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Measurement and the Illusion of Quantum Collapse

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Abstract: Models of quantum measurements are presented, including observers subject to the laws of quantum mechanics. Observers perform multiple measurements in an attempt to disprove the collapse hypothesis. It is shown that under reasonable assumptions the effort cannot succeed. Under a unitary evolution the end state is a correlation between the result of the first measurement and the result(s) of the others, consistent with the observer's collapse hypothesis. The observer may conclude that the laws of nature include a non-unitary collapse mechanism, even if one does not actually exist. Discard of information is the crucial factor which distinguishes entanglement from measurement.

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1. Introduction

According to quantum mechanics, the state of nature evolves unitarily and deterministically. Measurement on the other hand appears to be stochastic, and only the probabilities of various results can be predicted. If we consider the measuring apparatus and the observer to be physical objects subject to deterministic evolution, an apparent contradiction arises. This is known as the measurement problem.

A possible solution is to assert that there is an unknown non-unitary process called collapse that occurs during measurement. Collapse determines which of the various possibilities occurs. This is the heart of the Copenhagen interpretation.

Another approach is to treat measurement and the observer quantum mechanically and show that the observer is lead to believe collapse occurs even though the evolution is deterministic. In this view, collapse is an illusion. This is the approach we analyze. Our observers may consider collapse in their experiments, but the actual evolution will be strictly unitary.

Unless otherwise specified, the Born rule, governing the frequency of results in collapse models and apparent frequency in unitary models, is not assumed. Instead, density operators are treated as first class objects which evolve unitarily. Without collapse, its components do not represent probabilies. In this view, a wavefunction is simply a density operator satisfing the purity condition.

Throughout this article, the term *state* always refers to a density operator. ρ is used for the system of interest, especially the observer's state of knowledge of the system. σ is used to represent the overall state of the laboratory, possibly including: system, probe, observer, and result register. Composite states are written in that order from left to right.

2. Preliminaries

The most general model of quantum measurement consists of a target system and a probe [1]. The probe is prepared in a pure reference state so the overall initial state can be written

$$\sigma_0 = \rho \otimes |0\rangle \langle 0| \tag{1}$$

where ρ is the initial state of the system. The probe is allowed to interact with the system for a time. This arbitrary unitary interaction may be written

$$U = \sum_{aa'} A_{aa'} \otimes |a\rangle \langle a'| \,. \tag{2}$$

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where $A_{aa'}$ acts in the target Hilbert space and a is an index for the probe basis. Since we will always act on the probe in the reference state, we define $A_a = A_{a0}$. The A_a satisfy

$$\sum_a A_a^\dagger A_a = I \tag{3}$$

but are otherwise arbitrary linear operators. After the interaction, the state becomes

$$\begin{aligned}\sigma_1 &= U\sigma_0 U^\dagger \\ &= \sum_{ab} A_a \rho A_b^\dagger \otimes |a\rangle\langle b| .\end{aligned} \tag{4}$$

The observer then inspects the probe to determine its state and thus gain information about the system. If the observer finds result a, she collapses her state of knowledge using a projection operator

$$\Pi_a = I \otimes |a\rangle\langle a| \tag{5}$$

and the final state of the system, according to the observer, becomes

$$\rho_a = N_a A_a \rho A_a^\dagger \tag{6}$$

where $N_a = 1/Tr(A_a \rho A_a^\dagger)$ is a normalization added because the projection changes the trace. Note that only the observer's state of knowledge has collapsed. σ remains unchanged. Based on the collapse hypothesis and the Born rule, the observer expects to see result a with probability

$$P(a) = Tr(A_a \rho A_a^\dagger). \tag{7}$$

A special case occurs if the measurement is repeatable. This occurs if the A_a are themselves projection operators. If the collapse projection (5) is applied, a repeatable measurement is referred to as a von Neumann measurement. In practice the system and probe bases can be coarse grained by the a index (each a value represents a set of basis vectors) without disturbing repeatability. For simplicity we will ignore coarse graining.

The observer performs a second measurement with a new probe to test the collapse hypothesis. The overall state becomes

$$\sigma_2 = \sum_{abcd} B_c A_a \rho A_b^\dagger B_d^\dagger \otimes |c\rangle\langle d| \otimes |a\rangle\langle b| \tag{8}$$

and the observer's state of knowledge becomes

$$\rho_{ac} = N_{ac} B_c A_a \rho A_a^\dagger B_c^\dagger. \tag{9}$$

The observer is instructed to either perform a series of secondary measurements identical to the first or create a collection of identically prepared systems with enough duplication to determine if collapse occurs with some fixed minimum statistical confidence. In the repeatable case, she will expect subsequent measurement results to be identical to the first. In the duplicated case, among the copies where the first measurement has result a, she expects, based on the collapse hypothesis and the Born rule, a fraction

$$P(c|a) = Tr(B_c \rho_a B_c^\dagger) \tag{10}$$

will have result c for the second measurement.

