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[Yaakov Friedman](#)^{*} and [Tzvi Scarr](#)

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Article

A Unified Relativistic Theory of Electromagnetism and Gravity

Yaakov Friedman ^{1,*} and Tzvi Scarr ^{2,*}

¹ Jerusalem College of Technology, Jerusalem, Israel; friedman@g.jct.ac.il

² Jerusalem College of Technology, Jerusalem, Israel; scarr@g.jct.ac.il

* Correspondence: friedman@g.jct.ac.il

‡ These authors contributed equally to this work.

Abstract: This article lays the foundation for *Extended Relativity (ER)* – a unified geometric approach to the electromagnetic and gravitational field and relativistic dynamics in them. We explain here the basic tenets of *ER*, including the *influenced spacetime* of an object and the *Extended Principle of Inertia*. These two new ideas enable a unified theory of these fields. We introduce *local scaling functions* for describing the geometry of a spacetime influenced by a field and construct a simple, universal local scaling function which unifies electromagnetism and gravity. We recover the full electromagnetic theory, including Maxwell's equations. For gravitation, *ER* passes all of the tests of General Relativity. Despite the non-linearity of relativistic gravitation, we obtain a method of combining fields from multiple sources. Finally, we identify the differences in the relativistic treatment of the electromagnetic field by Special Relativity and of the gravitational field by General Relativity and indicate how these differences are resolved in *ER*.

Keywords: unification; relativity of spacetime; Extended Principle of Inertia; geometrization of spacetime; local scaling function of curved spacetime; four-potential of a field; tensor of a field; relativistic dynamics; superposition in gravitation

1. Introduction

There are two relativistic theories properly describing the electromagnetic field, the gravitational field, and the motion of objects in these fields. Both theories were introduced by A. Einstein. Special Relativity (*SR*) describes the electromagnetic field [1,2], and General Relativity (*GR*) describes the gravitational field [3–5].

This special issue, *Relativity Based on Symmetry – Part II*, presents significant new achievements in the unification of the relativistic treatment of electromagnetic and gravitational fields, a theory we call *Extended Relativity (ER)*. At the time of the publication of Part I, *ER* handled electromagnetism and gravity via completely distinct models. To achieve the unified version of *ER* presented here, we applied the geometric approach also to electromagnetism. This involved several new ideas, which will be presented here. In its new version, *ER* now unifies not only electromagnetism and gravity, but also the propagation of light in media. This article describes the new *ER*, starting from its basic principles.

For the reader's convenience, we recall the main results of Part I and discuss the new results presented here.

Using the symmetry contained in Galileo's Principle of Relativity, it is shown in [6] that the Lorentz transformations are the true transformations between inertial frames. These transformations define a group of automorphisms $\varphi : D \rightarrow D$ of the ball $D = \{\mathbf{v} : |\mathbf{v}| < c\}$ of relativistically admissible 3D velocities, that is, 3D velocities less than c , the speed of light in vacuum. The generators of these automorphisms are used to write a relativistic dynamics equation for the motion of a charged particle in an electromagnetic field. This was the beginning of a geometric approach to electrodynamics.

The dynamics equation derived in [6] is three dimensional, handles only the electromagnetic force and does not handle gravity at all. The new *ER* is four dimensional, fully Lorentz covariant and unifies gravity and electromagnetism.

In Part I, gravity is handled separately, in [7]. As in *GR*, the *ER* approach, in its then-current form, was based on the existence of a metric on flat spacetime. For the gravitational field of a static, spherically symmetric source, the *ER* metric is equivalent to the Schwarzschild metric and thus satisfies Einstein's field equations and passes the tests of *GR*. Indeed, the trajectories of *ER* coincide with those of *GR*.

Two experiments were proposed in [8] to test whether *GR* or *ER* is the correct theory. The first experiment tests the relativistic time dilation beyond the weak-field post-Newtonian approximation. The second experiment measures the one-way speed of light toward and away from a gravitational source. These experiments have yet to be performed, so the jury is still out.

ER's quantum model is still in its infancy and begins with the articles [9] and [10] from Part I. Here, in Part II, we focus on the unification of electromagnetism and gravity.

