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Article

An Improved Ambiguity-free Unfolded Co-prime Linear Array Design for DOA Estimation

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Abstract: This paper considers the direction of arrival (DOA) estimation problem of phase ambiguity for unfolded coprime linear array (UCLA). The existing most common stacking subarray-based method can tackle the phase ambiguity problem. However, the method is not always true. For a given DOA, it has its corresponding steering vector, but if there are two DOAs which have the same steering vectors with the given DOA for different subarrays, the phase ambiguity problem still arises. A modified method, which defines a decision variable and uses the classic beamforming technique to estimate the DOAs is proposed. However, this method needs additional algorithm to distinguish the true DOAs besides multiple signal classification (MUSIC) spectrum. And sometimes this method is not reliable if the decision variable is not chosen appropriately. Therefore, based on the UCLA, we reconstruct the array to design an improved UCLA called IUCLA. With moving the reference element, the linear relationship among the directional vectors can be broken. In this way, we can directly utilize the MUSIC algorithm to estimate with no additional algorithm. The proposed method can solve the phase ambiguity problem and it does not demand other technique to decrease the complexity. Moreover, the Cramer-Rao Bound (CRB), as the lower bound of the unbiased estimation, is provided and numerical simulations are given to demonstrate the effectiveness and superiority of the proposed method.

Keywords: direction of arrival estimation; improved array arrangement design; improved unfolded coprime linear array; cramer-rao bound; multiple signal classification

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1. Introduction

Direction of arrival (DOA) estimation using array processing has focused much attention in these years and has been widely applied in many fields, such as wireless communication, radar, sonar, medicine and other engineering applications [1–9]. In the past decades, numerous subspace based DOA estimation algorithms, e.g., multiple signals classification (MUSIC) [10–17] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [18–22], have been proposed for uniform linear arrays (ULA) [23,24] and attracted much attention due to their high resolution and performance. However, the adjacent antenna element spacing is required to be less than half wavelength so that the phase ambiguity problem [25] is avoided. But it limits the array aperture and consequently influences the mutual coupling [26] between the elements and the estimation performance. Therefore, a non-uniform linear array has been recently proposed to increase the degrees of freedom (DOF) of the array, known as a sparse array. The sparse array breaks the limit of half wavelength. Coprime arrays [27,28] have aroused wide attention due to the improvement of DOA estimation performance. Meanwhile, due to the larger array aperture, less mutual coupling effect arises. A coprime linear array (CLA) consists of two ULAs with the inter element spacing larger than half-wavelength, so a higher resolution, larger array aperture and less mutual coupling effect can be attained [29,30]. Recently, the coprime arrays have been thoroughly investigated. A subarray-

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based method to solve the ambiguity problem is proposed in [31]. The algorithm considers the total array as two subarrays and processes the two subarrays separately. By combining the estimates and finding the nearest spectral peaks from MUSIC spectrums of two subarrays, the phase ambiguity problem can be eliminated. However, this method suffers the severe computational complexity due to the global angular searching. Concerning this, a partial spectral search method is proposed in [32] to decrease the computational complexity, which employs the linear relationship among ambiguous DOA estimates and searches over a small sector. However, these mentioned methods estimate the DOAs by combing the results of the two subarrays of the CLA, so some problems result in:

(i)DOF is limited by the subarray which has the smaller number of elements; (ii)only self information of two subarrays is exploited but mutual information is wasted, as a result, DOA estimation performance is affected and optimal DOA estimates can not be achieved; (iii)these methods need further process to eliminate the ambiguous angles.

To solve these problem, an unfolded coprime linear array (UCLA) is proposed, which unfolds the two subarrays in two opposite directions so that the array aperture is extended [33]. For the UCLA, the array is processed as one total array. In this way, both self information and mutual information is utilized. Ambiguity problem is suppressed by stacking the two directional matrices of two subarrays. Also, the method can achieve the full DOFs due to the employment of the whole array. Nevertheless, this technique is not always true. When the source signals satisfy some relations, the method will not work. In the case of three signals, for the two subarrays, when it exists two different signals having the same steering vectors as the given DOA, the phase ambiguity still exists. Aiming to tackle the problem, Yang et. al. proposed a beamforming based technique by defining a decision variable [34] to eliminate the phase ambiguity. However, this method employs other technique to distinguish the real DOAs besides MUSIC spectral searching, which can increase the computational complexity. Meanwhile, this method does not always work and it usually depends on the decision variable. When the decision variable is small, phase ambiguity problem still occurs.

To solve the ambiguity problem, increase the detecting accuracy and decrease the computational complexity, we design to rearrange the position of the reference sensor so that the linear relation of the directional vectors is broken. Therefore, the MUSIC algorithm can be employed directly without ambiguous angles arising. Meanwhile, we can get the DOAs estimation and do not need any other technique to estimate. The method can achieve the full DOFs of the array. We compared the proposed method and the original methods and we notice that the proposed method can effectively tackle the phase ambiguity problem.

The paper is organized as follows: Section II introduces the array configuration and the data model of the received signals. Section III, we detailed derive the ambiguity problem and present the proposed method, and analyze the DOFs and Cramer-Rao Bound (CRB) of the proposed method. In Section IV, it shows extensive simulation results and demonstrates the superiority of the proposed algorithm. And finally, we give the conclusions in Section VI.

The main contribution of this paper can be concluded as follows.

(1)The proposed method can effectively eliminate the phase ambiguity problem by rearranging the reference element spacing and breaking the spectral function with the directional matrix.

(2)The designed array treats the array as a whole so that it can achieve the full DOFs hence improves estimation performance.

(3)CRB, as the lower bound for unbiased estimation, is provided as a standard to measure the estimation performance. And it is validated that the designed array can achieve the lower CRB.

(4)The designed array can achieve DOA estimation with an excellent DOA estimation performance and it does not need additional algorithms.

Notations: Throughout this paper, we use uppercase (lowercase) bold characters to denote matrices (vectors). $(\bullet)^T$, $(\bullet)^*$ and $(\bullet)^H$ represent the transpose, conjugate and transpose conjugate operators, respectively. And $E\{\bullet\}$ represents the expectation operator.

2. Array Signal Model

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As depicted in Figure 1, we consider an UCLA with $T=M_1+M_2-1$ sensors. For the array, it is composed of two uniform linear subarrays. One subarray has M_1 sensors and the other one has M_2 sensors, respectively. And M_1 and M_2 are a pair of coprime integers. Meanwhile, it has $M_1<M_2$. It can be noticed that the two subarrays are overlapping at the position of $(0,0)$, so we can compute the total sensors of the array as $T=M_1+M_2-1$. In the Figure 1, the inter element spacing of the subarray 1 can be denoted as $d_1=M_2d=M_2\lambda/2$. And the inter element spacing of the subarray 2 can be denoted as $d_2=M_1d=M_1\lambda/2$, where $d=\lambda/2$ and λ means the wavelength.

