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A Geometric and Visual Perspective on the Four Color Map Theorem and K5 Non-Planarity and Their Connection

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A Geometric and Visual Perspective on the Four Color Map Theorem and K5 Non- Planarity and Their Connection

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Abstract: In this paper I have depicted proof of Four color map theorem and K5 non planarity in a visual and geometric way and shown how both theorems are connected and originally depict the same problem and deeply connected to some aspect of geometry. This unique approach sheds new light on these mathematical concepts, making them more accessible and intuitive. Furthermore, I demonstrate the deep connection between these theorems and geometry, revealing how geometric representations of graphs play a crucial role in understanding these fundamental problems in graph theory. I have developed a novel method for representing graphs in graph theory, offering a fresh perspective on how we visualize and analyze graph structures. This approach involves representing each vertex as a distinct circle, providing an intuitive and visually engaging way to explore various graph configurations. In this demonstration, I will illustrate this method by showcasing how one, two, three, and four vertices can be represented as groups of circles, allowing for a clearer understanding of their connectivity and relationships. The Four Color Theorem, a long-standing problem in graph theory and combinatorics, asserts that any planar graph can be colored with no more than four colors without adjacent vertices sharing the same color. Traditional approaches to this theorem have primarily focused on combinatorial methods. This paper introduces a novel geometric interpretation, representing the graph vertices as circles in a plane, with edges as tangents between these circles.

Keywords: geometry; graph theory; four color map theorem; K5 non-planarity; kissing number problem; planar graphs; visual representation; contradiction proofs; coloring; mathematical connections

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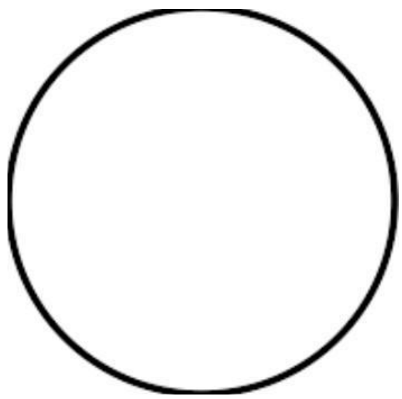
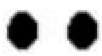
1. One Vertex N1 or K1:

If we represent it as circle:



2. Two Vertex isolated N2:



If we represent it as circle:

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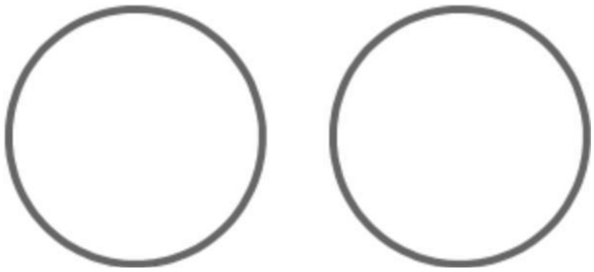
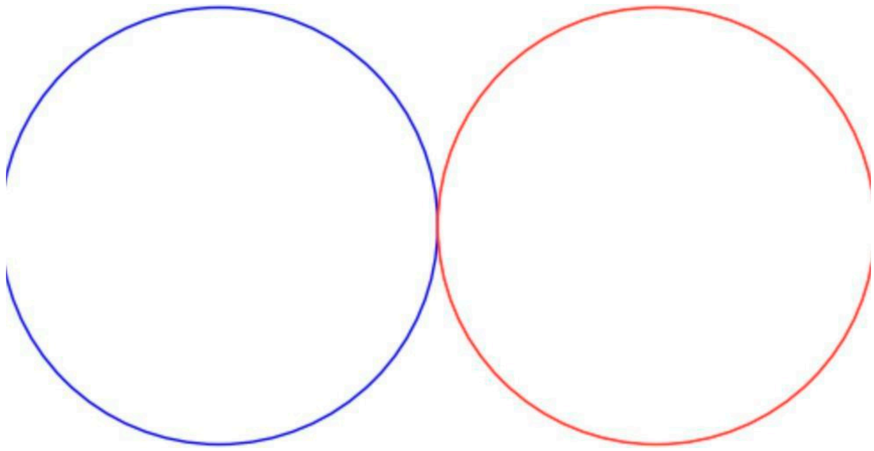


2.1. Two vertices adjacent to each other K2:

If we represent it as circle:



3.1. Three vertices adjacent to each other K3:

If we represent it as circle:

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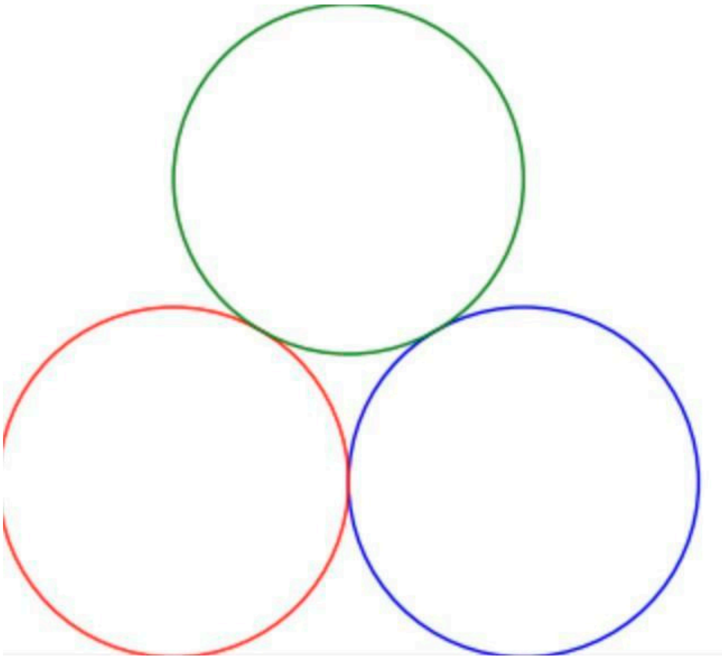
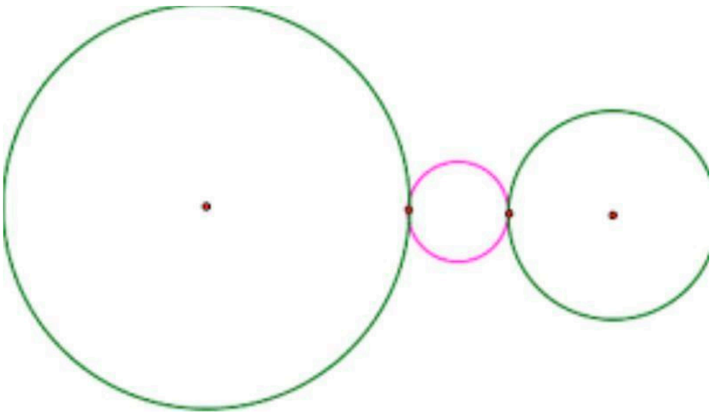


3.2. Also another variety of 3 vertices where only middle vertex connect two other two vertices:

If we represent it as circle:



4.1. Four vertices in a simple rectangular arrangement:

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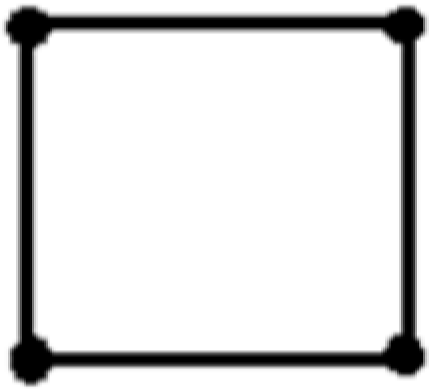
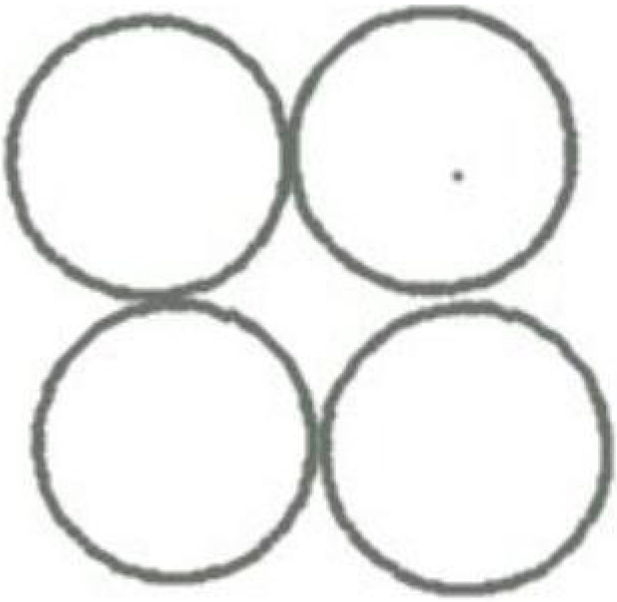


If we represent it as circle:



4.2. Four vertices in K4 rectangular arrangement:

If we represent it as circle:

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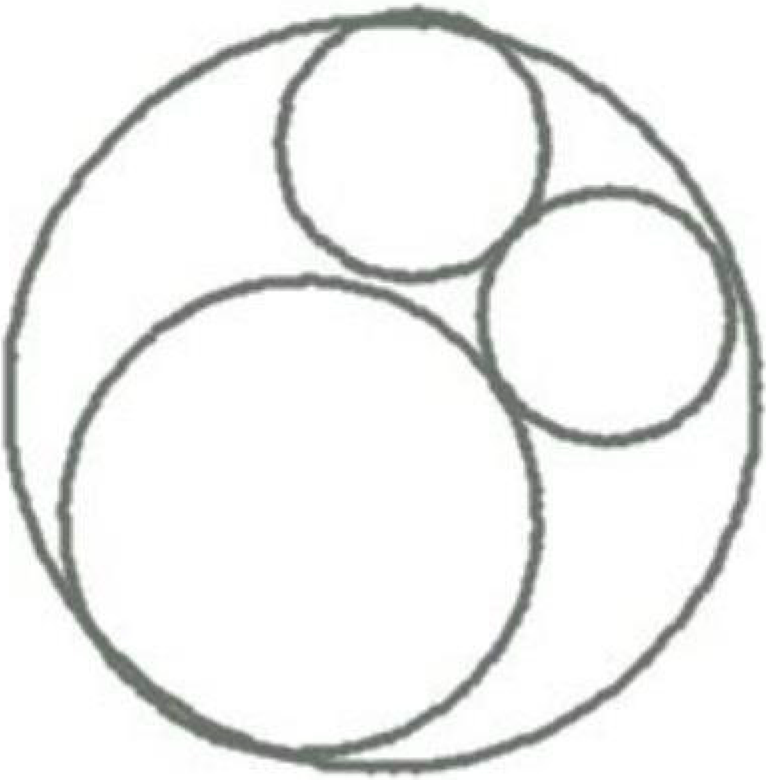
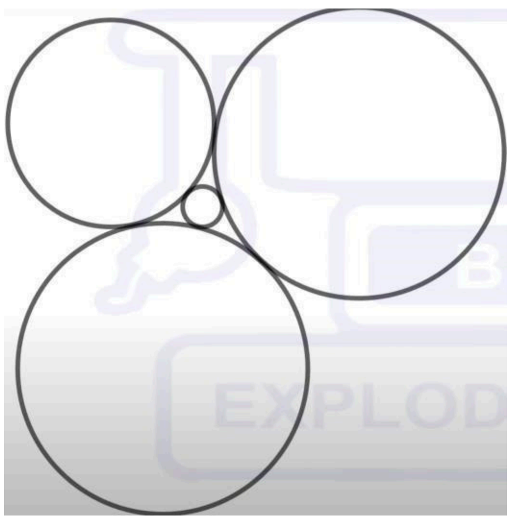


OR



Background:

The Four Color Theorem has been a fundamental problem in graph theory for over a century. It posits that any planar graph can be colored with no more than four colors such that no two adjacent regions share the same color.

Abstract:

This paper presents a unique approach to the Four Color and K5 non planarity and draws a conclusion that both are basically the same problem and their connection goes to geometry of interaction properties of a group of circles(Kissing number problem) . Using a geometric representation of planar graphs with circles and tangents. We explore the implications of the kissing number problem in two dimensions to provide a new perspective on the theorem.

The Four Color Theorem in this Context:

The theorem states that no more than four colors (or planes, in your interpretation) are needed to ensure that no two adjacent circles (vertices) share the same color (or are on the same plane).

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Lemma: Kissing Number in 2D Plane

In a 2D plane, the maximum number of equal-sized circles that can touch another circle without any of them overlapping is 4. The four color map theorem is deeply connected with this geometrical nature of interaction of circles or any closed graphs in a 2d plane.

Proof by Contradiction(Four color map theorem):

Assume that a planar graph needs at least five colors. This would mean, in your circle model, that there exists a set of five circles where each circle must touch all the others.

However, as per the kissing number problem in 2D, a maximum of four circles can mutually touch one another.

This contradiction would suggest that the initial assumption (needing at least five colors) is false, hence supporting the Four Color Theorem.

Proof by Contradiction (K5 non planarity):

Assume that a planar graph needs at least five colors. This would mean, in your circle model, that there exists a set of five circles where each circle must touch all the others. However, as per the kissing number problem in 2D, a maximum of four circles can mutually touch one another. This contradiction would suggest that the initial assumption (needing at least five colors) is false, hence supporting the non-planarity of K5.

Connection Between K5 and Four Color Map Theorem:

It's worth noting that the non-planarity of K5 and the Four Color Map Theorem are deeply connected. Both problems revolve around the concept of coloring, and the kissing number

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problem in 2D helps illustrate this connection. While the Four Color Theorem traditionally applies to the coloring of regions (faces) formed by the edges of a planar graph, the non- planarity of K5 demonstrates the limitations of such colorings, reinforcing the importance of the Four Color Theorem.

Closing and Conclusion:

In this paper, we have explored a fascinating intersection between geometry and graph theory, shedding light on the intriguing concept of the "kissing number" of circles in a 2D plane. We began by introducing the fundamental idea of representing planar graphs using groups of circles, where circles symbolize vertices, and tangents between them represent edges. This visual representation not only simplifies complex graph structures but also offers new insights into long-standing problems.

Our journey led us to the heart of the "kissing number" problem, a classic question in mathematics. We established the essential theorem that in a 2D plane, the maximum number of circles that can touch another circle without overlapping is 4. This simple yet profound result has far-reaching implications in various fields, from geometry to network design.

By providing a lemma and proof, we have contributed to the comprehensive understanding of this intriguing problem. We have clarified that the size of the circles need not be uniform; it is their relative positions and non-overlapping nature that define the kissing

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