2. Outline of the Approach

First and foremost, we want to derive a *simple* theory. Of course, a theory must be sophisticated enough to describe all known phenomena. Hence, we take Einstein's advice:

"A physical theory should be as simple as possible, but not simpler." - Albert Einstein

For example, an object's motion is represented by a worldline in an inertial (lab) frame rather than on a manifold. This makes the math much simpler.

Next, *ER* uses a bottom-up approach. To explain this, consider the following two examples, one from the business world, and one from Artificial Intelligence (AI).

There are two approaches to business management – top-down and bottom-up. In the top-down approach, all of the major decisions are made at the top, by the executives. Then they tell the lower levels what has been decided. The bottom-up approach starts at the bottom. The lowest level employees, who are closest to the customer end of the business, inform the upper levels what is needed.

The second examples comes from AI. Suppose one wants to teach an AI what a "chair" is. The original approach was to tell the AI as many properties of chairs as possible. A chair has a place to sit, a back, legs, etc. This is top-down. More recently, it has been discovered that a bottom-up approach works better. One shows the AI dozens, if not hundreds, of pictures of different chairs. The AI then forms its own concept of chair.

Returning to physics, the standard description of action at a distance uses a top-down approach. One assumes the existence of a field. The field is a solution of the field equations, which determine the field at any spacetime point, given from the sources of the field. From the field, one then derives the equations of motion, which give the acceleration of objects moving in the field. Thus, the approach is top-down, from the field to objects affected by it.

The *ER* approach is bottom-up. We start by considering how an inanimate object moves in a spacetime influenced by fields. Since the object has no mechanism to change its velocity *of its own accord*, we came to the conclusion that objects move according to an *Extended Principle of Inertia*. As the name implies, this extends Galileo's Principle of Inertia and states that an object's worldline is a geodesic in its influenced spacetime. We introduce a *local scaling function (LSF)* which describes the geometry of the influenced spacetime and leads to a natural definition of the "length" of a worldline. In order to lead to a valid relativistic theory, the *LSF* must satisfy certain properties. Guided by simplicity, we construct a simple *LSF*, which depends on two four-vector-valued functions $A(x)$ and $I(x)$.

Using the Euler-Lagrange equations, with proper time as the evolution parameter on the object's worldline, we obtain the equation of motion in our inertial lab frame. This equation defines the object's four-acceleration, which depends on its spacetime position, its four-velocity and (at most) one intrinsic parameter of the object. The dependence on position is an expression of the influence of the field and is defined by a tensor. The tensor associated to $A(x)$ coincides with the standard electromagnetic field strength tensor, and $A(x)$ is the four-potential of the field. The resulting equation of motion

coincides with the standard equation of motion of charges in an electromagnetic field, and the field equations are exactly Maxwell's equations. For a single source, one can compute $A(x)$ explicitly using the Newtonian limit and Lorentz covariance. In *ER*, like in EM, the EM field tensor of a collection of sources is the sum of the tensors of each source. As a result, the acceleration of a moving charge in a combined field is the sum of the accelerations caused by the field of each source. In other words, *ER* reproduces the usual superposition of electromagnetic fields.

For a single-source gravitational field, one can compute $l(x)$ explicitly using the Newtonian limit and Lorentz covariance. The function $l(x)$ is a four-potential for the field, since its derivatives define a gravitational field tensor. The equation of motion of an object in the field is obtained by contracting the field tensor with the object's four-velocity. This equation of motion passes all the tests of *GR*. The field tensor is not linear in the source, but it does decompose into a semi-classical part and a dilation component. Moreover, the dilation component may be computed directly from the semi-classical part. The semi-classical tensor of the combined field is the sum of the semi-classical tensors of each source. This is a relativistic extension of the superposition principle of Newtonian gravity. One then computes the dilation component for the combined field directly from the combined semi-classical tensor. The tensor of the field is the sum of the semi-classical and the dilation tensors. Since the dilation tensor is very local, it can be neglected when we are very far from the sources.

