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Article

Troubles with Gravitational Frequency Shift Derived from Energy Conservation

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Abstract: In physics, thought experiments are impressive heuristic tools. They are valuable instruments to help scientists find new results and to teach students the known ones. However, as we shall show, they should always be received with prudence, even when they are a shortcut to 'prove' well-established results. Here, we show that the most widely known thought experiments devised to derive the gravitational frequency shift from energy conservation are, in fact, problematic. When properly set and correctly read, those thought experiments reveal that the existence of the gravitational frequency shift is, in fact, at odds with energy conservation. We also propose two new simple thought experiments, one using the conservation of energy and the other the conservation of linear momentum, that corroborate that conclusion, showing that those conservation principles do not imply the gravitational frequency shift. We think that our results may be of some epistemological interest and could serve as a warning sign on how thought experiments should be received and trusted.

Keywords: special relativity; general relativity; gravitational frequency shift; conservation of energy; linear momentum conservation; thought experiments

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1. Introduction

In 1907, Einstein introduced the equivalence principle [1]. He used it to 'extrapolate' the effects of special relativity to systems at rest in a gravitational field via their alleged equivalence to uniformly accelerated systems. In that paper, Einstein first derived the gravitational redshift, the gravitational time dilation, and other effects of gravity on electromagnetic processes, like the variable velocity of light and the gravitational light deflection.

His first attempt to extend special relativity to gravitation was, according to Einstein himself, not particularly satisfying, and he returned to the topic in 1911, providing a much simpler derivation of the gravitational time dilation, redshift, and light deflection. Let us briefly review this second derivation of the gravitational redshift [2].

Consider two material systems, S_1 and S_2, at rest in a local, uniform gravitational field $\mathbf{a}$ (Figure 1). S_1 and S_2 are separated by a distance d. Consider further a reference frame K_0. System K_0 is a free-falling (gravitation-free) system located near S_2 with an initial instantaneous velocity relative to S_2 equal to zero.

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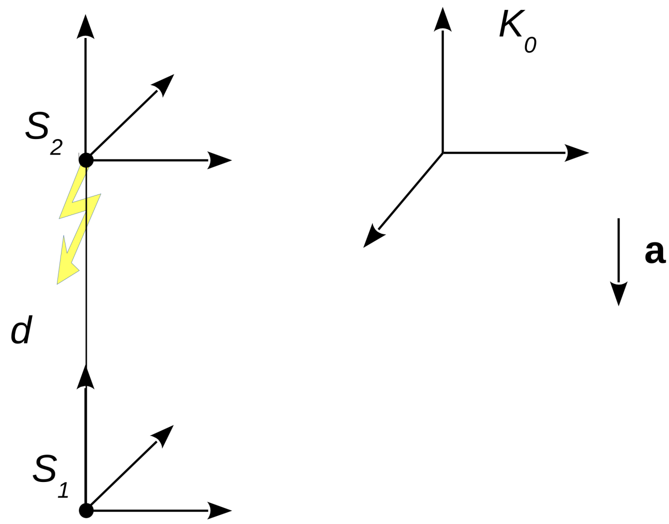

Figure 1. Material systems S_1 and S_2 are at rest in a local, uniform gravitational field $\mathbf{a}$. The reference frame K_0 is a free-falling (gravitation-free) system located near S_2 with zero initial velocity relative to S_2. According to the equivalence principle, this is equivalent to systems S_1 and S_2 accelerating upward with acceleration $-\mathbf{a}$ and frame K_0 being inertial and at rest.

Suppose further that a ray of light of frequency ν_2 is emitted by S_2 towards S_1 when the relative velocity of the free-falling frame K_0 with respect to S_2 and S_1 is still equal to zero. The ray of light reaches S_1 after a time nearly equal to d/c. According to the principle of equivalence, this situation is physically equivalent to one in which K_0 is at rest, and S_2 and S_1 accelerate with acceleration $-\mathbf{a}$ and initial velocity equal to zero. When the ray of light arrives at S_1, the velocity of S_1 relative to the stationary frame K_0 is equal to $v = ad/c$. Therefore, *in the view of any observer in frame* K_0, the ray of light received at S_1 has a frequency ν_1 as follows

$$\nu_1 = \nu_2 \left(1 + \frac{v}{c}\right) = \nu_2 \left(1 + \frac{ad}{c^2}\right), \tag{1}$$

where the second term is the Doppler formula for $v \ll c$.