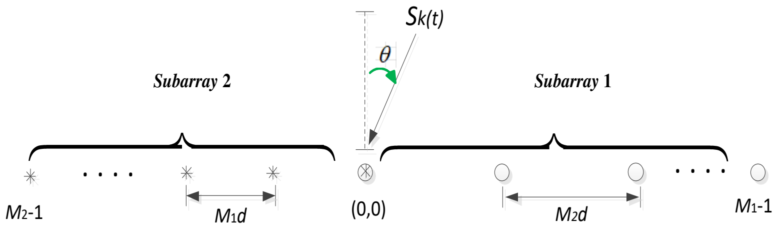

Figure 1. Unfolded Coprime Linear Array (UCLA).

Assume that there are K narrowband sources impinging on the array which locates at $\Theta=[\theta_1,\theta_2,\cdots,\theta_K]$ with signal powers σ_1^2, $\sigma_2^2,\cdots,$ σ_K^2. $\theta_k\in(-\pi/2,\pi/2)$ denotes the k-th signal where $K<T$ and $k\in[1,2,\cdots,K]$. Also, we suppose that these signals are far-field uncorrelated. The received signal at time t is denoted as [10]

$$\begin{aligned}\mathbf{x}(t)&=\begin{bmatrix}\mathbf{x}_1(t)\\ \mathbf{x}_2(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix}\mathbf{s}(t)+\begin{bmatrix}\mathbf{n}_1(t)\\ \mathbf{n}_2(t)\end{bmatrix}\\ &=\mathbf{As}(t)+\mathbf{n}(t)\end{aligned} \tag{1}$$

where $\mathbf{A}=[\mathbf{A}_1^T,\mathbf{A}_2^T]^T$. $\mathbf{A}_1$ and $\mathbf{A}_2$ are steering matrices of the two subarrays, respectively. $\mathbf{A}_1=[\mathbf{a}_1(\theta_1),\mathbf{a}_1(\theta_2),\cdots,\mathbf{a}_1(\theta_K)]$ and $\mathbf{A}_2=[\mathbf{a}_2(\theta_1),\mathbf{a}_2(\theta_2),\cdots,\mathbf{a}_2(\theta_K)]$. $\mathbf{a}_1(\theta_k)=[1,e^{jM_2\pi\sin\theta_k},\cdots,e^{j(M_1-1)M_2\pi\sin\theta_k}]^T$ is steering vector for the subarray 1, and $\mathbf{a}_2(\theta_k)=[e^{-j(M_2-1)M_1\pi\sin\theta_k},e^{-j(M_2-2)M_1\pi\sin\theta_k},\cdots,$ $e^{-jM_1\pi\sin\theta_k}$, $1]^T$ is steering vector for the subarray 2, respectively. $\mathbf{s}(t)=[s_1(t),\ s_2(t),\cdots,s_K(t)]^T$ is the signal emitted by the k-th target at time t, where $t=1,2,\cdots,L$ and L means the sampling number. $\mathbf{n}(t)$ stands for the additive white Gaussian noise and obeys the normal distribution $N(0,\sigma_k^2)$ and is independent from source signals. $\mathbf{n}_1(t)$ is the noise vectors of subarray 1. And $\mathbf{n}_2(t)$ denotes the noise vectors of subarray 2.

3. Ambiguity Problem Demonstration and Resolution

This section introduces the cause of phase ambiguity problem and illustrates the existing methods. Then the proposed improved array is presented and the full DOFs are achieved.

Preview

Generally, to avoid ambiguity problem, the inter element spacing is usually set to be no larger than half-wavelength, however, in this way, the array aperture is restricted. Coprime array, which is made up of two uniform linear arrays and the inter element spacing is larger than half wavelength.

Figure 2 depicts the relationship between the element spacing and the number of DOA estimation. It considers that there is one signal coming from $\theta_1=24°$. From the picture, we can get

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that when the element spacing is set to be $d=\lambda/2$, no ambiguous angle results in. And when $d=3\lambda/2$ or $d=5\lambda/2$, the ambiguous angles result in.

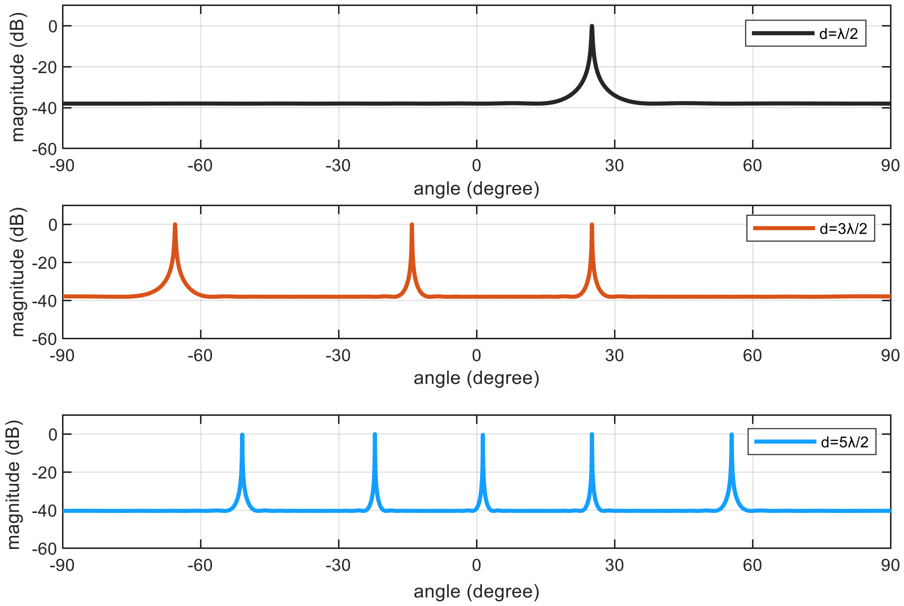

Figure 2. The relation between the phase ambiguity problem and the inter element spacing.

Case 1

In the case 1, we consider that there are two signals coming from $\Theta=[\theta_1,\theta_2]$.

Assume that there are two signals θ_1 and θ_2 which are real angles. The corresponding directional vectors $\mathbf{a}(\theta_1)$ and $\mathbf{a}(\theta_2)$ can be attained. And we can get

$$\mathbf{a}(\theta_1)=\left[\mathbf{a}_1^T(\theta_1),\mathbf{a}_2^T(\theta_1)\right]^T \tag{2}$$

$$\mathbf{a}(\theta_2)=\left[\mathbf{a}_1^T(\theta_2),\mathbf{a}_2^T(\theta_2)\right]^T \tag{3}$$

Assume θ_1' and θ_2' are ambiguous angles of θ_1 and θ_2 .Therefore, corresponding steering vectors $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$ can be attained. Then we can get

$$\mathbf{a}(\theta_1')=\mathbf{a}(\theta_1)=\left[\mathbf{a}_1^T(\theta_1'),\mathbf{a}_2^T(\theta_1')\right]^T=\left[\mathbf{a}_1^T(\theta_1),\mathbf{a}_2^T(\theta_1)\right]^T \tag{4}$$

$$\mathbf{a}(\theta_2')=\mathbf{a}(\theta_2)=\left[\mathbf{a}_1^T(\theta_2'),\mathbf{a}_2^T(\theta_2')\right]^T=\left[\mathbf{a}_1^T(\theta_2),\mathbf{a}_2^T(\theta_2)\right]^T \tag{5}$$