3. Influenced Spacetime

The room I am sitting in is full of influences. To mention just a few, there's the gravitational field of the Earth, electric fields generated by currents running through my computer, electromagnetic radiation from cellular phone towers, and a prism refracting the incoming sunlight into a rainbow. Despite this multitude of influences, any given object in the room only "notices" some of them. I, for example, notice only gravity. I am oblivious to the presence of the other fields. They don't affect me, at least not noticeably. For me, they don't exist. My *influenced spacetime* is determined solely by gravity.

There are two cellphones on my desk – a brand new model and an older phone. Each phone is influenced by electromagnetic radiation, but only by the specific frequency assigned to it. For my new phone, radiation of my old phone's frequency doesn't exist, and vice-versa. My new phone has a GPS locator which can display the phone's latitude and longitude. It also has an accelerometer and a gyroscope, so it knows when to rotate the screen display. The old phone has neither of these features. It is unaware of the GPS satellites and its own orientation. Thus, the two phones are affected by different sets of influences. Each one has its own influenced spacetime. The photons of sunlight streaming through my window are influenced by the prism. This influence clearly depends on each photon's frequency.

For another example, consider two wood screws, one with right-hand threads, and the other with left-hand threads. In addition, the right-hand screw has a pitch (distance between threads) of 1.25 mm, while the left-hand screw has a pitch (distance between threads) of 2.5 mm. Rotating the right-hand screw one full revolution causes the screw to *enter* the wood a depth of 1.25 mm. The same rotation applied to the left-hand screw causes it to *exit* the wood a distance of 2.5 mm. If we consider rotations to be the only influence on the screws, then the influenced spacetime of each screw depends on its orientation and pitch.

In *ER*, an object's influenced spacetime is determined solely by the fields which affect it and (at most) one intrinsic parameter of the object. The influenced spacetime of a massive, uncharged object is determined by solely by gravity. By the Equivalence Principle, not even the object's mass affects its motion. A charged particle is influenced by electromagnetic fields and one intrinsic parameter – its charge-to-mass ratio q/m . The charge q expresses not only the *strength* of the field's influence, but also the *direction* of influence, since q can be positive or negative. A neutral particle doesn't notice EM fields at all.

The next question is: How does an object move in its influenced spacetime?

4. The Extended Principle of Inertia

Galileo's Principle of Inertia (later codified as Newton's First Law) states that

"A body continues in its state of rest, or moves with constant velocity in a straight line, unless acted upon by a force."

Such motion is known as *free motion*, or motion with zero acceleration. The explanation of this law is that an inanimate object is unable to change its velocity of its own accord.

The worldline of free motion is a straight line. In Euclidean geometry, the distance between any two points on such a worldline is minimal. In Minkowski spacetime, the "distance" is the interval $ds = \sqrt{c^2 dt^2 - dx^2}$ between the two spacetime points. If the distance $\sqrt{dx^2}$ is minimal, then the interval is maximal. This is due to the signature of the Minkowski metric. In either case, the worldline of free motion has extremal length. A line of extremal length between any two points is called a *geodesic*.

In the presence of forces, our moving object still cannot change its velocity, but in our inertial frame, we don't observe its worldline as extremal. In fact, we measure nonzero acceleration. Nevertheless, in the object's influenced spacetime, its worldline *is* extremal. This is the *Extended Principle of Inertia*:

Since an inanimate object is unable to change its velocity, its worldline is a geodesic in its influenced spacetime.

That is, every object has constant velocity, or zero acceleration in its influenced spacetime.

We prefer to work with "zero acceleration" rather than constant velocity, because having zero acceleration is a *local* property. The claim that the velocity is constant during the motion requires comparing the velocity at different spacetime points, which is non-trivial in curved spacetime.

According to the Extended Principle of Inertia, all objects move along geodesics in their influenced spacetime. To determine geodesics, we must therefore define the notion of the length of a worldline between two points on it. In *GR*, the distance between two nearby points is defined by a metric. The idea of a metric is an extension of the Pythagorean Theorem to curved spacetime. However, as we will see later, the metric approach cannot describe the spacetime influenced by an electromagnetic field. Thus, we introduce a new, broader category of objects, called *local scaling functions*, which is wide enough to describe the geometries of spacetime influenced by both electromagnetic and gravitational fields. It also describes the geometry for light propagating in media.

