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[Espen Haug](#)^{*} and [Stephane Wojnow](#)

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Article

Investigation of the Casimir Effect in the Hubble Universe and Black Holes

Espen Gaarder Haug ^{1,*}  and Stéphane Wojnow ² 

¹ Norwegian University of Life Sciences, Christian Magnus Falsensvei 18, Aas, Norway Ås, Norway; espenhaug@mac.com

² Independent Researcher, Limoges, France; wojnow.stephane@gmail.com

* Correspondence: espenhaug@mac.com

Abstract: The Casimir effect can either work over very short distances or at very low temperatures. At the cosmic scale, we naturally deal with long distances, but at the same time, we encounter extraordinarily low temperatures, namely the CMB (Cosmic Microwave Background) temperature. Recently, there has been increased interest in the Casimir effect and its implications in cosmology. Here, we briefly demonstrate that the luminosity and radiation pressure of the CMB is mathematical identical to a theoretical Casimir effect within the Hubble sphere, or actually our solution seems to be valid for any Schwarzschild black hole. The Casimir effect for every Schwarzschild black hole is: $F_{ca} = G \frac{m_p^2}{l_p^2} \frac{1}{11520\pi} = k_e \frac{q_p^2}{l_p^2} \frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G} \frac{1}{11520\pi} \approx 3.34 \times 10^{39} \text{ N}$. This also include the Hubble sphere if we treat it as a Schwarzschild black hole that is related to increased interest in black hole cosmology.

Keywords: Hubble sphere; Casimir effect; luminosity; radiation pressure; quantum cosmology; vacuum energy

1. The Hubble-Casimir Effect and its Relation to the CMB luminosity

The total Casimir pressure effect is described by the formula (as seen in [1] and elaborated upon by Balian and Duplantier [2]):

$$P_{ca} = \frac{1}{A} (\bar{X}_0 + \bar{X}_T) = -\frac{\pi^2}{240} \frac{\hbar c}{D^4} - \frac{\pi^2}{45} \frac{1}{\beta} \frac{1}{(\beta \hbar c)^3} + \frac{1}{\beta} \frac{\pi}{D^3} e^{-\alpha} + \dots \quad (1)$$

Here, $\beta = \frac{1}{k_b T}$, where k_b is the Boltzmann constant, T is the temperature, D is the distance between the two plates, and $\alpha = \frac{\pi \beta \hbar c}{D}$. Typically, when one thinks of the Casimir effect, it is associated with plates placed at extremely short distances. However, as noted by Balian, Duplantier, and others, the Casimir pressure effect is valid at very short distances or, conversely, at very low temperatures. The longer the distance over which the Casimir effect operates, the lower the temperature must be.

The second term in the equation above represents the black body pressure in what is known as an infinite geometry.

$$P_{ca} = -\frac{\pi^2}{45} \frac{1}{\beta} \frac{1}{(\beta \hbar c)^3} \quad (2)$$

where again $\beta = \frac{1}{k_b T}$.

Investigating the Casimir effect in relation to cosmology is not very common. However, there are multiple recent papers on the Casimir effect in relation to cosmology, as seen in Leonhardt [3,4]. One reason is that the Casimir effect is typically related to vacuum, and most of the universe is a vacuum with very low energy, the cosmological vacuum energy.

In cosmology, we certainly do not work on short distances; rather, we focus on very long distances and over very large surfaces areas (in the Hubble sphere). However, we work with extraordinarily low temperatures, specifically the Cosmic Microwave Background (CMB) temperature, which is likely as

close as one can get to absolute zero in any observation. The CMB temperature has been measured more accurately and is one of the most precise factors in cosmology. It is approximately $2.72K$, as seen in [5–8]. Whether one can really study the Casimir effect at distances equal to the Hubble radius with only the CMB temperature is a question we leave for further study, but we will assume so here.

Let us now consider the Hubble radius, denoted as $R_H = \frac{c}{H_0}$, for the distance between the plates and the CMB temperature $T_{cmb} \approx 2.72K$. This gives:

$$P_{ca} = \frac{dF_c}{dS} = \frac{1}{A}(\bar{X}_0 + \bar{X}_T) = -\frac{\pi^2}{240} \frac{\hbar c}{R_H^4} - \frac{\pi^2}{45} \frac{1}{\beta} \frac{1}{(\beta \hbar c)^3} + \frac{1}{\beta} \frac{\pi}{R_H^3} e^{-\alpha} \approx -1.39 \times 10^{-14} J \cdot m^{-3} \quad (3)$$

where $\alpha = \frac{\pi \beta \hbar c}{R_H}$ and $\beta = \frac{1}{k_b T_{cmb}}$. We have used a Hubble parameter given by the recent study of Kelly and et. al [9]: $H_0 = 66.6^{+4.1}_{-3.3} (km/s)/Mpc$.