Correspondingly, we have

$$\begin{cases}\mathbf{a}_1(\theta_1)=\mathbf{a}_1(\theta_1') \\ \mathbf{a}_2(\theta_1)=\mathbf{a}_2(\theta_1') \\ \mathbf{a}_1(\theta_2)=\mathbf{a}_1(\theta_2') \\ \mathbf{a}_2(\theta_2)=\mathbf{a}_2(\theta_2')\end{cases} \tag{6}$$

Then we have

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$$\begin{cases} M_2\pi\sin\theta_1 = M_2\pi\sin\theta_1' + 2k_1\pi \\ M_1\pi\sin\theta_1 = M_1\pi\sin\theta_1' + 2k_2\pi \\ M_2\pi\sin\theta_2 = M_2\pi\sin\theta_2' + 2k_1\pi \\ M_1\pi\sin\theta_2 = M_1\pi\sin\theta_2' + 2k_2\pi \end{cases} \tag{7}$$

where $k_1 = -(M_2 - 1), \cdots, -1, 1, \cdots, (M_2 - 1)$ and $k_2 = -(M_1 \quad -1),\ \cdots, -1, 1, \cdots, (M_1 - 1)$ 。
Then we can get

$$\begin{cases} \sin\theta_1 = \sin\theta_1' + 2k_1 / M_2 \\ \sin\theta_1 = \sin\theta_1' + 2k_2 / M_1 \\ \sin\theta_2 = \sin\theta_2' + 2k_1 / M_2 \\ \sin\theta_2 = \sin\theta_2' + 2k_2 / M_1 \end{cases} \tag{8}$$

Then we have

$$\frac{2k_1}{M_2} = \frac{2k_2}{M_1} \tag{9}$$

Due to the coprime property of M_1 and M_2, the Eq. (9) is not satisfied. That is to say, in the case of two signals of θ_1 and θ_2, phase ambiguity problem does not occur.

Figure 3 depicts spectrums without ambiguity by the method proposed in [33]. The two signals are $\theta_1 = 10°, \theta_2 = 37°$. SNR and the snapshots are set to be $\mathrm{SNR} = 5\mathrm{dB}$ and $L = 200$, respectively. We can draw a conclusion that the method proposed in [33] can tackle the phase ambiguity problem with case 1.

Case 2

In the case 2, consider three signals with $\Theta = [\theta_1, \theta_2, \theta_3]$. Similarly, we suppose θ_1' and θ_2' to be ambiguous angles of θ_1 and θ_2, respectively. Corresponding directional vectors can be denoted as $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$. If the relationship that the sine function of the third signal θ_3 equals to $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$ is satisfied, the phase ambiguity problem results in. Figure 4 shows the spectrums of the method in [33], where it has three target signals with $\theta_1 = 10°$ and $\theta_2 = 20°, \theta_3 = 30°$. We set the number of snapshot to be $L = 200$ and $\mathrm{SNR} = 5\mathrm{dB}$. It is depicted clearly in Figure 3 that the method proposed in [33] still has no difficulty to resolve the three source signals. However, this method is not always true. When these three source signals satisfy relationship that the sine function of third signal θ_3 equals to $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$, the method in [33] will not work and we will illustrate in case 3.

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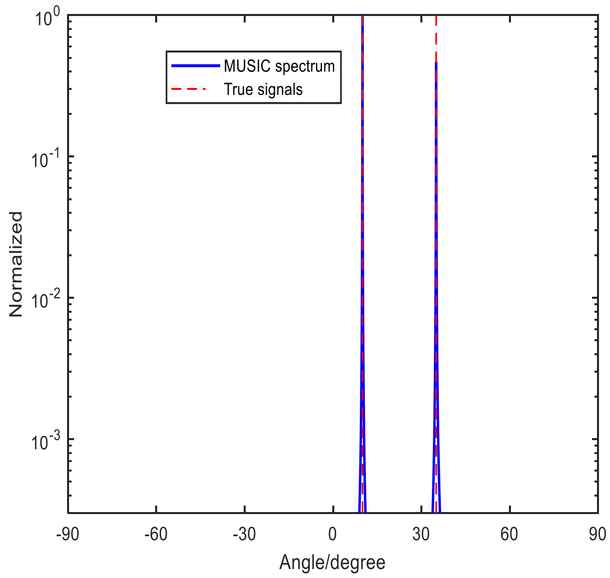

Figure 3. No ambiguous angle arises with 2 source signals, where $\theta_1 = 10°, \theta_2 = 37°$. .

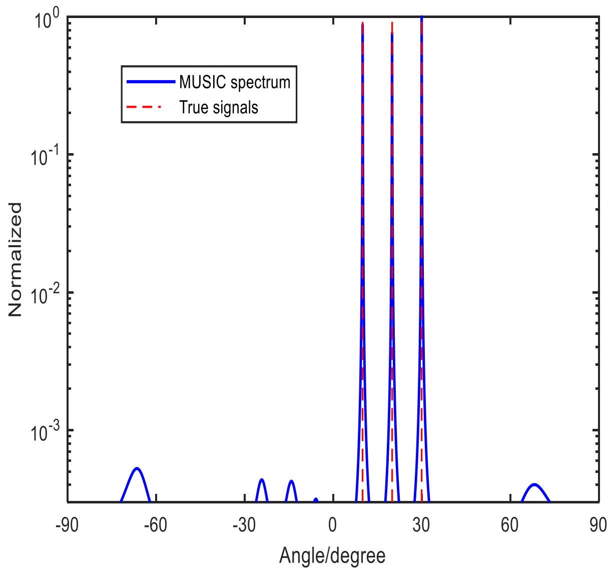

Figure 4. No ambiguous angle arises with the given 3 source signals, where $\theta_1 = 10°, \theta_2 = 20°, \theta_3 = 30°$.
.

Case 3

From the case 2, we notice that the method in [33] can effectively detect three signals without the ambiguous angles, whereas it is not always working. Then we illustrate the case 3 in the following.

Consider three target signals impinging the array with $\Theta = [\theta_1, \theta_2, \theta_3]$. Similar to case 2, we suppose θ_1' and θ_2' to be ambiguous angles of θ_1 and θ_2, respectively. Corresponding directional vectors can be denoted as $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$. When it satisfies that the sine function of the third signal θ_3 equals to $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$, the ambiguous angle will arise. In other words, phase ambiguity problem results in, which is invalidated in the following.

