Empty space-times admitting a three parameter group of motions, *Ann. Math.* **1951**, *53*, 472–490; Gen. relativ. Gravit. **2004**, *36*, 2699–2719. [CrossRef]
11. Heckman, O.; Schücking, E. **1962**, in *Relativistic Cosmology in Gravitation: an introduction to current research*, ed., L. Witten (1962) (New York and London: J. Wiley).
12. Ellis, G.F.R.; MacCallum, M.A.H. A class of homogeneous cosmological models, *Comm. Math. Phys.* **1969**, *12*, 108–141. [CrossRef]
13. Huang, W. Anisotropic Cosmological Models with Energy Density Dependent Bulk Viscosity, *J. Math. Phys.* **1990**, *31*, 1456–1462. [CrossRef]
14. Chimento, L.P.; et al. Cosmological solutions with nonlinear bulk viscosity, *Class. Quantum Gravit.* **1997**, *14*, 3363–3375. [CrossRef]
15. Saha, B. Anisotropic cosmological models with perfect fluid and dark energy reexamined, *Int. J. Theor. Phys.* **2006**, *45*, 983–995. [CrossRef]
16. Saha, B. Anisotropic Cosmological Models with a Perfect Fluid and a Term, *Astrophys. Space Sci.* **2006**, *302*, 83–91. [CrossRef]
17. Sengupta, R.; Paul, P.; Paul, B.C.; Ray, S. Inflation in anisotropic brane universe using tachyon field, *Int. J. Mod. Phys. D* **2019**, *28*, 1941010. [CrossRef]
18. Collins, C.B. Global structure of the "Kantowski–Sachs" cosmological models, *J. Math. Phys.* **1977**, *18*, 2116–2124. [CrossRef]
19. Weber, E. Kantowski–Sachs cosmological models approaching isotropy, *J. Math. Phys.* **1984**, *25*, 3279–3285. [CrossRef]
20. Linde, A.D. Initial conditions for inflation, *Phys. Lett. B* **1985**, *162*, 281–286. [CrossRef]
21. Bradley, M.; Dunsby,P.K.S.; Forsberg, M.; Keresztes, Z. Perturbations of Kantowski-Sachs models, *Class. Quantum Gravit.* **2012**, *29*, 095023.
22. Kamenshchik, A.; Moschella, U.; Pasquier, V. An alternative to quintessence. *Phys. Lett. B* **2001**, *511*, 265-268. [CrossRef]
23. Bento, M.C.; Bertolami, O.; Sen, A.A. Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification. *Phys. Rev. D* **2002**, *66*, 043507. [CrossRef]
24. Pourhassan, B.; Kahya, E.O. Extended Chaplygin gas model. *Res. Phys.* **2014**, *4*, 101. [CrossRef]
25. Hulke, N.; Singh, G.P.; Bishi, B.K.; Singh, A. Variable Chaplygin gas cosmologies in $f(R,T)$ gravity with particle creation. *New Astron.* **2020**, *77*, 101357. [CrossRef]
26. Bilic, N.; Tupper, G.B.; Viollier, R. Dark Matter, Dark Energy and the Chaplygin Gas. arXiv: astro-ph/0207423. [CrossRef]
27. Benaoum, H.B. Modified Chaplygin Gas Cosmology. *Adv. High Energy Phys.* **2012**, *2012*, 357802. [CrossRef]
28. Benaoum, H.B. Modified Chaplygin Gas Cosmology with Bulk Viscosity in D Dimensions. *Int. J. Mod. Phys. D* **2014**, *23*, 1450082. [CrossRef]
29. Bandyopadhyay, T. Thermodynamics of Gauss-Bonnet brane with modified Chaplygin gas. *Astrophys. Space Sci.* **2012**, *341*, 689-693. [CrossRef]
30. Panigrahi, D.; Chatterjee, S. Viability of Variable Generalised Chaplygin gas - a thermodynamical approach. *Int. J. Mod. Phys. D* **2012**, *21*, 1250079. [CrossRef]
31. Wu, Y.-B.; Li, S.; Lu, J.-B.; Yang, X.-Y. The Modified Chaplygin gas as a unified dark sector model. *Mod. Phys. Lett. A* **2007**, *22*, 783-790. [CrossRef]
32. Bilic, N.; Tupper, G.B.; Violler, R. Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas. *Phys. Lett. B* **2002**, *535*, 17-21. [CrossRef]
33. Liu, D.-J.; Li, X.-Z. Cosmic Microwave Background Radiation constraints on a modified Chaplygin gas model. *Chin. Phys. Lett.* **2005**, *22*, 1600. [CrossRef]
34. Lixin, X.; Yuting, W.; Hyerim, N. Modified Chaplygin gas as a unified dark matter and dark energy model and cosmic constraints. *Eur. Phys. J. C* **2012**, *72*, 1931. [CrossRef]

35. Matarrese, S.; Colpi, M.; Gorini, V.; Moschella, U. Ed. *Dark Matter and Dark Energy: A challenge for modern cosmology* (Springer, 2010).