For ad, Einstein substituted the gravitational potential Φ of S_2, that of S_1 is taken as zero, and assumed that the relation (1), deduced for a homogeneous gravitational field, would also hold for other forms of field. Then, Einstein arrived at the well-known (approximated) formula for the gravitational redshift (in this example, it is actually a blueshift)

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right). \tag{2}$$

From this formula, Einstein also derived the gravitational time dilation formula. Suppose that, during the time interval Δt_2 (as measured by a clock at rest at S_2), S_2 emits n waves. Then, from the definition of frequency, we have $n = \nu_2 \Delta t_2$. Let S_1 receive these same n waves during the time interval Δt_1 (as measured by a clock at rest at S_1). Then, again, according to the definition of frequency, we have $n = \nu_1 \Delta t_1 = \nu_2 \Delta t_2$. Hence, equation (2) leads to the gravitational time dilation formula

$$\Delta t_2 = \Delta t_1 \left(1 + \frac{\Phi}{c^2}\right). \tag{3}$$

In section 2 of the 1911 paper, Einstein showed that, in general, energy is affected by a gravitational field and that, like the inertial mass, the gravitational mass of a body increases by E/c^2 when the body absorbs an amount of energy equal to E. In that derivation, the setup is the same as in Figure 1.

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Einstein used the approximated relativistic energy transformation law $E_1 = E_2\left(1 + \frac{v}{c}\right)$ [3] and, again, the equivalence principle. Moreover, by devising a clever thought experiment, he proved the following:

> [...] hence, energy must possess a *gravitational* mass which is equal to its *inertial* mass. If a mass M_0 is suspended from a spring balance in the system K' [the system moving with acceleration $-a$], the balance will indicate the apparent weight $M_0 a$ because of the inertia of M_0. If the quantity of energy E is transferred to M_0, the spring balance will indicate $\left(M_0 + \frac{E}{c^2}\right) a$, in accordance with the principle of the inertia of energy. According to our basic assumption [the principle of equivalence], exactly the same thing must happen if the experiment is repeated in the system K, i.e., in the gravitational field. [emphasis in the original]

It cannot escape us that the discovery that energy is affected by a gravitational field, together with the quantum formula for the energy of a photon of frequency ν, $E = h\nu$, and some algebra, again gives the relation (2) for the gravitational frequency shift. However, although Einstein's discovery of mass and energy dependence on gravitation is a crucial assumption, and his idealized experiment inspired the subsequent thought experiments analyzed in this paper, this last derivation does not strictly count as a derivation of the gravitational frequency shift from energy conservation. In fact, it is still a derivation from special relativity and the principle of equivalence. Instead, the typical (archetypal) derivations from energy conservation can be found, for instance, in the books by Born [4], Feynman, Leighton, and Sands [5], Weinberg [6], Misner, Thorne, and Wheeler [7], Rindler [8], and Schutz [9] (and many minor physics textbooks).

In the following section, we first explicitly list all the assumptions necessary for the derivation from energy conservation and outline the typical thought experiment widely used in literature, which extends those assumptions to the photon. We briefly discuss the criticism of these kinds of derivation advanced, among others, by Weinberg [6] and Okun, Selivanov, and Telegdi [10]. We agree with their reservations in applying the mentioned assumptions to the photon. However, we recall that a derivation from energy conservation that does not require these assumptions for the photon is possible. As representatives of such derivations, we analyze in more detail the thought experiments presented in [5], [6], and [7]. We show that, when revised and corrected, they, too, appear unfortunately troublesome for the very existence of the gravitational frequency shift.

That early attempts for a clear derivation of the gravitational redshift were fraught with errors and ambiguities is not new, see Scott [12], but here we can say more. In section 3, we prove that, in fact, energy conservation does not imply gravitational frequency shift. We do that with a simple thought experiment that does not need to assume any gravitational mass or gravitational potential energy of the photon. In that proof, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body. In the Appendix, we also present a more direct thought experiment that uses the conservation of linear momentum and arrives at the same conclusion.

In the last section, we briefly recap the significance of our results.

2. Gravitational frequency shift and the conservation of energy

The derivation of the gravitational frequency shift from energy conservation is generally seen as a confirmation of that phenomenon alternative to (and independent of) the classical derivation from special relativity and the principle of equivalence, and it can be found in many textbooks on general relativity.