Suppose that the third angle θ_3 whose corresponding sine function equals to $\mathbf{a}(\theta_1')$ and $\mathbf{a}(\theta_2')$. It has

$$\begin{cases} \sin\theta_3 = \sin\theta_1 + 2(-k_1)/M_2 \\ \sin\theta_3 = \sin\theta_2 + 2(-k_2)/M_1 \end{cases} \tag{10}$$

Then we can get the relationship which exists among θ_1 , θ_2 and θ_3

$$\begin{cases} M_2\pi\sin\theta_3 = M_2\pi\sin\theta_1 + 2(-k_1)\pi \\ M_1\pi\sin\theta_3 = M_1\pi\sin\theta_2 + 2(-k_2)\pi \end{cases} \tag{11}$$

where $k_1 = -(M_2 - 1),\cdots,-1,1,\cdots,(M_2-1)$ and $k_2 = -(M_1 -1),\ \cdots,-1,1,\cdots,\ (M_1-1)$ 。

It has

$$\begin{cases} \mathbf{a}_1(\theta_3)=\mathbf{a}_1(\theta_1) \\ \mathbf{a}_2(\theta_3)=\mathbf{a}_2(\theta_2) \end{cases} \tag{12}$$

so we have

$$\begin{cases} \mathbf{a}_1(\theta_1)+\mathbf{a}_1(\theta_2)-\mathbf{a}_1(\theta_3)=\mathbf{a}_1(\theta_2) \\ \mathbf{a}_2(\theta_1)+\mathbf{a}_2(\theta_2)-\mathbf{a}_2(\theta_3)=\mathbf{a}_2(\theta_1) \end{cases} \tag{13}$$

Then we define

$$\begin{cases} \mathbf{a}_1(\theta_4)=\mathbf{a}_1(\theta_1)+\mathbf{a}_1(\theta_2)-\mathbf{a}_1(\theta_3) \\ \mathbf{a}_2(\theta_4)=\mathbf{a}_2(\theta_1)+\mathbf{a}_2(\theta_2)-\mathbf{a}_2(\theta_3) \end{cases} \tag{14}$$

It has

$$\begin{cases} \mathbf{a}_1(\theta_4)=\mathbf{a}_1(\theta_2) \\ \mathbf{a}_2(\theta_4)=\mathbf{a}_2(\theta_1) \end{cases} \tag{15}$$

It is obvious that the forth angle θ_4 arises. That is to say, phase ambiguity problem results in. Simulation is presented to demonstrate the analysis.

Figure 5 depicts the scenery that three signals $\theta_1 = 12.37°$, $\theta_2 = 30°$ and $\theta_3 = 64.16°$ come to the array. The SNR and the number of the snapshots are set to be $\text{SNR} = 5\text{dB}$ and $L = 200$, respectively. It can be found that these three signals satisfy $\mathbf{a}_1(\theta_3)=\mathbf{a}_1(\theta_1)$ and $\mathbf{a}_2(\theta_3)= \mathbf{a}_2(\theta_2)$. And it is validated that the method in [33] is not effective. From Figure 5, we can notice that other than the three signals are detected while it exists the forth spectrum. Aiming at solving the ambiguity problem, Yang et. al. proposed a modified method by defining a decision variable[34]. Meanwhile, the method combines the beamforming technique with MUSIC. However, this method is not always effective. Figure 6 depicts that five signals come to the array. And by using the method in [34], the five signals can be detected successfully, but it still has ambiguous angle, which can increase the ineffectiveness of the DOA estimation performance. To tackle this problem, we design an improved ambiguity–free array, where there are three coprime integers. By moving the reference element, the linear combination relation of the steering vectors can be broken. In this way, MUSIC algorithm can be applied directly and no additional algorithm needs. The method is illustrated in the following part.

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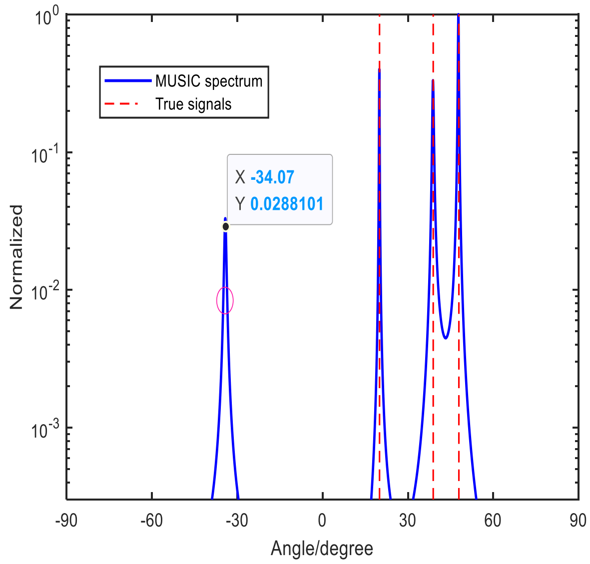

Figure 5. With the method in [33], ambiguous angle arises with 3 source signals which satisfy Eq. (10).

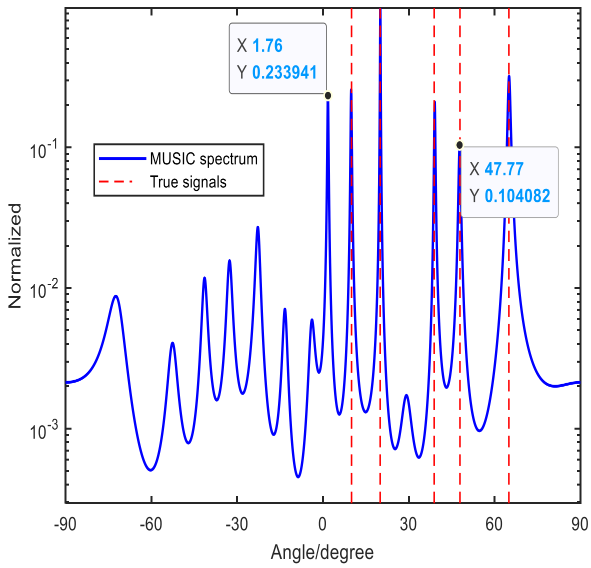

Figure 6. Using the method in [34] of beamforming technique sometimes is not effective.

4. Proposed Method for DOA Estimation

This section presents the proposed method to tackle the phase ambiguity problem with case 3. In the proposed method, we consider to rearrange reference sensor. Figure 7(a) presents an UCLA, which is composed of two subarrays, including subarray 1 with M_1 sensors and subarray 2 with M_2 sensors. Figure 7(b) presents a rearranged reference point instead of (0, 0). From the Figure 7(b), for the new array, it still consists of two subarrays which contain M_1 and M_2 sensors, respectively. And position of the reference element is changed from (0, 0) to $(M_3 d, 0)$, where M_3 is the third coprime integer besides M_1 and M_2. Due to the change of the reference sensor, it can be found that the steering vectors have changed correspondingly. In this way, two different directional vectors can be attained. By using this property, the resulted ambiguity can be broken.