The observer is to report the final result by setting the value of a 3 level result register whose states are labeled 'u', 'c', and 'n' for "unset", "collapsed", and "not collapsed", respectively.

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3. Repeated measurements

In the repeated case the A_a are projections (and hermitian) and the state after N measurements is given by

$$\sigma_2 = \sum_{ab} A_a \rho A_b \otimes (\otimes \Pi_{i=1}^{N} |a_i\rangle \langle b_i|) \otimes |u\rangle \langle u| \tag{11}$$

where the state of the result register is included. The product notation indicates each term in the sum includes a tensor product of N copies of $|a\rangle \langle b|$, one for each probe. When the observer examines the probes, they are entangled with the system degrees of freedom, and the contributions may interfere. The proper contributions are given by the partial trace over the decoupled degrees of freedom[1]. Since the observer is finished with the system, it can be discarded[2] and the reduced state is given by

$$\begin{aligned} \sigma_r &= \sum_{ab} Tr(A_a \rho A_b)(\otimes \Pi_{i=1}^{N} |a_i\rangle \langle b_i|) \otimes |u\rangle \langle u| \\ &= \sum_{a} \rho_{aa}(\otimes \Pi_{i=1}^{N} |a_i\rangle \langle a_i|) \otimes |u\rangle \langle u| , \end{aligned} \tag{12}$$

where ρ_{aa} is a single component of the density operator written in the (projection) basis of the A's. There are several things to note about this result. First, if we were to assume the Born rule, ρ_{aa} would be the probability the observer finds result a. Second, σ_r is in general a mixed state even if the initial state ρ was pure. Third, σ_r no longer represents the entire laboratory: the system has been discarded. Discarding information is a crucial aspect of measurement and we will discuss it further below.

The observer now applies a unitary evolution to set the result state to 'c' if all the probes have the same value. Since this is true in all terms of the sum, the result state $|c\rangle \langle c|$ factors out as a decoupled tensor product, and we can say with certainty that the observer is confident that collapse occurred. Note that this conclusion is independent of the Born rule.

4. Duplicated measurements

To analyze the duplicated case we need to model the observer more explicitly. We also require the Born rule, since the observer is comparing statistical results to a confidence standard and is not certain to conclude that collapse occurred for any finite number of copies. In each measurement we model the observer's memory similar to a second probe. We assume the bases of the probe and memory are aligned, and the examination of the probe is a repeatable/projective measurement.

First consider a single copy of the experiment. After the first interaction (result (4)) the observer examines the probe and the state becomes

$$\begin{aligned} \sigma_{1o} &= \sum_{cd} P_c \sigma_1 P_d \otimes |c\rangle \langle d| \\ &= \sum_{cd} P_c \sum_{ab} A_a \rho A_b^\dagger \otimes |a\rangle \langle b| P_d \otimes |c\rangle \langle d| \\ &= \sum_{cd} A_c \rho A_d^\dagger \otimes |c\rangle \langle d| \otimes |c\rangle \langle d| , \end{aligned} \tag{13}$$

where the projection operators P act in the Hilbert space of the probe. The first $|c\rangle \langle d|$ represents the probe and the second the observer's memory.

Next we discard the probe and trace over its degrees of freedom. The result is

$$\sigma_{1r} = \sum_{a} A_a \rho A_a^\dagger \otimes |a\rangle \langle a| . \tag{14}$$

[1] Note that it is possible to derive the partial trace without assuming the Born rule.
[2] If the observer interacts with the system further, there is no inconsistency but a different analysis is required.

If we were to now discard the system the result would be

$$\sigma_{1p} = \sum_a P(a) \otimes |a\rangle \langle a| \,, \tag{15}$$

a mixed state weighted by probabilities. But we need to retain the system for the second measurement.

After the second interaction and the examination and discard of the probe, the state becomes

$$\sigma_{2r} = \sum_{ad} B_d A_a \rho A_a^\dagger B_d^\dagger \otimes |a\rangle \langle a| \otimes |d\rangle \langle d| \,. \tag{16}$$

where $|a\rangle \langle a|$ represents the observer's memory of the first measurement and $|d\rangle \langle d|$ the second.

At this point we discard the system. Using (6), (7), and (10) the result is

$$\sigma_{2p} = \sum_{ad} P(d|a) P(a) \, |a\rangle \langle a| \otimes |d\rangle \langle d| \,. \tag{17}$$

where the probability functions should be interpreted as numerical values, since we have not yet applied the Born rule to the final state.