5. Local Scaling Functions

In *ER*, the motion of an object is described by a worldline $x(\sigma)$ in the Minkowski space of a far away observer. The Minkowski metric tensor is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Raising and lowering of indices is done via $\eta_{\mu\nu}$. The Minkowski dot product of two four-vectors x, y is defined by

$$x \cdot y = \eta_{\mu\nu} x^\mu y^\nu.$$

Note that we abbreviate $x \cdot x$ by x^2 .

It is known that the dot product of two four-vectors is a Lorentz-invariant scalar. *ER* is covariant under the extended Lorentz group, also called the Poincaré group, which describes the true transformation of spacetime and four-vectors between inertial frames. The parameter σ on the worldline may be chosen freely, but later we will use only the proper time τ of the moving object, which is a Lorentz-invariant scalar.

To define the length of a worldline, it is enough to define the distance between two close points x and $x + \epsilon u$ on the worldline, where u is a four-vector denoting the direction from x to $x + \epsilon u$. If we normalize u , meaning $u^2 = 1$, then the distance between these points in Minkowski space is ϵ . However, in the moving object's influenced spacetime, the distance between x and $x + \epsilon u$ may differ from ϵ by some scaling factor, which may depend on the point x and the direction u . We thus arrive at the following definition.

Definition The *local scaling function (LSF)* $L(x, u)$ is a scalar-valued function of a spacetime position x and a four-vector u , with the meaning that the distance between two points $P = x$ and $Q = x + \epsilon u$ in an object's influenced spacetime is $L(x, u)\epsilon$ if ϵ is small.

In other words, $L(x, u) = \lim_{\epsilon \rightarrow 0} \frac{D(x, x + \epsilon u)}{\epsilon}$, where $D(x, y)$ denotes the distance in the object's spacetime between two spacetime points x and y .

The *length* of $x(\sigma)$, from σ_1 to σ_2 , is defined to be

$$S[x(\sigma)] = \int_{\sigma_1}^{\sigma_2} L\left(x(\sigma), \frac{dx}{d\sigma}\right) d\sigma. \quad (1)$$

Note that in (1), we have substituted *on a given worldline*, that is, $x = x(\sigma)$ and $u = dx/d\sigma$. Thus, after substitution, the local scaling function $L(x, u)$ describes infinitesimal distances from the point $x(\sigma)$ in the direction of the local tangent to the worldline.

A worldline $x(\sigma)$ is a *geodesic* if, for any two points $x(\sigma_1)$ and $x(\sigma_2)$ on the worldline, $S[x(\sigma)]$ is extremal among all worldlines connecting $x(\sigma_1)$ and $x(\sigma_2)$. To find geodesics, we first define, for $\mu = 0, 1, 2, 3$, the *unit-free μ -momentum*

$$p_\mu(\sigma) = \left. \frac{\partial L(x, u)}{\partial u^\mu} \right|_{x=x(\sigma), u=dx(\sigma)/d\sigma}. \quad (2)$$

Theorem The worldline $x(\sigma)$ is a geodesic if and only if for every μ , we have

$$\frac{d}{d\sigma} p_\mu(\sigma) = \left. \frac{\partial L(x, u)}{\partial x^\mu} \right|_{x=x(\sigma), u=dx(\sigma)/d\sigma}. \quad (3)$$

Equations (3) are known as the *Euler-Lagrange equations*. The notation in (2) and (3) means that one first differentiates L by either x^μ or u^μ and then substitutes $x = x(\sigma)$ and $u = dx(\sigma)/d\sigma$.

Corollary 1 The acceleration of an object with respect to σ in its influenced spacetime defined by the LSF $L(x, u)$ is

$$a(x(\sigma)) = \frac{d}{d\sigma} p_\mu(\sigma) - \left. \frac{\partial L(x, u)}{\partial x^\mu} \right|_{x=x(\sigma), u=dx(\sigma)/d\sigma}. \quad (4)$$

Proof In Minkowski space, $L(x, u) = \sqrt{u^2}$, implying that $p(\sigma) = dx(\sigma)/d\sigma$ and $\frac{\partial L(x, u)}{\partial x^\mu} = 0$. This implies that $a(x(\sigma)) = dx^2(\sigma)/d\sigma^2$ is the four-acceleration, with respect to σ , on the worldline. In a general, influenced spacetime defined by an arbitrary $L(x, u)$, the definition of a geodesic implies that the acceleration on any worldline should vanish. By the theorem, $a(x(\sigma)) = 0$ along a geodesic.