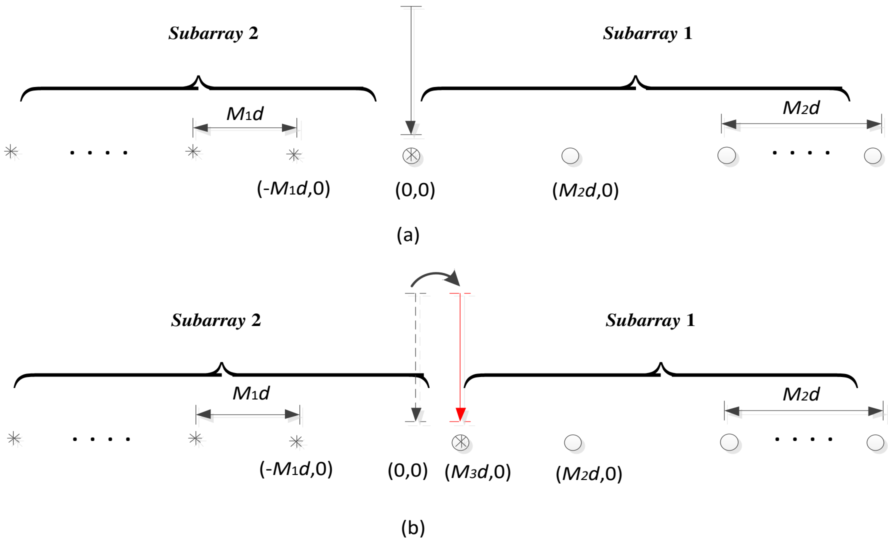

Figure 7. (a)The unfolded coprime linear array. (b)The designed improved unfolded coprime linear array.

In the rearranged array, denote the directional vectors of two subarrays with the k-th signal as

$$\mathbf{a}_{11}(\theta_k)=[e^{j3M_2\pi\sin\theta_k},e^{jM_2\pi\sin\theta_k},\cdots,e^{j(M_1-1)M_2\pi\sin\theta_k}]^T \quad (16)$$

$$\mathbf{a}_{22}(\theta_k)=[e^{-j(M_2-1)M_1\pi\sin\theta_k},e^{-j(M_2-2)M_1\pi\sin\theta_k},\cdots,\ e^{-jM_1\pi\sin\theta_k},\ e^{-j3M_2\pi\sin\theta_k}]^T \quad (17)$$

So we can ger the steering vectors of source signal θ_3

$$\mathbf{a}_{11}(\theta_3)=[e^{j3M_2\pi\sin\theta_3},e^{jM_2\pi\sin\theta_3},\cdots,e^{j(M_1-1)M_2\pi\sin\theta_3}]^T \quad (18)$$

$$\mathbf{a}_{22}(\theta_3)=[e^{-j(M_2-1)M_1\pi\sin\theta_3},e^{-j(M_2-2)M_1\pi\sin\theta_3},\cdots,\ e^{-jM_1\pi\sin\theta_3},\ e^{-j3M_2\pi\sin\theta_3}]^T \quad (19)$$

$\mathbf{a}(\theta_k)$ denotes the directional vector of the total array .

$$\mathbf{a}(\theta_k)=\left[\mathbf{a}_{11}^T(\theta_k),\mathbf{a}_{22}^T(\theta_k)\right]^T \quad (20)$$

Similar to Eq. (1), we can get the received signal

$$\begin{aligned}\mathbf{x}_{iu}(t)&=\begin{bmatrix}\mathbf{x}_{iu1}(t)\\ \mathbf{x}_{iu2}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A}_{11}\\ \mathbf{A}_{22}\end{bmatrix}\mathbf{s}(t)+\begin{bmatrix}\mathbf{n}_{11}(t)\\ \mathbf{n}_{22}(t)\end{bmatrix}\\ &=\mathbf{A}_{iu}\mathbf{s}(t)+\mathbf{n}_{iu}(t)\end{aligned} \quad (21)$$

where $\mathbf{A}_{iu}=[\mathbf{A}_{11}^T,\mathbf{A}_{22}^T]^T$. $\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ are steering matrices of the two subarrays for IUCLA, respectively. $\mathbf{A}_{11}=[\mathbf{a}_{11}(\theta_1),\ \mathbf{a}_{11}(\theta_2),\cdots,\mathbf{a}_{11}(\theta_K)]$ and $\mathbf{A}_{22}=[\mathbf{a}_{22}(\theta_1),\mathbf{a}_{22}(\theta_2),\cdots,\mathbf{a}_{22}(\theta_K)]$, where $\mathbf{a}_{11}(\theta_k)$ and $\mathbf{a}_{22}(\theta_k)$ are denoted as Eq. (16) and Eq. (17). $\mathbf{n}_{iu}(t)=[\mathbf{n}_{11}^T,\mathbf{n}_{22}^T]^T$ is the total array noise vector. The corresponding total covariance matrix can be computed with L snapshots

$$\hat{\mathbf{R}}_{iu}=(1/L)\sum_{l=1}^{L}\mathbf{X}_{iu}\mathbf{X}_{iu}^H \quad (22)$$

The eigenvalue decomposition result of the total covariance matrix $\hat{\mathbf{R}}_{iu}$ can be denoted as

$$\hat{\mathbf{R}}_{iu}=\hat{\mathbf{E}}_{siu}\hat{\mathbf{D}}_{siu}\hat{\mathbf{E}}_{siu}^H+\hat{\mathbf{E}}_{niu}\hat{\mathbf{D}}_{niu}\hat{\mathbf{E}}_{niu}^H \quad (23)$$

where $\hat{\mathbf{E}}_{siu}$ and $\hat{\mathbf{E}}_{niu}$ are the signal subspave and noise subspace matrix. And $\hat{\mathbf{D}}_{siu}$ and $\hat{\mathbf{D}}_{niu}$ include the eigenvalues.

Referring to the orthogonality between the signal subspace and the noise subspace, the spectral peak function of MUSIC can be denoted as [10]

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$$f(\theta)=\frac{1}{\mathbf{a}_{iu}^{H}(\theta)\mathbf{E}_{niu}\mathbf{E}_{niu}^{H}\mathbf{a}_{iu}(\theta)} \tag{24}$$

where $\mathbf{a}_{iu}(\theta)=\left[\mathbf{a}_{11}^{T}(\theta),\mathbf{a}_{22}^{T}(\theta)\right]^{T}$.

Referring to the derivation above, when there are three sinals coming to the array and they satisfy $\mathbf{a}_{11}(\theta_3)=\mathbf{a}_{11}(\theta_1)$ and $\mathbf{a}_{22}(\theta_3)=\mathbf{a}_{22}(\theta_2)$, phase ambiguity problem arises. In the following, we focus on proving and resolving the ambiguity problem.

Proof: Assume $\mathbf{a}_{11}(\theta_3)=\mathbf{a}_{11}(\theta_1)$. $\mathbf{a}_{11}(\theta_3)$ and $\mathbf{a}_{11}(\theta_1)$ represent the steering vectors of θ_3 and θ_1 with subarray 1, respectively. It has

$$\begin{cases}3M_2\pi\sin\theta_3=3M_2\pi\sin\theta_1+2k_1\pi\\ M_2\pi\sin\theta_3=M_2\pi\sin\theta_1+2k_1\pi\end{cases} \tag{25}$$

where $k_1=(-M_2,M_2)$. So it exists $\mathbf{a}_{11}(\theta_3)\neq\mathbf{a}_{11}(\theta_1)$.