Further the second term, the black body radiation pressure at infinite geometry is equal to:

$$P_{ca} = -\frac{\pi^2}{45} \frac{1}{\beta} \frac{1}{(\beta \hbar c)^3} \approx -1.39 \times 10^{-14} J \cdot m^{-3} \quad (4)$$

we mention this as it is evident the second term is the main contributor when dealing with the Hubble scale. If we multiply P_{ca} with the surface area of the Hubble sphere we get the Casimir force of $F_{ca,H} = P_{ca} A_H = -1.39 \times 10^{-14} \times \frac{4}{3} \pi R_H^2 \approx 3.34 \times 10^{39} N$.

Next, we will analyze the radiation pressure related to CMB luminosity within the Hubble sphere. Based on the Stefan-Boltzmann [10,11] law, we have a theoretical blackbody luminosity derived from the CMB temperature:

$$L_{cmb} = 4\pi R_H^2 \sigma T_{cmb}^4 = 4\pi \left(\frac{c}{H_0} \right)^2 \sigma \times 2.72^4 \approx 7.5 \times 10^{47} W \quad (5)$$

Alternatively, this can be predicted using a new equation recently published by Haug and Wojnow [12], where they have derived the CMB luminosity from the Stefan-Boltzmann law, resulting in:

$$L_{cmb} = \frac{\hbar c^6}{15360\pi G^2 m_p^2} \approx 7.50338 \times 10^{47} W \quad (6)$$

The formula is valid for any Schwarzschild black hole size [13] and is remarkably independent of the mass of the black hole, unlike the Bekenstein-Hawking luminosity [14], which is mass-dependent. Based on the Stefan-Boltzmann law, the radiation pressure is given by:

$$P_{rad} = \frac{L}{4\pi R^2 c} \quad (7)$$

For a Hubble sphere, when we input the CMB luminosity as L_{cmb} , Haug and Wojnow [12] have demonstrated that this leads to:

$$P_{cmb} = \frac{\hbar c}{46080\pi^2 R_s^2 l_p^2} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} \quad (8)$$

Here, we have used the Hubble radius as the Schwarzschild radius, which is permissible when treating the Hubble sphere as a Schwarzschild black hole. The idea of the Hubble sphere as a black hole was likely first introduced in 1972 by Pathria [15], but is an actively discussed alternative view on cosmology to this day, see for example [16–22], this could also be highly relevant for the wider class of $R_h = ct$ cosmology models, see for example [23–28].

This results in a radiation pressure of:

$$P_{\text{cmb}} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} \approx 1.39 \times 10^{-14} \text{ J} \cdot \text{m}^{-3} \quad (9)$$

where $R_H = \frac{c}{H_0}$. Notably, this is identical to the Casimir pressure calculated previously (Equation 3 and 4). When multiplied by the area of the Hubble surface, we obtain:

$$P_{\text{cmb}} A_H = \frac{\hbar c}{46080\pi^2 R_H l_p^2} 4\pi R_H^2 = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G} \frac{1}{11520\pi} \approx 3.34 \times 10^{39} \text{ N}. \quad (10)$$

The Planck gravitational force is given by

$$F_{N,p} = G \frac{m_p m_p}{l_p^2} = \frac{\hbar c}{l_p^2} \quad (11)$$

which is also identical to the Coulomb [29] force for two Planck charges, as we have

$$F_{C,p} = k_e \frac{q_p q_p}{l_p^2} = \frac{\hbar c}{l_p^2} \quad (12)$$

where q_p is the Planck charge which again is equal to $q_p = \frac{e}{\sqrt{\alpha}}$, where e is the elementary charge and α is the fine structure constant. This means we also can re-write equation 10 as

$$P_{\text{cmb}} A_H = \frac{\hbar c}{11520\pi l_p^2} = F_{N,p} \frac{1}{11520\pi} = F_{C,p} \frac{1}{11520\pi}. \quad (13)$$