36. Jawad, A.; Ilyas, A.; Rani, S. Dynamics of Modified Chaplygin gas inflation on the Brane with bulk viscous pressure. *Int. J. Mod. Phys. D* **2017**, *26*, 1750031. [CrossRef]
37. Safsafi, A.; Khay, I.; Salamate, F.; Chakir, H.; Bennai, M. *Adv. High Energy Phys.* **2018**, *73*, 1. [CrossRef]
38. P.T. Silva and O. Bertolami, Expected constraints on the generalized Chaplygin equation of state from future supernova experiments and gravitational lensing statistics. *Astron. Astrophys.* **2003**, *599*, 829-838. [CrossRef]
39. Dev, A.; Jain, D.; Alcaniz, J.S. Constraints on Chaplygin quartessence from the CLASS gravitational lens statistics and supernova data. *Astron. Astrophys.* **2004**, *417*, 847-852. [CrossRef]
40. Bertolami, O.; Silva, P.T. Gamma-ray bursts as dark energy–matter probes in the context of the generalized Chaplygin gas model. *Mon. Not. R. Astron. Soc.* **2006**, *365*, 1149-1159. [CrossRef]
41. Thakur, P.; Ghosh, S.; Paul, B.C. Modified Chaplygin gas and constraints on its B parameter from cold dark matter and unified dark matter energy cosmological models. *Mon. Not. R. Astron. Soc.* **2009**, *397*, 1935-1939. [CrossRef]
42. Chakraborty, S.; Debnath, U.; Ranjit, C. Observational constraints of Modified Chaplygin gas in loop quantum cosmology. *Eur. Phys. J. C* **2012**, *72*, 2101. [CrossRef]
43. Alvarado, R. *Adv. Stud. Theor. Phys.* **2016**, *10*, 421.
44. Tripathy, S.K.; Behera, D.; Mishra, B. Unified dark fluid in Brans–Dicke theory. *Eur. Phys. J. C* **2015**, *75*, 149. [CrossRef]
45. Mishra, B.; Esmaeili, F.MD.; Ray, S. Cosmological models with variable anisotropic parameter in $f(R,T)$ gravity. *Ind. J. Phys.* **2021**, *95*, 2245-2254. [CrossRef]
46. Weinberg, S. *Cosmology* (Oxford Univ. Press, 2008).
47. Tripathy, S,K,; Mishra, B. Phantom cosmology in an extended theory of gravity. *Chin. J. Phys.* **2020**, *63*, 448-458. [CrossRef]
48. Mukhopadhyay, U.; Ray, S.; Usmani, A.A.; Ghosh, P.P. Time variable Λ and the accelerating universe. *Int. J. Theor. Phys.* **2011**, *50*, 752-759. [CrossRef]
49. Rapetti, D.; Allen, S.W.; Amin, M.A.; Blandford, R.D. A kinematical approach to dark energy studies. *Mon. Not. R. Astron. Soc.* **2007**, *375*, 1510-1520. [CrossRef]
50. Kumar, S. Observational constraints on Hubble constant and deceleration parameter in power-law cosmology. *Mon. Not. R. Astron. Soc.* **2012**, *422*, 2532-2538. [CrossRef]