Similarly, we can get that $\mathbf{a}_{22}(\theta_3)\neq\mathbf{a}_{22}(\theta_2)$, where $\mathbf{a}_{22}(\theta_3)$ and $\mathbf{a}_{22}(\theta_1)$ represent the steering vectors of θ_3 and θ_1 with subarray 2, respectively. And we can get that spectral peak function is broken, in this way, we can get the accurate DOAs estimation without ambiguous angle θ_4, which means that the phase ambiguity problem is solved.

DOF Analysis

In this part, we will provide the DOF performance of the proposed method. The method can achieve the full DOFs. The Figure 8 depicts there are three signals coming to the array with $M_1=3,\ M_2=2$. And the signals are $\theta_1=10°,\theta_2=27.35°$ and $\theta_3=35.01°$.

The Figure 9 depicts there are seven signals coming to the array with $M_1=5,\ M_2=4$. And the signals are denoted as $\theta_1=-30°,\theta_2=-10°,\theta_3=10°,\theta_4=30°,\theta_5=35°,\theta_6=40°,$ and $\theta_7=50°$. From the Figure 8 and the Figure 9, we can see the proposed method can achieve the full DOF.

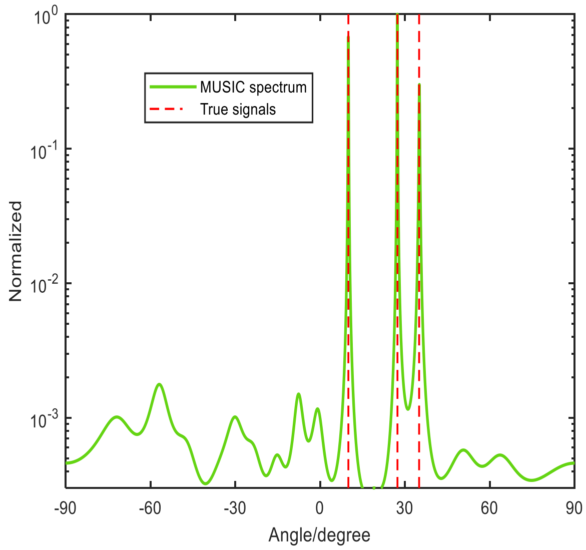

Figure 8. The reconstructed array configuration can achieve the full DOFs of 3 sources signals.

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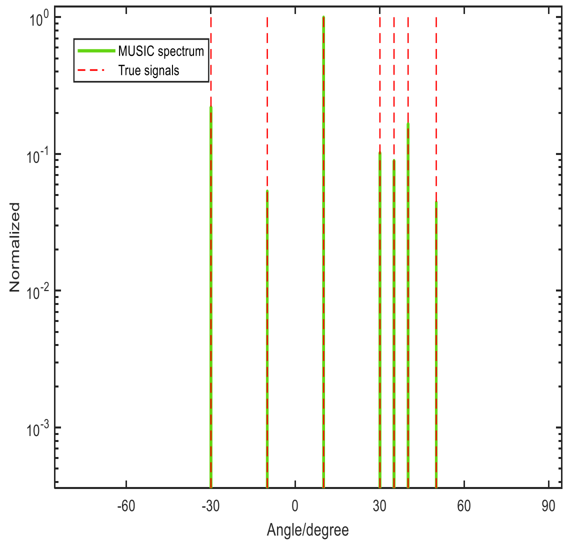

Figure 9. The reconstructed array configuration can achieve the full DOFs of 7 sources signals.

Cramer-Rao Bound

The Cramer-Rao Bound (CRB), which is the lower bound for unbiased estimation, is provided as a standard to measure the estimation performance [35,36]. In this section, we derive the CRB of the designed IUCLA.

Construct the steering matrix of IUCLA as

$$\mathbf{A}_\alpha = \begin{bmatrix} \mathbf{A}_{11} \\ \mathbf{A}_\chi \end{bmatrix} \tag{26}$$

where $\mathbf{A}_\chi$ represents a sub-matrix containing the second row to the last one of $\mathbf{A}_{22}$ since these two subarrays of IUCLA share the same element at the original point.

Referring to [36], we can get the CRB as

$$\mathbf{CRB} = \frac{\sigma_n^2}{2L}\left\{\mathrm{Re}\left[\mathbf{D}^H[\mathbf{I} - \mathbf{A}_\alpha(\mathbf{A}_\alpha^H\mathbf{A}_\alpha)^{-1}\mathbf{A}_\alpha^H]\mathbf{D} \oplus \mathbf{R}_s\right]\right\}^{-1} \tag{27}$$

where $\mathbf{R}_s = \frac{1}{L}\sum_{t=1}^{L}\mathbf{s}(t)\mathbf{s}^H(t)$, $\mathbf{D} = \left[\frac{\partial \mathbf{a}_{\alpha,1}}{\partial\theta_1}, \frac{\partial \mathbf{a}_{\alpha,2}}{\partial\theta_2}, \ldots, \frac{\partial \mathbf{a}_{\alpha,K}}{\partial\theta_K}\right]$ and $\mathbf{a}_{\alpha,k}$ denotes the k-th column of $\mathbf{A}_\alpha$.

5. Simulation Results And Discussion

In the simulation section, we validate the reliability of the proposed method compared with the methods in [33] and [34], where we employ an UCLA. And subarray 1 is with $M_1 = 5$ sensors. And subarray 2 is with $M_2 = 7$ sensors. Also, we present the computational complexity comparison of the methods mentioned above and CRB comparison of UCLA, CLA and the designed IUCLA.

A. Reliability Comparison

Example 1: Assume three signals $\theta_1 = 12.37°, \theta_2 = 30°$ and $\theta_3 = 64.16°$ coming to the array. It can be noticed that these signals satisfy the Eq. (10). In the simulation, we set $\mathrm{SNR} = \text{-}5\mathrm{dB}$ and the number of snapshot $L = 200$. We provide the simulation result of method in [33] with the proposed method. From Figure 10(a), we can see that using the designed method has no ambiguity when three sources satisfy the Eq. (10), whereas the ambiguity-free method still has ambiguity problem. It concludes that the proposed method can tackle the phase ambiguity problem of case2.