When the experiment is copied, the state will be a tensor product of N copies of σ_{2p}. Taking the sum outside the product we get

$$\sigma_{2rr} = \sum_{\{a_i d_i\}} \otimes \Pi_{i=1}^N (P(d_i|a_i) P(a_i) \, |a_i\rangle \langle a_i| \otimes |d_i\rangle \langle d_i|) \tag{18}$$

A unitary evolution is performed to set the result register based on whether the counts in each term match the confidence criterion. To get a final decision, we trace over the observer's degrees of freedom and write the state of the register as

$$\sigma_f = P(c) \, |c\rangle \langle c| + (1 - P(c)) \, |n\rangle \langle n| \,. \tag{19}$$

where

$$P(c) = \sum_{\{a_i d_i\} \in \mathfrak{C}} \Pi_{i=1}^N (P(d_i|a_i) P(a_i)) \tag{20}$$

and $\mathfrak{C}$ is the set of all combined results that meet the confidence criterion. For large N the terms matching the collapse criteria will have the largest probabilities so that $\lim_{N\to\infty} P(c) = 1$. We now apply the Born rule to the entire laboratory. For finite N the likelihood our observer reports a result inconsistent with her collapse hypothesis is given by the usual statistical calculation, just as she would expect.

5. Siloed evolution

In the general case of an arbitrary measurement on a single system followed by an arbitrary evolution, we start from (14) which we write as

$$\sigma_U = \sum_a \rho_a \otimes |a\rangle \langle a| \,, \tag{21}$$

where ρ_a is shorthand for $A_a \rho A_a^\dagger$. It is not normalized, but otherwise behaves as a density operator. $|a\rangle \langle a|$ may represent the observer's memory or a register where the result is written for later examination. If the collapse projection is applied the state will be

$$\sigma_C = N_a \rho_a \otimes |a\rangle \langle a| \,. \tag{22}$$

If we assume the memory/register is undisturbed, the evolution is given by

$$\begin{aligned}\sigma_U(t) &= \sum_a U\rho_a U^\dagger \otimes |a\rangle\langle a| \\ \sigma_C(t) &= N_a U\rho_a U^\dagger \otimes |a\rangle\langle a|.\end{aligned} \tag{23}$$

In the unitary case, the evolution is siloed into histories which are each consistent with the initial measurement. Any experiments performed by the observer, or even by us, will maintain this consistency. If we assume the Born rule, the two evolutions will lead to the same distribution of results when collated by memory/register values. The collapse hypothesis leads to no detectable differences.

6. Discarded information

To achieve siloing, it is important that the discarded subsystem carry the result information. We can write the trace operation as

$$\sigma_{ab}^{reduced} = \sum_\alpha \sigma_{a\alpha,b\alpha}^{full}. \tag{24}$$

Each pair of subscripts in σ^{full} are indices for the tensor product of the retained and discarded Hilbert spaces. If the entanglement between these spaces is faithful, σ^{full} will be zero when a and α or b and α correspond to different measurement values. Then $\sigma^{reduced}$ will be diagonal (block diagonal in the coarse grained case). The discarded subsystem may be considered to encode "which way" information, destroying interference [2].

In the opposite case where nothing is discarded, all bets are off. It is possible in principle to reverse the measurement exactly by applying the inverse unitary evolution. In that case the observer's memory will be restored to its reference state. Or a unitary can be applied that will allow a measurement in a complementary basis instead. Similar things could also happen if a discarded probe is recovered and there was no other discard. If there is no discard at all, the result is just entanglement and not measurement.

In many cases the discarded information is only lost effectively. For example a particle entering a detector may be absorbed. It is still present, but its information becomes distributed in the environment. If it were possible to recover that information coherently, the measurement could in principle be reversed. The case of environmental information loss has been studied extensively [3], but the perspective has been somewhat different, focusing primarily on decoherence of the target system and the emergence of classical physics for macroscopic systems.

In practice there are many layers of probes and *all* of them are likely to discard entangled information. A particle interacts with a detector which takes a binary state. The detector interacts with electronics to set a counter state. The counter sets a display state. Our eyes perceive the visual field of the display and send signals to the brain. Ultimately all observations must be aligned with human senses' preferred bases. The observer will then reason about what his observations mean for the system. Such reasoning is a universal aspect of measurement.

7. Conclusion

The picture that emerges for measurement in a unitary universe is as follows: a system entangles with an observer through multiple layers of interaction, information is discarded, and the resulting correlated state of system and memory is block diagonal among various results which evolve in parallel such that an imagined collapse projection does not impact correlated observations.

In this picture, the rules of quantum mechanics imply that an observer governed by unitary, deterministic evolution will be unable to refute the collapse hypothesis, and may reasonably operate under the assumption that the hypothesis is true.

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