Let us first list all the premises and commonly held beliefs explicitly or tacitly assumed in the derivation from energy conservation. They are crucial for the acceptance of its physical validity:

1) Not only can mass be converted into energy, but *every kind of energy* has mass as well (or can always be converted into mass) via the mass-energy equivalence formula $E = m_0 c^2$, where m_0 is the rest mass [1,2];

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2) Inertial mass is equivalent to gravitational mass;
3) The energy of a light photon with frequency ν is $E = h\nu$, where h is the Planck's constant;
4) The principle of conservation of energy.

An example of derivation is the following. An 'infinitesimal' version of it can be found, for instance, in the book by Rindler [8]. A receiver R is placed straight above an emitter of photons E at a distance d. Both are stationary in a uniform gravitational field g. The emitter E emits a photon of frequency ν, and energy $E = h\nu$, towards R. Photons do not have rest mass, but for the sake of derivation, it is assumed that the emitted photon has an 'effective' gravitational mass m equal to its inertial mass obtained from the mass-energy equivalence, $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$ (assumptions 1, 2, and 3). Since the emitted photon needs to climb a height d in the uniform gravitational field, its energy E' at the receiver R is lower than E. Due to the conservation of energy (assumption 4), we necessarily have that

$$E' = E - mgd, \tag{4}$$

where the potential energy mgd is the energy 'spent' by the photon climbing the distance d.

Equation (4) can be rewritten as follows,

$$\nu' = \frac{E'}{h} = \frac{E - mgd}{h} = \frac{h\nu - \frac{h\nu}{c^2}gd}{h} = \nu\left(1 - \frac{gd}{c^2}\right), \tag{5}$$

which is the sought-out gravitational frequency shift formula (1) (if the positions of E and R are reversed, the minus sign becomes a plus sign in the equation).

As far as this author knows, that type of derivation received few criticisms, with some notable exceptions like Weinberg, who affirms that the concept of gravitational potential energy for a photon is without foundation [6]. Like Weinberg, Okun, Selivanov, and Telegdi argue that any explanation of the gravitational frequency shift in terms of gravitational mass and gravitational potential energy of the photon is incorrect and misleading [10].

We fully agree with these authors since the photon has no rest mass, and the appeal to its gravitational mass and potential energy is a loose and not legitimate argument. However, it is possible to come up with a gravitational frequency shift derivation from energy conservation that does not appeal to those concepts like, for instance, the derivations in Feynman, Leighton, and Sands [5], Weinberg [6], Misner, Thorne, and Wheeler [7], Schutz [9], Koks [11], and Earman and Glymour [13].

As a representative of such derivations, consider first the thought experiment by Misner, Thorne, and Wheeler. They recount Einstein's 1911 realization of the interaction between light and gravity as follows (the speed of light is set as $c = 1$):

> That a photon must be affected by a gravitational field Einstein (1911) showed from the law of conservation of energy, applied in the context of Newtonian gravitation theory. Let a particle of rest mass m start from rest in a gravitational field g at point $\mathcal{A}$ and fall freely for a distance h to point $\mathcal{B}$. It gains kinetic energy mgh. Its total energy, including rest mass, become
>
> $$m + mgh.$$
>
> Now, let the particle undergo an annihilation at $\mathcal{B}$, converting its total rest mass plus kinetic energy into a photon of the same energy. Let this photon travel upward in the gravitational field to $\mathcal{A}$. If it does not interact with gravity, it will have its original energy on arrival at $\mathcal{A}$. At this point it could be converted by a suitable apparatus into another particle of rest mass m (which could then repeat the whole process) plus an excess energy mgh that costs nothing to produce. To avoid this contradiction of the principal [*sic*] of conservation of energy, which can also be stated in purely classical terms, Einstein saw that the photon must suffer a red shift.

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In this derivation, nowhere reference is made to the gravitational mass or the gravitational potential energy of the photon. Energy has a mass only after absorption by a non-relativistic and macroscopic material body (the apparatus that converts it into a particle in the last part of the process). That is allowed by the widely-held interpretation of the mass-energy equivalence.