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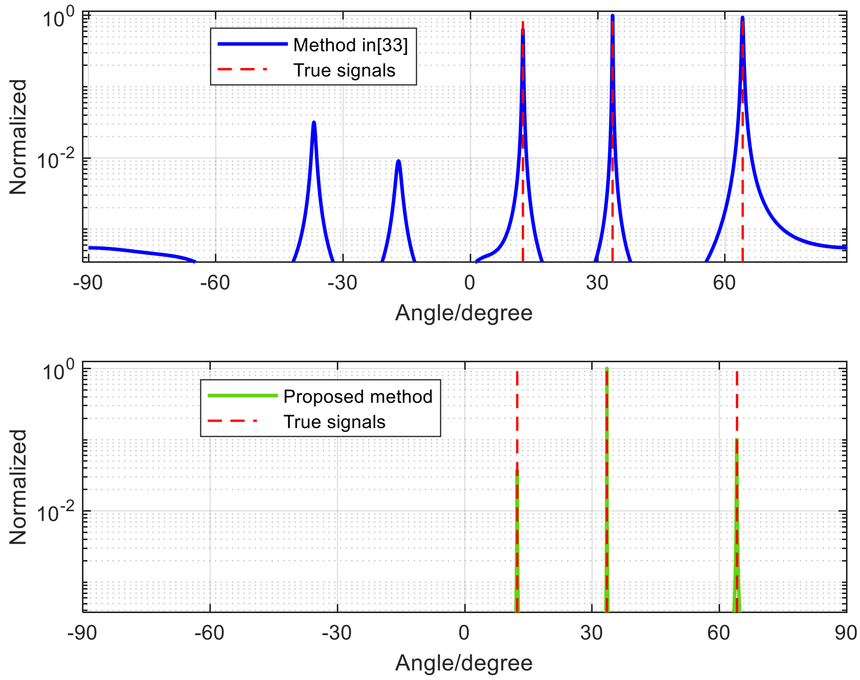
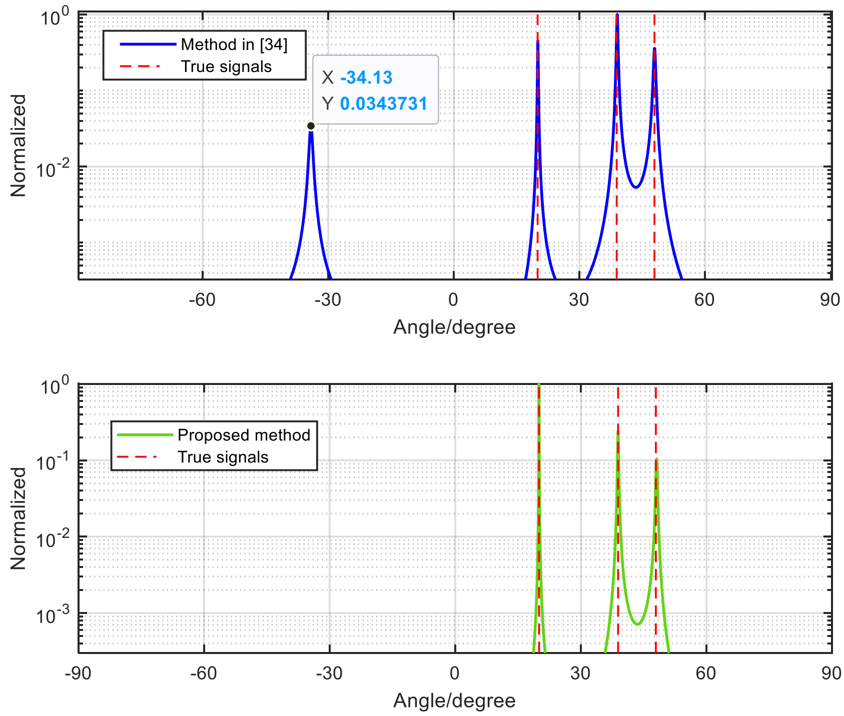
Example 2: Assume three signals $\theta_1 = 20°, \theta_2 = 38.88°$ and $\theta_3 = 47.90°$ coming to the array. It can be noticed that these signals satisfy the Eq. (10). In the simulation, we set $\mathrm{SNR} = \text{-}5\mathrm{dB}$ and the number of snapshot $L = 200$. We provide the simulation result of method in [34] with the proposed method. From Figure 10(b), we can see that method in [34] is not that effective and the designed array using the spectral peak search algorithm has no ambiguity.



(a)



(b)

Figure 10. (a)Comparison of the proposed method with the method in [33] and (b)Comparison of the proposed method with the method in [34].

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Estimation Properties Analysis

For the same simulation scenario mentioned above, we utilize the Root Mean Square Error (RMSE) to validate the estimation accuracy of the proposed method, which is defined as [37]

$$RMSE=\sqrt{\sum_{p=1}^{Q}\sum_{k=1}^{K}\left(\hat{\theta}_{k,p}-\theta_{k}\right)^{2}/PK} \quad (28)$$

where P is the number of Monte Carlo simulations, $\hat{\theta}_{k,p}$ stands for the estimate of the p-th trial for the k-th theoretical angle θ_k. And in this paper, we set P=1000. Figure 11 and Figure 12 show the RMSE versus SNR and the number of snapshots, respectively. And the CRB is provided. It is observed that the estimaiton accuracy of the proposed method improves with the increase in SNR and the number of snapshots owing to its robustness against noise. And it is close to the CRB.

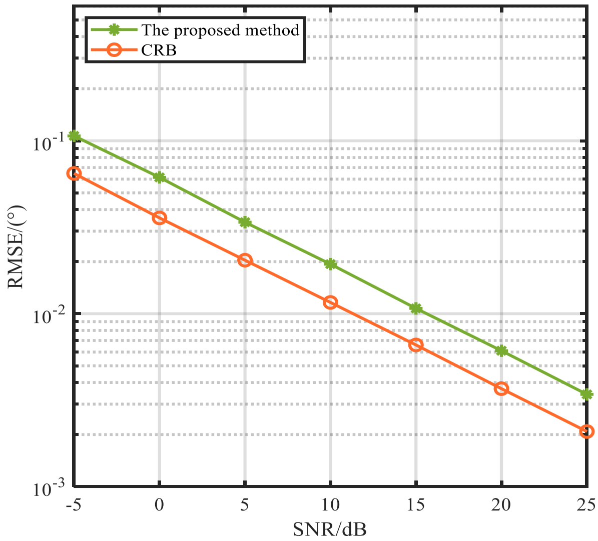

Figure 11. The RMSE versus SNR of the proposed method.

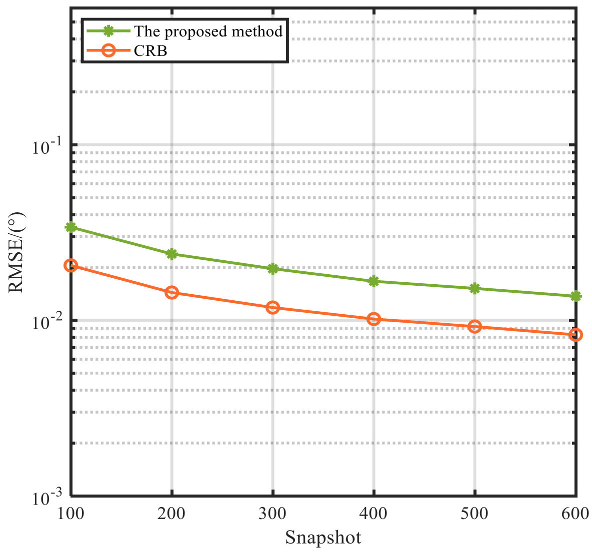

Figure 12. The RMSE versus snapshots of the proposed method.

