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Investigation of the Casimir Effect in the Hubble Universe and Black Holes

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Abstract: The Casimir effect can either work over very short distances or at very low temperatures. At the cosmic scale, we naturally deal with long distances, but at the same time, we encounter extraordinarily low temperatures, namely the CMB (Cosmic Microwave Background) temperature. Recently, there has been increased interest in the Casimir effect and its implications in cosmology. Here, we briefly demonstrate that the luminosity and radiation pressure of the CMB is mathematical identical to a theoretical Casimir effect within the Hubble sphere, or actually our solution seems to be valid for any Schwarzschild black hole. The Casimir effect for every Schwarzschild black hole is: $F_{ca} = G\frac{m_p^2}{l_p^2}\frac{1}{11520\pi} = k_e\frac{q_p^2}{l_p^2}\frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G}\frac{1}{11520\pi} \approx 3.34 \times 10^{39}\ N$. This also include the Hubble sphere if we treat it as a Schwarzschild black hole that is related to increased interest in black hole cosmology.

Keywords: Hubble sphere; Casimir effect; luminosity; radiation pressure; quantum cosmology; vacuum energy

1. The Hubble-Casimir Effect and its Relation to the CMB luminosity

The total Casimir pressure effect is described by the formula (as seen in [1] and elaborated upon by Balian and Duplantier [2]):

$$P_{ca} = \frac{1}{A}(\bar{X}_0 + \bar{X}_T) = -\frac{\pi^2}{240}\frac{\hbar c}{D^4} - \frac{\pi^2}{45}\frac{1}{\beta}\frac{1}{(\beta\hbar c)^3} + \frac{1}{\beta}\frac{\pi}{D^3}e^{-\alpha} + \cdots \tag{1}$$

Here, $\beta = \frac{1}{k_b T}$, where k_b is the Boltzmann constant, T is the temperature, D is the distance between the two plates, and $\alpha = \frac{\pi\beta\hbar c}{D}$. Typically, when one thinks of the Casimir effect, it is associated with plates placed at extremely short distances. However, as noted by Balian, Duplantier, and others, the Casimir pressure effect is valid at very short distances or, conversely, at very low temperatures. The longer the distance over which the Casimir effect operates, the lower the temperature must be.

The second term in the equation above represents the black body pressure in what is known as an infinite geometry.

$$P_{ca} = -\frac{\pi^2}{45}\frac{1}{\beta}\frac{1}{(\beta\hbar c)^3} \tag{2}$$

where again $\beta = \frac{1}{k_b T}$.

Investigating the Casimir effect in relation to cosmology is not very common. However, there are multiple recent papers on the Casimir effect in relation to cosmology and gravity, as seen in a series of interesting papers [3–8]. One reason is that the Casimir effect is typically related to vacuum, and most of the universe is a vacuum with very low energy, the cosmological vacuum energy.

In cosmology, we certainly do not work on short distances; rather, we focus on very long distances and over very large surfaces ares (in the Hubble sphere). However, we work with extraordinarily low temperatures, specifically the Cosmic Microwave Background (CMB) temperature, which is likely as close as one can get to absolute zero in any observation. The CMB temperature has been measured

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more accurately and is one of the most precise factors in cosmology. It is approximately $2.72K$, as seen in [9–12]. Whether one can really study the Casimir effect at distances equal to the Hubble radius with only the CMB temperature is a question we leave for further study, but we will assume so here.

Let us now consider the Hubble radius, denoted as $R_H = \frac{c}{H_0}$, for the distance between the plates and the CMB temperature $T_{cmb} \approx 2.72K$. This gives:

$$P_{ca} = \frac{dF_c}{dS} = \frac{1}{A}(\bar{X}_0 + \bar{X}_T) = -\frac{\pi^2}{240}\frac{\hbar c}{R_H^4} - \frac{\pi^2}{45}\frac{1}{\beta}\frac{1}{(\beta\hbar c)^3} + \frac{1}{\beta}\frac{\pi}{R_H^3}e^{-\alpha} \approx -1.39 \times 10^{-14}\ J \cdot m^{-3} \tag{3}$$

where $\alpha = \frac{\pi\beta\hbar c}{R_H}$ and $\beta = \frac{1}{k_b T_{cmb}}$. We have used a Hubble parameter given by the recent study of Kelly and et. al [13]: $H_0 = 66.6^{+4.1}_{-3.3}\ (km/s)/Mpc$. For a sphere, the Casimir effect is typically different from that between two plates. However, since the curvature of the Hubble sphere's surface is nearly flat, and because today's universe is considered almost flat based on observations, we suggest the hypothesis that the Hubble surface can be treated as flat plates, especially when we consider pressure per square meter. A square meter on the Hubble surface will have curvature that is negligible compared to what we are examining. However there could also be other ways to interpret the mathematical findings we soon get to.