This value remains constant and is the same for every Schwarzschild black hole. Consequently, we have:

$$P_{\text{cmb}} A_H = F_{ca,H} = G \frac{m_p^2}{l_p^2} \frac{1}{11520\pi} = k_e \frac{q_p^2}{l_p^2} \frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} \approx 3.34 \times 10^{39} \text{ N} \quad (14)$$

where $F_{ca,H}$ stands for the Casimir force acting on the Hubble sphere. The change in the surface area can be seen as from Planck mass micro black hole to the Hubble sphere, this is consistent with growing black hole cosmology models of the $R_H = ct$ type, and likely also a steady state black hole that can be derived from the Haug-Spavieri metric, something we have to come back to in a future paper.

This implies that there could indeed be a Casimir effect within the Hubble sphere and for any Schwarzschild black hole, as the value remains constant at approximately $3.34 \times 10^{39} \text{ N}$.

All of this should also be seen and investigated further in connection with the recent rapid theoretical progress in understanding the CMB temperature. Tatum et al. [30] already heuristically suggested the formula to predict the CMB temperature as follows:

$$T_{\text{cmb}} = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M m_p}} = \frac{\hbar c^3}{k_b 4\pi G \sqrt{R_s 2l_p}} \quad (15)$$

The formula was first formally mathematically proven by Haug and Wojnow [31] by deriving the CMB temperature from the Stefan-Boltzmann law. Haug and Tatum [32] demonstrated that the CMB temperature formula above could be derived from a more general geometric mean approach, and Haug [33] also showed that the CMB temperature can be derived based on the quantized minimum bending of light at the horizon of a black hole. All in all, it seems that more and more pieces of what we can call the Hubble puzzle are falling into place, and all this can be applied to any Schwarzschild black hole, and can likely relatively easily be extended to Reiser-Nordström [34,35], Kerr [36], Kerr-Newman [37,38] and Haug-Spavieri [39] black holes also.

2. Possible interpretation

We have found that the Casimir force for the Hubble sphere is given by $F_{ca} \approx G \frac{m_p^2}{l_p^2} \frac{1}{11520\pi} = k_e \frac{q_p^2}{l_p^2} \frac{1}{11520\pi} \approx 3.34 \times 10^{39} \text{ N}$, which is close to the Planck gravitational force and also the Coulomb force between two Planck charges over the distance of the Planck length. This is an enormous force. However, we think this can be seen as an aggregated Casimir force coming from the entire Hubble surface $A = 4\pi R_H^2$, so the Casimir pressure is identical to the CMB pressure of only $P_{ca} = P_{cmb} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} = \frac{F_{ca}}{A_H} = J \cdot m^{-3} = N \cdot m^{-2} \approx -1.39 \times 10^{-14} \text{ N} \cdot m^{-2}$, which is very low. So even if the Casimir force is very strong for the Hubble sphere, it is divided over a large surface area, so the radiation pressure is very low.

As our formulas end up having Planck units in them, this indicates a strong connection between gravity and Planck units. Einstein [40] already in 1916 suggested that the next step in the development of gravity theory would be a quantum gravity theory. Already in 1918, Eddington [41] suggested that such a theory had to be linked to the Planck length (Planck units). Quantum gravity is an actively researched area to this day, and perhaps even our findings of the Casimir effect for the Hubble sphere can one day be incorporated into such theories, but that is outside the scope of this paper.

Another interesting question is how the Casimir force can be identical to the gravitational force and the Coulomb force, as we have demonstrated in the derivations above. It is important to be aware that this is naturally only applicable to black holes. One possible interpretation is that in black holes, at least close to the singularity, mass is so densely packed together that everything is possible. Therefore, gravity and electromagnetism, along with all other forces, become united into one force, as predicted by multiple researchers. However, this should naturally be further investigated and, at this point, is at best only a hypothesis. Still, it's a hypothesis that may gain strength with this paper.

3. Conclusion

We have tried to demonstrate that radiation pressure, derived from CMB luminosity, which is again linked to CMB temperature, exhibits close similarities to the Casimir force. We have named this phenomenon the Casimir-Hubble effect. Alternatively, it could be referred to as the black hole Casimir effect, as it should apply to all black holes. We have shown that for any Schwarzschild black hole we have: $F_{ca} = G \frac{m_p^2}{l_p^2} \frac{1}{11520\pi} = k_e \frac{q_p^2}{l_p^2} \frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G} \frac{1}{11520\pi} \approx 3.34 \times 10^{39} \text{ N}$. That is in a black hole the Casimir force is equal to the gravity force for two Planck mass particles or the Coulomb force of two Planck charges, after we multiply these by $\frac{1}{11520\pi}$.