Computational Complexity Analysis

We compute the complexity of the proposed method. And compare the complexity with methods in [33] and [34]. The complexity of the propoded method is similar as the method in [33],

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which is denoted as is $O(T^2L+T^3+GT(T-K))$, where $T=M_1+M_2-1$. And $G=180°/\tau$ is the number of spectrum searching, where τ is the searching step and we usually set $\tau=0.1°$. L is the number of the snapshots. And the method in [34] needs additional algorithm to distinguish the true DOAs besides MUSIC spectrum. So the complexity exceeds the method in [33] and the proposed method, which is $O(T^2L+T^3+GT(T-K)+QT^2)$. Q is the number of searching of the additional beamforming technique. Table 1 presents the computational complexity comparison and the running time of the methods above, which is computed by the MATLAB R2015b under the condition of Intel (R) Xeon (R) CPU E430 @3.10GHz and 8GB random access memory, where $L=200, K=3,$ $(\theta_1,\phi_1)=(20°,38.88°,47.90°)$, $M_1=5, M_2=4,$ which presents clearly that the proposed method is similar to the method in [33], and outperforms the method in [34]. Figure 13 depicts the complexity comparison versus the number of the subarray 2, where the number of the subarray 1 is $M_1=5$. As the proposed method does not need additional procedure to identify the targets, it shows clearly that its complexity is much lower than the method in [34] and near to the method in [33].

Table 1. Comparison of computational complexity.

Algorithms	Computational Complexity	Running Time
Method in [33]	$O(T^2L+T^3+GT(T-K))$	4.11ms
Method in [34]	$O(T^2L+T^3+GT(T-K)+QT^2)$	6.01ms
The proposed	$O(T^2L+T^3+GT(T-K))$	4.12ms

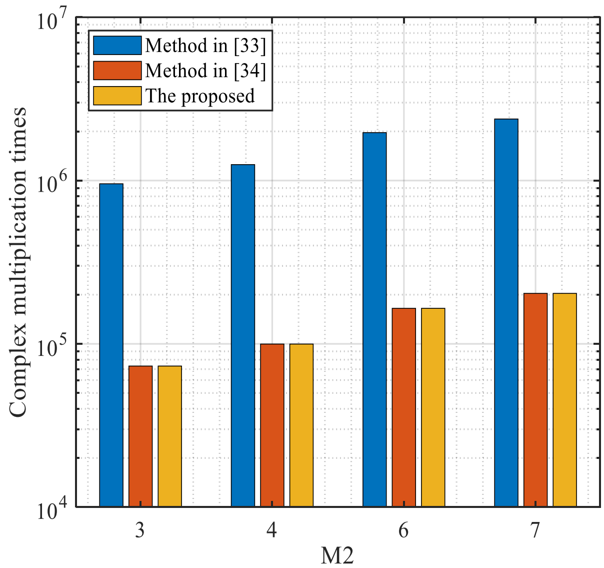

Figure 13. The RMSE versus snapshots of the proposed method.

CRB Analysis

We compare the estimation performance of the proposed method with IUCLA, UCLA and conventional ULA. In the simulation, for fair comparison, assume all arrays with the same number of sensors. We can conclude from Figure 14 and Figure 15 that the CRB and estimation performance of the proposed method with the 10 sensors is superior than the other two arrays.

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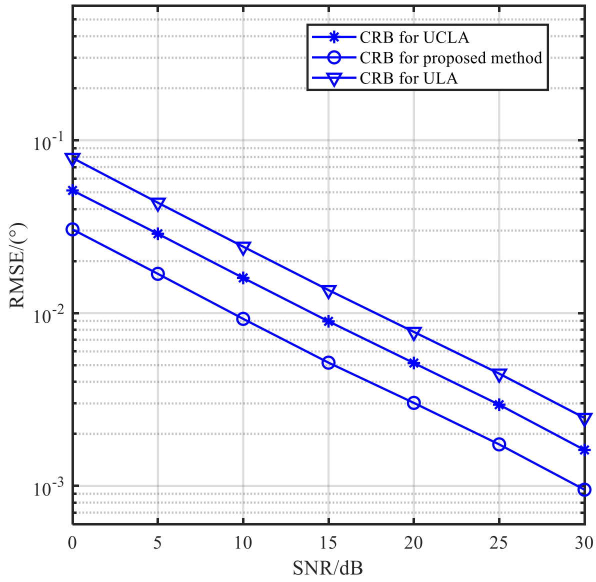

Figure 14. The RMSE versus SNR based on different arrays.

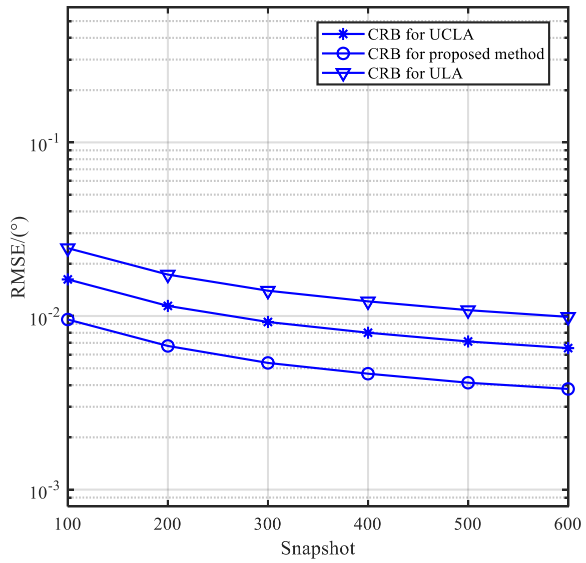

Figure 15. The RMSE versus Snapshot based on different arrays.

6. Conclusions

Coprime array, due to its advantages, attracts much attention in recent years. Since the element spacing is larger than half wavelength, the phase ambiguity problem results in. An ambiguity-free method is proposed in [33], which can tackle the ambiguity problem. In the case of two signals, this method can detect the targets effectively, neverthless, this method is not always true. In the case of three signals of which two signals have the same directional vector as the third DOA, ambiguous angle will arise. Aiming at resolving this problem, Yang. et. al proposed a modified method [34] which defines a decision variable and combines the classic beamforming technique and MUSCI to eliminate the ambiguous angle. However, the accuracy of this method depends on the decision variable, and sometimes it is not true. Meanwhile, this method should need additional algorithm to detect the real DOAs other than MUSIC spectrum. So we design an improved ambiguity-free

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method by rearrangeing the reference sensor to change the relation of the directional vectors of subarrays. And MUSIC algorithm can be applied directly with no additional algorithm. Simulation results demonstrate that the proposed method can estimate DOAs effectively without ambiguous angles compared with the original methods.

Author Contributions: The main idea was proposed by Pan Gong and Xiaofei Zhang. Xiaofei Zhang conceived the experiments and provided many valuable suggestions. Pan Gong conducted experiments and collected data. Pan Gong wrote the manuscript and all the authors participated in amending the manuscript.

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