Further the second term, the black body radiation pressure at infinite geometry is equal to:

$$P_{ca} = -\frac{\pi^2}{45}\frac{1}{\beta}\frac{1}{(\beta\hbar c)^3} \approx -1.39 \times 10^{-14}\ J \cdot m^{-3} \tag{4}$$

we mention this as it is evident the second term is the main contributor when dealing with the Hubble scale. If we multiply P_{ca} with the surface area of the Hubble sphere we get the Casimier force of $F_{ca,H} = P_{ca}A_H = -1.39 \times 10^{-14} \times \frac{4}{3}\pi R_H^2 \approx 3.34 \times 10^{39} N$.

Next, we will analyze the radiation pressure related to CMB luminosity within the Hubble sphere. Based on the Stefan-Boltzmann [14,15] law, we have a theoretical blackbody luminosity derived from the CMB temperature:

$$L_{cmb} = 4\pi R_H^2 \sigma T_{cmb}^4 = 4\pi \left(\frac{c}{H_0}\right)^2 \sigma \times 2.72^4 \approx 7.5 \times 10^{47}\ W \tag{5}$$

Alternatively, this can be predicted using a new equation recently described by Haug and Wojnow [16], where they have derived the CMB luminosity from the Stefan-Boltzmann law, resulting in:

$$L_{cmb} = \frac{\hbar c^6}{15360\pi G^2 m_p^2} \approx 7.50338 \times 10^{47}\ W \tag{6}$$

The formula is valid for any Schwarzschild black hole size [17] and is remarkably independent of the mass of the black hole, unlike the Bekenstein-Hawking luminosity [18], which is mass-dependent. Based on the Stefan-Boltzmann law, the radiation pressure is given by:

$$P_{\text{rad}} = \frac{L}{4\pi R^2 c} \tag{7}$$

For a Hubble sphere, when we input the CMB luminosity as L_{cmb}, Haug and Wojnow [16] have demonstrated that this leads to:

$$P_{\text{cmb}} = \frac{\hbar c}{46080\pi^2 R_s^2 l_p^2} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} \tag{8}$$

Here, we have used the Hubble radius as the Schwarzschild radius, which is permissible when treating the Hubble sphere as a Schwarzschild black hole. The idea of the Hubble sphere as a black hole was likely first introduced in 1972 by Pathria [19], but is an actively discussed alternative view on

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cosmology to this day, see for example [20–26], this could also be highly relevant for the wider class of $R_h = ct$ cosmology models, see for example [27–32] .

This results in a radiation pressure of:

$$P_{\text{cmb}} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} \approx 1.39 \times 10^{-14} J \cdot m^{-3} \tag{9}$$

where $R_H = \frac{c}{H_0}$. Notably, this is identical to the Casimir pressure calculated previously (Equation 3 and 4). When multiplied by the area of the Hubble surface, we obtain:

$$P_{\text{cmb}} A_H = \frac{\hbar c}{46080\pi^2 R_H l_p^2} 4\pi R_H^2 = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G} \frac{1}{11520\pi} \approx 3.34 \times 10^{39} \ N. \tag{10}$$

The Planck gravitational force is given by

$$F_{N,p} = G \frac{m_p m_p}{l_p^2} = \frac{\hbar c}{l_p^2} \tag{11}$$

which is also identical to the Coulomb [33] force for two Planck charges, as we have

$$F_{C,p} = k_e \frac{q_p q_p}{l_p^2} = \frac{\hbar c}{l_p^2} \tag{12}$$

where q_p is the Planck charge which again is equal to $q_p = \frac{e}{\sqrt{\alpha}}$, where e is the elementary charge and α is the fine structure constant. This means we also can re-write equation 10 as

$$P_{\text{cmb}} A_H = \frac{\hbar c}{11520\pi l_p^2} = F_{N,p} \frac{1}{11520\pi} = F_{C,p} \frac{1}{11520\pi}. \tag{13}$$

This value remains constant and is the same for every Schwarzschild black hole. Consequently, we have:

$$P_{\text{cmb}} A_H = F_{ca,H} = G \frac{m_p^2}{l_p^2} \frac{1}{11520\pi} = k_e \frac{q_p^2}{l_p^2} \frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} \approx 3.34 \times 10^{39} \ N \tag{14}$$

where $F_{ca,H}$ stands for the Casimir force acting on the Hubble sphere. The change in the surface area can be seen as from Planck mass micro black hole to the Hubble sphere, this is consistent with growing black hole cosmology models of the $R_H = ct$ type, and likely also a steady state black hole that can be derived from the Haug-Spavieri metric, something we have to come back to in a future paper.