Unfortunately, even Misner, Thorne, and Wheeler's argument is problematic [16]. If a particle of rest mass m starts from rest in a gravitational field g at point $\mathcal{A}$ and falls freely for a distance h to point $\mathcal{B}$, that particle possesses also an energy equal to mgh already at point $\mathcal{A}$. It is called gravitational potential energy. Therefore, owing to mass-energy equivalence (assumption 1; see also Appendix A), at point $\mathcal{A}$, that particle already has a total mass/energy equal[1] to $m + mgh$. Now, if the energy of the photon produced in the particle annihilation at point $\mathcal{B}$ and traveling upward does not have its original value on arrival at $\mathcal{A}$ (i.e., $m + mgh$), the mass of the particle created by the suitable apparatus at the end of the process would not have the same mass as the original particle (again, $m + mgh$), and the total energy/mass would not be conserved. When Misner, Thorne, and Wheeler say that the particle "gains kinetic energy mgh" on arrival at point $\mathcal{B}$, and "its total energy, including rest mass, becomes $m + mgh$", they seem to forget that the particle already has gravitational potential energy mgh, and total energy $m + mgh$, just before starting to fall. That is demanded by the principle of conservation of energy. The same analysis with a few adjustments also applies to the derivations in Schutz [9] and Koks [11] with the same conclusion.

Even if Misner, Thorne, and Wheeler do not explicitly mention the Planck-Einstein formula $E = h\nu$, the fact that the energy can be converted into a single photon or a finite (and definite) number of photons is a tacit but important further assumption. For if it were possible to convert energy into light in a 'continuous' way, the conservation of energy could still be re-established: in principle, if the emitter at point $\mathcal{B}$ continuously emitted a higher frequency radiation (higher intrinsic energy) for an interval Δt and the receiver at point $\mathcal{A}$ continuously received a lower frequency radiation (lower intrinsic energy) for a suitably longer interval $\Delta t' > \Delta t$, the total amount of energy could still be conserved (and the gravitational time dilation would necessarily get back into the game). However, the quantization of energy in light transmission has solid theoretical and experimental corroboration.

Weinberg presented a derivation from energy conservation slightly different from that given by Misner, Thorne, and Wheeler but equally problematic. It could be of some interest to go into detail. Weinberg writes (again, the speed of light is set as $c = 1$) [6]:

> Incidentally, the gravitational red shift of light rising from a lower to a higher gravitational potential can to some extent be understood as a consequence of quantum theory, energy conservation, and the "weak" Principle of Equivalence. When a photon is produced at point 1 by some heavy nonrelativistic apparatus, an observer in a locally inertial coordinate system moving with the apparatus will see its internal energy and hence its inertial mass change by an amount related to the photon frequency ν_1 he observes, that is, by
>
> $$\Delta m_1 = -h\nu_1$$
>
> where $h = 6.625 \times 10^{-27}$ erg sec is Planck's constant. Suppose that the photon is then absorbed at point 2 by a second heavy apparatus; an observer in a freely falling system will

[1] If assumption 1 holds, it can be shown that, in a uniform gravitational field g, the mass m_h of a particle at height h is $m_h = me^{\frac{gh}{c^2}}$, where m is the proper mass at the height taken as zero. The total energy E_{tot}, proper mass plus gravitational potential energy, at height h is given by $E_{tot} = mc^2e^{\frac{gh}{c^2}}$. For small distances h, we have $m_h \simeq m + \frac{mgh}{c^2}$ and $E_{tot} \simeq mc^2 + mgh$ (a similar result is also present in [10]). By assuming $c = 1$, like in [7], we have that the mass and total energy of the particle at the height h (point $\mathcal{A}$) are $m + mgh$.

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see the apparatus change in inertial mass by an amount related to the photon frequency ν_2 he observes, that is, by

$$\Delta m_2 = h\nu_2$$

However, the total internal plus gravitational potential energy of the two pieces of apparatus must be the same before and after these events, so

$$0 = \Delta m_1 + \phi_1 \Delta m_1 + \Delta m_2 + \phi_2 \Delta m_2$$

and therefore

$$\frac{\nu_2}{\nu_1} = \frac{1+\phi_1}{1+\phi_2} \simeq 1 + \phi_1 - \phi_2$$

in agreement with our previous result. (Also, it makes no difference whether the photon frequencies are measured in locally inertial systems, because the gravitational field in any other frame will affect the rate of the observer's standard clock in the same way as it affects the ν's.)