Conflicts of Interest: The authors declares no conflict of interest.

References

1. H. B. G. Casimir. On the attraction between two perfectly conducting plates. *Proc. Kon. Ned. Akad. Wet.*, 51: 793, 1948.
2. R. Balian and B. Duplantier. Geometry of the casimir effect. *arXiv page 13*, 2004. URL <https://doi.org/10.48550/arXiv.quant-ph/0408124>.
3. U. Leonhardt. The case for a Casimir cosmology. *Philosophical Transactions Of The Royal Society A*, 378: 20190229, 2020. URL <https://doi.org/10.1098/rsta.2019.0229>.
4. U. Leonhardt. Casimir cosmology. *International Journal of Modern Physics A*, 37:2241006, 2022. URL <https://doi.org/10.1142/S0217751X22410068>.
5. D. J. Fixsen and et. al. The temperature of the cosmic microwave background at 10 GHz. *The Astrophysical Journal*, 612:86, 2004. URL <https://doi.org/10.1086/421993>.
6. D. J. Fixsen. The temperature of the cosmic microwave background. *The Astrophysical Journal*, 707:916, 2009. URL <https://doi.org/10.1088/0004-637X/707/2/916>.

7. P. Noterdaeme, P. Petitjean, R. Srianand, C. Ledoux, and S. López. The evolution of the cosmic microwave background temperature. *Astronomy and Astrophysics*, 526, 2011. URL <https://doi.org/10.1051/0004-6361/201016140>.
8. S. Dhal, S. Singh, K. Konar, and R. K. Paul. Calculation of cosmic microwave background radiation parameters using coBE/firas dataset. *Experimental Astronomy* (2023), 612:86, 2023. URL <https://doi.org/10.1007/s10686-023-09904-w>.
9. P. L. Kelly and et. al. Constraints on the Hubble constant from supernova Refsdal's reappearance. *Science*, 380:6649, 2023. URL <https://doi.org/10.1126/science.abh1322>.
10. L. Boltzmann. Ableitung des stefanschen gesetzes, betreffend die abhängigkeit der wärmestrahlung von der temperatur aus der electromagnetischen lichttheori. *Annalen der Physik und Chemie*, 22:291, 1879.
11. Stefan J. Über die beziehung zwischen der wärmestrahlung und der temperatur. *Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften in Wien*, 79:391, 1879.
12. E. G. Haug and S. Wojnow. The blackbody CMB temperature, luminosity, and their relation to black hole cosmology, compared to Bekenstein-Hawking luminosity. *Hal archive*, 2024. URL <https://hal.science/hal-04369924>.
13. K. Schwarzschild. über das gravitationsfeld einer kugel aus inkompressibler flussigkeit nach der Einsteinschen theorie. *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse für Mathematik, Physik, und Technik*, page 424, 1916.
14. S. Hawking. Black hole explosions. *Nature*, 248, 1974. URL <https://doi.org/10.1038/248030a0>.
15. R. K. Pathria. The universe as a black hole. *Nature*, 240:298, 1972. URL <https://doi.org/10.1038/240298a0>.
16. W. M. Stuckey. The observable universe inside a black hole. *American Journal of Physics*, 62:788, 1994. URL <https://doi.org/10.1119/1.17460>.
17. N. Popławski. The universe in a black hole in Einstein–Cartan gravity. *The Astrophysical Journal*, 832:96, 2016. URL <https://doi.org/10.3847/0004-637X/832/2/96>.
18. E. T. Tatum. *A Heuristic Model of the Evolving Universe Inspired by Hawking and Penrose in New Ideas Concerning Black Holes and the Universe*, Edited by Eugene Tatum. 2019. URL <https://doi.org/10.5772/intechopen.87019>.
19. O. Akhavan. The universe creation by electron quantum black holes. *Acta Scientific Applied Physics*, 2:34, 2022. URL <https://actascientific.com/ASAP/pdf/ASAP-02-0046.pdf>.
20. C. H. Lineweaver and V. M. Patel. All objects and some questions. *American Journal of Physics*, 91(819), 2023.
21. E. G. Haug and G. Spavieri. New exact solution to Einsteins field equation gives a new cosmological model. *Researchgate.org Pre-print*, 2023. URL <http://dx.doi.org/10.13140/RG.2.2.36524.44161>.
22. S. Wojnow. Alternative cosmology: Λ CDM-like predictions today. *HyperScience International Journal*, 3:4, 2023. URL <https://doi.org/10.55672/hij2023pp24-30>.
23. F. Melia and Shevchuk A. S. H. The $r_h = ct$ universe. *Monthly Notices of the Royal Astronomical Society*, 419: 2579, 2012. URL <https://doi.org/10.1111/j.1365-2966.2011.19906.x>.
24. F. Melia. The $r_h = ct$ universe without inflation. *Astronomy & Astrophysics*, 553, 2013. URL <https://doi.org/10.1051/0004-6361/201220447>.
25. F. Melia. The linear growth of structure in the $r_h = ct$ universe. *Monthly Notices of the Royal Astronomical Society*, 464:1966, 2017. URL <https://doi.org/10.1093/mnras/stw2493>.
26. E. T. Tatum and U. V. S. Seshavatharam. How a realistic linear $r_h = ct$ model of cosmology could present the illusion of late cosmic acceleration. *Journal of Modern Physics*, 9:1397, 2018.
27. M. V. John. $r_h = ct$ and the eternal coasting cosmological model. *Monthly Notices of the Royal Astronomical Society*, 484, 2019. URL <https://doi.org/10.1093/mnrasl/sly243>.
28. M. V. John and K. B. Joseph. Generalized chen-wu type cosmological model. *Physical Review D*, 61:087304, 2000. URL <https://doi.org/10.1103/PhysRevD.61.087304>.
29. C. A. Coulomb. Premier mémoire sur l'électricité et le magnétisme. *Histoire de Académie Royale des Sciences*, pages 569–577, 1785.
30. E. T. Tatum, U. V. S. Seshavatharam, and S. Lakshminarayana. The basics of flat space cosmology. *International Journal of Astronomy and Astrophysics*, 5:16, 2015. URL <http://dx.doi.org/10.4236/ijaa.2015.52015>.
31. E. G. Haug and S. Wojnow. How to predict the temperature of the CMB directly using the Hubble parameter and the planck scale using the Stefan-Boltzman law. *Research Square, Pre-print, under consideration by journal*, 2023. URL <https://doi.org/10.21203/rs.3.rs-3576675/v1>.