This implies that there could indeed be a Casimir effect within the Hubble sphere and for any Schwarzschild black hole, as the value remains constant at approximately 3.34×10^{39} N.

All of this should also be seen and investigated further in connection with the recent rapid theoretical progress in understanding the CMB temperature. Tatum et al. [34] already heuristically suggested the formula to predict the CMB temperature as follows:

$$T_{\text{cmb}} = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M m_p}} = \frac{\hbar c^3}{k_b 4\pi G \sqrt{R_s 2 l_p}} \tag{15}$$

The formula was first formally mathematically proven by Haug and Wojnow [35] by deriving the CMB temperature from the Stefan-Boltzmann law. Haug and Tatum [36] demonstrated that the CMB temperature formula above could be derived from a more general geometric mean approach, and Haug [37] also showed that the CMB temperature can be derived based on the quantized minimum bending of light at the horizon of a black hole. All in all, it seems that more and more pieces of what we can call the Hubble puzzle are falling into place, and all this can be applied to any Schwarzschild black hole, and can likely relatively easily be extended to Reisner-Nordström [38,39], Kerr [40], Kerr-Newman [41,42] and Haug-Spavieri [43] black holes also.

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2. Possible interpretation

We have found that the Casimir force for the Hubble sphere is given by $F_{ca} \approx G\frac{m_p^2}{l_p^2}\frac{1}{11520\pi} = k_e\frac{q_p^2}{l_p^2}\frac{1}{11520\pi} \approx 3.34 \times 10^{39}\ N$, which is close to the Planck gravitational force and also the Coulomb force between two Planck charges over the distance of the Planck length. This is an enormous force. However, we think this can be seen as an aggregated Casimir force coming from the entire Hubble surface $A = 4\pi R_H^2$, so the Casimir pressure is identical to the CMB pressure of only $P_{ca} = P_{\rm cmb} = \frac{\hbar c}{46080\pi^2 R_H^2 l_p^2} = \frac{F_{ca}}{A_H} = J \cdot m^{-3} = N \cdot m^{-2} \approx -1.39 \times 10^{-14}\ N \cdot m^{-2}$, which is very low. So even if the Casimir force is very strong for the Hubble sphere, it is divided over a large surface area, so the radiation pressure is very low.

As our formulas end up having Planck units in them, this indicates a strong connection between gravity and Planck units. Einstein [44] already in 1916 suggested that the next step in the development of gravity theory would be a quantum gravity theory. Already in 1918, Eddington [45] suggested that such a theory had to be linked to the Planck length (Planck units). Quantum gravity is an actively researched area to this day, and perhaps even our findings of the Casimir effect for the Hubble sphere can one day be incorporated into such theories, but that is outside the scope of this paper.

Another interesting question is how the Casimir force can be identical to the gravitational force and the Coulomb force, as we have demonstrated in the derivations above. It is important to be aware that this is naturally only applicable to black holes. One possible interpretation is that in black holes, at least close to the singularity, mass is so densely packed together that everything is possible. Therefore, gravity and electromagnetism, along with all other forces, become united into one force, as predicted by multiple researchers. However, this should naturally be further investigated and, at this point, is at best only a hypothesis. Still, it's a hypothesis that may gain strength with this paper.

3. Conclusion

We have tried to demonstrate that radiation pressure, derived from CMB luminosity, which is again linked to CMB temperature, exhibits close similarities to the Casimir force. We have named this phenomenon the Casimir-Hubble effect. Alternatively, it could be referred to as the black hole Casimir effect, as it should apply to all black holes. We have shown that for any Schwarzschild black hole we have: $F_{ca} = G\frac{m_p^2}{l_p^2}\frac{1}{11520\pi} = k_e\frac{q_p^2}{l_p^2}\frac{1}{11520\pi} = \frac{\hbar c}{11520\pi l_p^2} = \frac{c^4}{G}\frac{1}{11520\pi} \approx 3.34 \times 10^{39}\ N$. That is in a black hole the Casimier force is equal to the gravity force for two Planck mass particles or the Coulomb force of two Planck charges, after we multiply these by $\frac{1}{11520\pi}$.

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