Corollary 2 (The Law of Conservation) If the local scaling function is independent of x^μ , then the μ -momentum p_μ is conserved along geodesics.

6. A Simple, Universal Local Scaling Function

To determine a more explicit form for $L(x, u)$, we explore the properties that the LSF must have. First, notice that from definition (1), the length of a worldline ostensibly depends on the parameter σ . However, the motion is defined by the worldline, as a collection of events, which is independent of the parametrization. Thus, in order for the physics to be independent of the parametrization, the length $S[x(\sigma)]$ of a worldline $x(\sigma)$ in the influenced spacetime defined by $L(x, u)$ must be independent of the choice of parameter σ . What property of the LSF will ensure this?

Let σ' be another parametrization of the same worldline. Define a function $f(\sigma) = \sigma'$ such that $x(\sigma) = x'(\sigma'(\sigma))$. Since the parametrizations preserve order, $f'(\sigma) > 0$. Now $\frac{dx}{d\sigma} = f'(\sigma) \frac{dx'}{d\sigma'}$ and $d\sigma = \frac{1}{f'(\sigma)} d\sigma'$, implying that

$$L\left(x(\sigma), \frac{dx}{d\sigma}\right) d\sigma = L\left(x'(\sigma'), f'(\sigma) \frac{dx'}{d\sigma'}\right) \frac{1}{f'(\sigma)} d\sigma'.$$

Thus, in order to have $L\left(x(\sigma), \frac{dx}{d\sigma}\right) d\sigma = L\left(x'(\sigma'), \frac{dx'}{d\sigma'}\right) d\sigma'$, we need

$$L\left(x'(\sigma'), f'(\sigma) \frac{dx'}{d\sigma'}\right) = f'(\sigma) L\left(x'(\sigma'), \frac{dx'}{d\sigma'}\right). \quad (5)$$

In other words, for any positive scalar a , we require that $L(x, au) = aL(x, u)$. Thus, in order that the geodesics will be independent of the parametrization, $L(x, u)$ must be *positive homogeneous in u of degree 1*. For such an $L(x, u)$, we are free to use any parameter. Indeed, a judicious choice of parameter can greatly simplify the calculations.

We do not want to assume that the *LSF* has a particular form, as it is done for Lagrangians. Rather, we want to define first the properties of this function that will lead to a physically acceptable relativistic theory.

The local scaling function $L(x, u)$ must satisfy the following properties:

1. $L(x, u)$ must be a Lorentz-invariant scalar-valued function in order to satisfy the Principle of Relativity.
2. $L(x, u)$ must be positive homogeneous in u of degree 1 to guarantee independence of the parametrization.
3. When the field strength tends to 0, we must have $L(x, u) = \sqrt{u^2}$, the local scaling function for Minkowski space.
4. $L(x, u)$ is allowed to depend on at most one parameter intrinsic to the object.

Based on these properties, we derive a simple, universal *LSF*. This function describes the geometry of an object's influenced spacetime. It is simple in the sense that it is minimal in the order of the independent variables. It is universal in the sense that it unifies electromagnetism and gravity. After we obtain a more explicit form of $L(x, u)$, we will apply the Euler-Lagrange equations to it and derive a relativistic dynamics equation which models all known electromagnetic and gravitational phenomena.

First, we deal with Property 1. In order for $L(x, u)$ to be a Lorentz-invariant scalar function, we can use the four-vector u . But since x is not a four-vector (since its coordinates change during translation), we need another four-vector associated to x . This means that we need a four-vector-valued, Lorentz-covariant function, say $A(x)$, defined on Minkowski space. Since the Minkowski dot product of two four-vectors in Minkowski space is a Lorentz-invariant scalar, the scalar-valued function $A(x) \cdot u$ will be Lorentz invariant. Now, from Properties 3 and 4, we obtain the first candidate for a simple *LSF*:

$$L_1(x, u) = \sqrt{u^2} + kA(x) \cdot u, \quad (6)$$

where k is an intrinsic parameter of the moving object. This *LSF* also satisfies Property 2. We show now that this *LSF* defines the influenced spacetime of charges moving in an EM field. Later, we show how to expand this function to incorporate also the influence of gravitational fields.