32. E. G. Haug and E. T. Tatum. The hawking Hubble temperature as a minimum temperature, the Planck temperature as a maximum temperature and the CMB temperature as their geometric mean temperature. *Hal archive*, 2023. URL <https://hal.science/hal-04308132>.
33. E. G. Haug. Planck quantized bending of light leads to the CMB temperature. *Hal archive*, 2023. URL <https://hal.science/hal-04357053>.
34. H. Reissner. Über die eigengravitation des elektrischen feldes nach der Einsteinschen theorie. *Annalen der Physics*, 355:106, 1916. URL <https://doi.org/10.1002/andp.19163550905>.
35. G. Nordström. On the energy of the gravitation field in Einstein's theory. *Koninklijke Nederlandsche Akademie van Wetenschappen Proceedings*, 20:1238, 1918.
36. R. P. Kerr. Gravitational field of a spinning mass as an example of algebraically special metrics. *Physical Review Letters*, 11:237, 1963. URL <https://doi.org/10.1103/PhysRevLett.11.237>.
37. E. T. Newman and A. I. Janis. Note on the kerr spinning-particle metric. *Journal of Mathematical Physics*, 6: 915, 1965. URL <https://doi.org/10.1063/1.1704350>.
38. E. Newman, E. Couch, K. Chinnapared, A.; Exton, A. Prakash, and R. Torrence. Metric of a rotating, charged mass. *Journal of Mathematical Physics*, 6:918, 1965. URL <https://doi.org/10.1063/1.1704351>.
39. E. G. Haug and G. Spavieri. Mass-charge metric in curved spacetime. *International Journal of Theoretical Physics*, 62:248, 2023. URL <https://doi.org/10.1007/s10773-023-05503-9>.
40. A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.
41. A. S. Eddington. *Report On The Relativity Theory Of Gravitation*. The Physical Society Of London, Fleetway Press, London, 1918.

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