Leaving aside the reference to the free-falling observer who will necessarily see a Doppler shift due to the motion relative to the stationary emitting apparatus, a thing that, in the humble opinion of this author, unnecessarily complicates the picture, Weinberg's derivation seems to violate the conservation of energy just from the beginning. First, he states that, upon the photon emission, the apparatus will change its *internal* energy by an amount $h\nu_1 = |\Delta m_1|$. But, then, he says that the variation of the *total energy* of the apparatus to consider upon emission is $|\Delta m_1 + \phi_1 \Delta m_1|$. Namely, the apparatus emits energy equal to $|\Delta m_1|$, but its total energy variation is $|\Delta m_1 + \phi_1 \Delta m_1| \neq |\Delta m_1|$. That already represents a violation of energy conservation. If we reestablish the conservation of energy ($|\Delta m_{1/2} + \phi_{1/2} \Delta m_{1/2}| = |h\nu_{1/2}|$), no gravitational frequency shift is implied.

A similar issue in the application of energy conservation also affects the derivation in Feynman, Leighton, and Sands [5]. They write:

> We know that the gravitational force on an object is proportional to its mass M, which is related to its total internal energy E by $M = E/c^2$. For instance, the masses of nuclei determined from the energies of nuclear reactions which transmute one nucleus into another agree with the masses obtained from atomic weights.
>
> Now think of an atom which has a lowest energy state of total energy E_0 and a higher energy state E_1, and which can go from the state E_1 to the state E_0 by emitting light. The frequency ω of the light will be given by
>
> $$\hbar\omega = E_1 - E_0 \tag{42.7}$$
>
> Now suppose we have such an atom in the state E_1 sitting on the floor, and we carry it from the floor to the height H. To do that we must do some work in carrying the mass $m_1 = E_1/c^2$ up against the gravitational force. The amount of work done is
>
> $$\frac{E_1}{c^2} gH \tag{42.8}$$
>
> Then we let the atom emit a photon and go into the lower energy state E_0. Afterward we carry the atom back to the floor. On the return trip the mass is E_0/c^2; we get back the energy
>
> $$\frac{E_0}{c^2} gH, \tag{42.9}$$
>
> so we have done a net amount of work equal to

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$$\Delta U = \frac{(E_1 - E_0)}{c^2} gH. \tag{42.10}$$

When the atom emitted the photon it gave up the energy $E_1 - E_0$. Now suppose that the photon happened to go down to the floor and be absorbed. How much energy would it deliver there? You might at first think that it would deliver just the energy $E_1 - E_0$. But that can't be right if energy is conserved, as you can see from the following argument. We started with the energy E_1 at the floor. When we finish, the energy at the floor level is the energy E_0 of the atom in its lower state plus the energy E_{ph} received from the photon. In the meantime we have had to supply the additional energy ΔU of Eq. (42.10). If energy is conserved, the energy we end up with at the floor must be greater than we started with by just the work we have done. Namely, we must have that

$$E_{ph} + E_0 = E_1 + \Delta U,$$

or

$$E_{ph} = (E_1 - E_0) + \Delta U. \tag{42.11}$$

It must be that the photon does not arrive at the floor with just the energy $E_1 - E_0$ it started with, but with a *little more energy*. Otherwise some energy would have been lost. If we substitute in Eq. (42.11) the ΔU we got in Eq. (42.10) we get that the photon arrives at the floor with the energy

$$E_{ph} = (E_1 - E_0)\left(1 + \frac{gH}{c^2}\right). \tag{42.12}$$

But a photon of energy E_{ph} has the frequency $\omega = E_{ph}/\hbar$. Calling the frequency of the emitted photon ω_0—which is by Eq. (42.7) equal to $(E_1 - E_0)/\hbar$—our result in Eq. (42.12) gives again the relation of (42.5) between the frequency of the photon when it is absorbed on the floor and the frequency with which it was emitted.

The weak link of the above inference chain appears to be the assumption (42.9). The total energy of the atom sitting on the floor is E_1. After being carried to the height H, its total energy becomes $E_1 + \frac{E_1}{c^2} gH$ (its rest energy plus the work done on the atom). With the emission of a photon of energy $\hbar\omega = E_1 - E_0$, the total energy becomes $(E_1 + \frac{E_1}{c^2} gH) - (E_1 - E_0) = E_0 + \frac{E_1}{c^2} gH$. According to the conservation of energy, that total energy must be conserved after the atom is carried back to the floor. Now, if we subtract the new rest energy E_0 of the atom from this total energy, we get back the correct energy, $\frac{E_1}{c^2} gH$, and the net amount of work we have done is $\Delta U = 0$. Therefore, according to equation (42.11), the photon must arrive at the floor with just the energy $E_1 - E_0$ it started with at the height H.