To derive the equation of motion in the influenced spacetime defined by $L_1(x, u)$, we apply the Euler-Lagrange equations. Use the proper time τ as the evolution parameter on the worldline, where

$$d\tau^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \quad (7)$$

and denote differentiation by τ with a dot.

In order to apply the Euler-Lagrange equations to the *LSF* (6), we first compute the momentum $p_\mu(x)$, defined by (2):

$$p_\mu = \dot{x}_\mu + kA_\mu(x). \quad (8)$$

Next, compute

$$\left. \frac{\partial L(x, u)}{\partial x^\mu} \right|_{x=x(\sigma), u=dx(\sigma)/d\sigma} = kA_{\nu, \mu}(x) \dot{x}^\nu. \quad (9)$$

The Euler-Lagrange equations (3) now imply that

$$\ddot{x}^\alpha = kF_V^\alpha(A) \dot{x}^\nu, \quad (10)$$

where $F_{\lambda\nu}(A) = A_{\nu, \lambda} - A_{\lambda, \nu}$ and $F_V^\alpha(A) = \eta^{\alpha\lambda} F_{\lambda\nu}(A)$. For $k = q/mc^2$, equation (10) is the equation of motion of a particle with charge-to-mass ratio q/m in an EM field with antisymmetric field strength tensor F_V^α (see also [2], equation (12.3)). Hence, $A(x)$ is a well-defined four-potential of the EM field. For a single-source field, $A(x)$ is the well-known Liénard-Wiechert four-potential.

If we apply equation (10) also to describe the motion of a planet in the gravitational field of the Sun, we obtain that the trajectory is a precessing ellipse. For Mercury, however, this precession differs from the observed one [12]. Thus, our $L_1(x, u)$ (6) cannot be used for the gravitational field. We must extend this function to be able to describe gravitation properly.

To extend the L_1 (6), we introduce a new Lorentz-covariant, four-vector-valued function $l(x)$ and call it the gravitational four-potential. As above, we can obtain a Lorentz-invariant scalar function in the form $l(x) \cdot u$. If we add it to the right-hand side of (6), the equation of motion will be similar to (10), and this will not solve the problem with the precession of Mercury. Thus, we must place this term under a square root. However, in order keep Property 2, we must take the square of this function. Thus, we obtain the following *LSF*

$$L(x, u) = \sqrt{(\eta_{\mu\nu} - l_\mu(x)l_\nu(x))u^\mu u^\nu} + kA_\mu(x)u^\mu. \quad (11)$$

The minus sign under the square root is needed to avoid superluminal velocities. As we will see, l is a null vector, and so has only three degrees of freedom. This is significantly less than the ten degrees of freedom of the metric in *GR*.

The function $L(x, u)$ (11) satisfies all the properties of a *LSF*. It was shown [11] and [12] that it defines properly the motion of massive and massless objects in EM and gravitational fields, and passes all *GR* tests. The tensorial form and superposition principle of the gravitational field will be presented in a forthcoming paper.

7. Comparison of *SR*, *GR*, and *ER*

Einstein dreamed of uniting his two relativity theories, *SR* and *GR*. He did not succeed, however, as there are many differences between the two theories. We discuss several of these obstacles to unification here as well as the way in which *ER* overcomes them.

1. *Effect of the field*. In *SR*, an electromagnetic (EM) field effects the motion of charges through the Lorentz force. In *GR*, on the other hand, the gravitational force curves spacetime, and the motion of objects is along geodesics in curved spacetime. This is an expression of Riemann's idea "force equal geometry". *ER* adopts this geometric approach and extends it to EM.
2. *The background space*. The EM field and the motion of charges in the field in *SR* is described in the Minkowski space (flat spacetime) of an inertial lab frame. On the other hand, *GR* represents the gravitational field and motion in it in curved spacetime, in the form of a four-dimensional pseudo-Riemannian manifold. Whitehead [13] developed a partial theory of gravity using inertial frames. In *ER*, both EM and gravitational fields and motion in them are described in an inertial frame of an observer far away from the sources.

3. *Covariance*. In Minkowski space, the Principle of Relativity demands that relativistic theories must be Lorentz covariant. Indeed, *SR*'s theory of EM is Lorentz covariant. *GR*, on the other hand, assumes general covariance, that is, invariance of the laws of physics under any continuous transformation that preserves non-zero determinants. However, there is no physical need for such wide covariance. In *ER*, we work in inertial frames, as mentioned above, and require only Lorentz covariance.
4. *Potential of the Field*. In *SR*, the EM field can be described by a four-potential $A_\mu(x)$ which depends on the spacetime point x . In *GR*, the curving of spacetime caused by a gravitational field is described by a metric, expressed as a rank two, 4×4 symmetric metric tensor $g_{\mu\nu}(x)$. In *ER*, the spacetime curving due to the EM or/and gravitational fields is described by a local scaling function (11). The Lorentz covariant four-potentials are $A_\mu(x)$ for the EM field and $l_\mu(x)$ for a single-source gravitational field.
5. *Field Tensor*. In *SR*, the 3D electric and magnetic field vectors are combined into a rank two, 4×4 antisymmetric EM field strength tensor $F_{\alpha\nu}(x)$, depending on the spacetime point x . The tensor

$$F_{\alpha\nu}(x) = A_{\nu,\alpha} - A_{\alpha,\nu}, \quad (12)$$

where $A_{\nu,\alpha} = \frac{\partial A_\nu(x)}{\partial x^\alpha}$. In *GR*, we have Christoffel symbols, defined as

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}). \quad (13)$$

In *ER*, in addition to the tensor $F_{\alpha\nu}(x)$ for EM, we have a tensor $G_{\alpha\mu\nu}$ for the gravitational field, defined as follows: The tensor $G_{\alpha\mu\nu}$ decomposes as a sum $G_{\alpha\mu\nu} = G_{\alpha\mu\nu}^{(s)} + G_{\alpha\mu\nu}^{(d)}$, where

$$G_{\alpha\mu\nu}^{(s)} = l_\alpha l_{\mu,\nu} - l_\mu l_{\nu,\alpha} + l_\nu l_{\alpha,\mu} \quad (14)$$

is called the *semi-classical tensor*, and

$$G_{\alpha\mu\nu}^{(d)} = G_{\beta\mu\nu}^{(s)} l^\beta l_\alpha \quad (15)$$

is called the *dilation tensor*.

6. *Evolution Parameter*. The proper time τ , defined by (7), is the evolution parameter in *SR* for motion in an EM field. The proper time is a Lorentz-invariant scalar and is independent of the field. In *GR*, on the other hand, the evolution parameter is the arc length with respect to the metric of the curved spacetime under consideration. This parameter is dependent on the field. This makes it difficult to combine the fields of multiple sources, as each source defines its own parameter. In *ER*, we use proper time as the evolution parameter in both EM and gravitational fields. This makes it possible to obtain the superposition of fields also for gravity.
7. *Dynamics*. In both EM and gravity, the acceleration of an object moving in a field depends on both the field and the velocity of the moving object. In EM, the four-acceleration \ddot{x} of a particle, with charge-to-mass ratio q/m and four-velocity \dot{x} , in an EM field with field strength tensor $F_{\alpha\nu}(x)$ is ([2], equation (12.3))

$$\ddot{x}_\alpha = \frac{q}{mc^2} F_{\alpha\nu}(x) \dot{x}^\nu. \quad (16)$$

That is, in EM, the acceleration is obtained by contracting the field strength tensor with the four-velocity of the moving object.

In *GR*, one first solves the field equations to find the metric tensor $g_{\mu\nu}$ and then computes the Christoffel symbols. *GR*'s equation of motion, known as the *geodesic equation*, is

$$\ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0. \quad (17)$$

Here, the acceleration is a contraction of the Christoffel symbols with the four-velocity.