3. Proof that energy conservation does not imply gravitational redshift

Here, we prove that the conservation of energy does not imply gravitational frequency shift. We do that with a simple thought experiment that does not need to assume any gravitational mass or gravitational potential energy of the photon. In this proof, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body (assumption 1), as we have done in the revision of some previous demonstrations. Incidentally, it can be shown that that must always be the case (see Appendix A).

Consider a body of mass m stationary at point $\mathcal{B}$ and a macroscopic apparatus stationary at point $\mathcal{A}$ at a height h above point $\mathcal{B}$ in a gravitational field g (Figure 2). Let the apparatus perform mechanical work on body m, raising it to point $\mathcal{A}$. The work done by the apparatus is equal to mgh,

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which is also equal to the gravitational potential energy of the body m relative to point $\mathcal{B}$. Now, if the mass is lowered back to point $\mathcal{B}$ and its potential energy conventionally (and entirely) converted into electrical energy and then into a single photon of energy mgh (ultimately emitted by a beacon), the energy of the photon must always be the same while climbing up the gravitational field back to point $\mathcal{A}$. The photon energy at point $\mathcal{A}$ must still be equal to mgh. That is demanded by the conservation of energy. Through photon absorption, the apparatus must regain the same energy expended at the beginning of the cycle on m. Therefore, owing to the Planck-Einstein formula, the photon frequency must be the same at points $\mathcal{A}$ and $\mathcal{B}$.

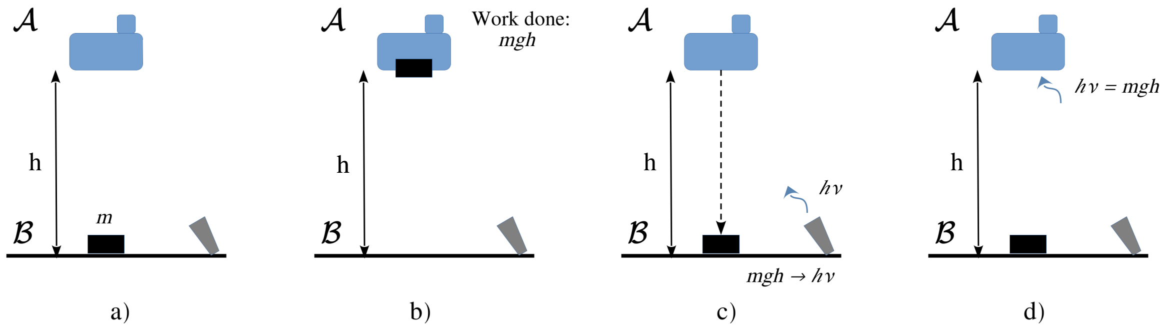

Figure 2. Pictorial representation of the thought experiment described in the text. In this case, mass-energy equivalence is not needed.

To emphasize the above conclusion, consider the cycle in reverse. The first step now consists of the crane emitting a photon of energy E' (frequency ν') suitably lower than mgh. The original energy E' is such that when the photon arrives at the beacon, it becomes equal to $E_b = mgh$ ($> E'$) owing to the standard gravitational redshift (blueshift in this case). In this way, E_b is what is exactly needed to raise the mass m to the crane at the height h. Then, the mass is released back to the initial position, and the energy coming from that release (mgh) goes into the crane reservoir. At the end of the cycle, the crane will gain positive energy ($mgh - E' > 0$) out of nowhere.

4. Concluding remarks

In physics, thought experiments can be powerful heuristic tools. However, as we have shown with the derivation of the gravitational frequency shift from energy conservation, they must be received with prudence. Sometimes, as they are conceived, they dangerously lend to being a rhetorical device to sustain what we already believe in or, at least, what seems reasonable to us from the beginning.

Our analysis and revision of, in particular, Misner, Thorne, and Wheeler's, Weinberg's, and Feynman, Leighton, and Sands's thought experiments but, above all, the proofs given in section 3 and Appendix A ultimately pit the existence of the gravitational frequency shift or, better, the dependence of the photon energy on gravitation against the conservation of energy (and linear momentum).

However, in light of the well-known and well-accepted experimental proofs of the gravitational redshift (e.g., Pound-type experiments [14,15]), it is hard to believe that what we have derived in the present paper will be felt as a confutation of the phenomenon. At any rate, we think that what we have presented may be instructive as a warning sign on how thought experiments should be received and trusted, and therefore, it could be of some epistemological interest and utility.