In *ER*, applying the Euler-Lagrange equations to the local scaling function for an EM field reproduces the standard EM field strength tensor $F_{\alpha\nu}(x)$ and equation (16) for the motion of charges in the field. Applying the Euler-Lagrange equations to the local scaling function for a gravitational field, and using the rank 3 tensor $G_{\alpha\mu\nu}$, the equation of motion is

$$\ddot{x}_\alpha = G_{\alpha\mu\nu}\dot{x}^\nu\dot{x}^\mu - G_{\beta\mu\nu}\dot{x}^\nu\dot{x}^\mu\dot{x}^\beta\dot{x}_\alpha, \quad (18)$$

which is a contraction of the field tensor with the four-velocity of the object.

8. *Superposition of Fields*. The issue here is how to combine fields from multiple sources. In EM, it is known that the EM field tensor of a collection of sources is the sum of the tensors of each source. From (16), it follows that the acceleration of a moving charge in a combined field is the sum of the accelerations caused by the field of each source.

GR has no method for the superposition of gravitational fields.

In *ER*, the semi-classical tensor of a field from several sources is the sum of the corresponding tensors from each source. This is a relativistic extension of the superposition principle in Newtonian gravity. The dilation tensor, related to the gravitational time dilation, is derived from the semi-classical tensor and use of (15).

9. *Field Equations*. In EM, Maxwell's equations define the field strength tensor $F_{\alpha\nu}(x)$ from the sources of the field.

In *GR*, Einstein's field equations, a system of second-order, non-linear differential equations define the metric from the sources of the gravitational field.

In *ER*, the field tensors are defined first for a single-source field, based on the retarded potential and the Newtonian limit. For a field generated by a collection of sources, the field tensor is obtained by the superposition principle.

We point out that all three theories, *SR*, *GR*, and *ER*, all have the correct Newtonian limit. The differences discussed above are summarized in Table 1.

Table 1. The comparison between *SR*, *GR* and *ER*.

	EM (<i>SR</i>)	Gravity (<i>GR</i>)	EM & Grav. (<i>ER</i>)
1. Field Effect	Force	Curving space	Curving space
2. Background	flat	curved	flat
3. Covariance	Lorentz	general	Lorentz
4. Four-potential	A_μ	$g_{\mu\nu}$	$L(x, u), A_\mu, l_\mu$
5. Tensor	$F_{\alpha\nu}$	$\Gamma_{\mu\nu}^\alpha$	$F_{\alpha\nu}, G_{\alpha\mu\nu}$
6. Parameter	proper time	metric arc length	proper time
7. Dynamics	eq. (16)	eq. (17)	eq. (16) and (18)
8. Superposition	Yes	No	Yes
9. Field equations	Maxwell's	Einstein's	By superposition

8. Summary

This paper is an introduction to the special issue *Relativity Based on Symmetry – Part II*. We presented here the principles of Extended Relativity (*ER*) – a unified relativity theory of the electromagnetic and the gravitational fields. In Section 2, we outlined the *ER* approach. In the two sections which follow, we introduced two new concepts: the *influenced spacetime* of a moving object and the *Extended Principle of Inertia*. These form the foundation of *ER*. In Section 5, we introduced a new description of the geometry of curved spacetime, called the *local scaling function*, which enables the unification of the two relativity theories. In Section 6, we derived a simple, universal local scaling function (11)

from the properties needed to obtain a physically acceptable relativistic theory. We recover [12] the full electromagnetic theory, including Maxwell's equations. For gravitation, *ER* passes all of the tests of *GR*. Gravity is non-linear in the sources. The desire for unity led us to a tensor description for the gravitational field and to a superposition principle. This enables one to combine gravitational fields from multiple sources. In Section 7, we presented the differences between the treatment of the electromagnetic field by Special Relativity and the gravitational field by General Relativity. We indicated how these differences are resolved within *ER*.

9. Conclusions

A necessary step to a Grand Unified Theory is the unification of the two macroscopic fields – EM and gravity. In this article, we presented the main ideas of *Extended Relativity*, which enables the unification of the relativistic description of electromagnetic and gravitational fields, as well as the dynamics in these fields. *ER* already models the propagation of light in media and some quantum effects. Thus, *ER* is a good candidate for grand unification. The next step is to extend *ER* to the quantum region.

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