Acknowledgments: Although not always in agreement with them, the author is indebted to Nils Erik Bomark, Andrea Erdas, Espen Gaarder Haug, and Gianfranco Spavieri for stimulating and fruitful discussions on the topic.

Appendix A

Here, we show that the gravitational potential energy of a body contributes to the total mass of the body, as illustrated in footnote 1 of the present paper. Consider the following ideal experiment. A

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closed wagon of mass M moves horizontally without friction in a vertical uniform gravitational field g at a constant velocity v (see Figure A1). Inside the wagon, attached to the floor $\mathcal{B}$, there is a particle of mass $m_{\mathcal{B}}$. At a certain point, mass $m_{\mathcal{B}}$ annihilates into a photon of energy $h\nu_{\mathcal{B}} = m_{\mathcal{B}}c^2$. Then, the photon travels upward toward ceiling $\mathcal{A}$ and is absorbed and converted by a suitable apparatus into another particle of mass $m_{\mathcal{A}}$. That particle is also attached to the wagon frame. The whole process happens exclusively inside the closed wagon. Owing to the conservation of energy, we must have that $h\nu_{\mathcal{B}} = m_{\mathcal{A}}c^2 + m_{\mathcal{A}}gh$ but, as is widely believed, the mass of the generated particle at point A does not include the equivalent mass of its gravitational potential energy $m_{\mathcal{A}}gh/c^2$.

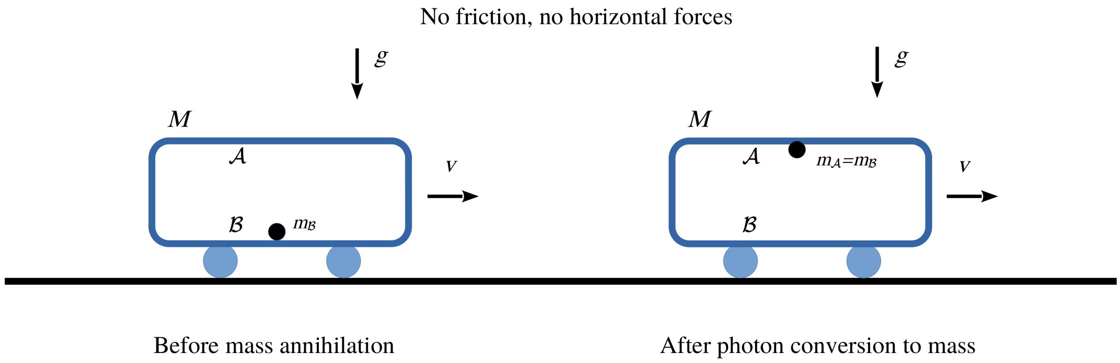

Figure A1. Pictorial representation of the thought experiment described in the Appendix.

In reality, the total mass of the particle generated at point A must be $m_{\mathcal{A}} + m_{\mathcal{A}}gh/c^2 = h\nu_{\mathcal{B}}/c^2 = m_{\mathcal{B}}$, and therefore, it must include the equivalent mass of its own gravitational potential energy. Any different scenario seems to violate the conservation of (the horizontal) linear momentum of the closed system wagon+particle. No horizontal external forces act upon the system, and no mass is ejected. Therefore, the total velocity v must be the same before and after the whole process. However, before the annihilation, the total horizontal linear momentum is $P_i = (M + m_{\mathcal{B}})v$ while, after the conversion of the photon energy into mass, the total horizontal linear momentum becomes $P_f = (M + m_{\mathcal{A}})v < P_i$. That is quite bizarre. On the other hand, by imposing the conservation of the horizontal linear momentum, we would have an equally strange consequence. Without any horizontal external force acting upon the wagon and without any mass ejection, we would see the wagon increase its velocity by itself at the end of the whole process.

Incidentally, the above argument confirms that there is no gravitational redshift with energy conservation: if the total mass of the particle generated at point $\mathcal{A}$ is still $m_{\mathcal{B}}$, the energy of the photon from which it derives is $m_{\mathcal{B}}c^2 = h\nu_{\mathcal{B}}$, namely the frequency of the photon at point $\mathcal{A}$ is the same as that at point $\mathcal{B}$, $\nu_A = \nu_B$.

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