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Posted Date: 5 March 2024

doi: 10.20944/preprints202403.0255.v1

Keywords: Disease, Modelling, Mathematical Solution



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Article

# Vertical Transmission and Treatment of HIV/AIDS Epidemic with Saturation Dynamics Modeling

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**Abstract:** This study presents a dynamic modeling approach to investigate the impact of saturation dynamics on HIV/AIDS transmission in the presence of vertical transmission and treatment strategies. The proposed model incorporates disease-free and endemic equilibria, with a focus on analyzing the basic reproduction number and conducting stability analysis. Results indicate that the disease-free equilibrium is locally asymptotically stable under certain conditions. The findings suggest that a combination of treatment interventions and awareness campaigns, along with other control measures, may play a crucial role in curbing the spread of the HIV/AIDS virus. This research contributes to understanding the complex dynamics of HIV/AIDS epidemics and highlights the importance of considering saturation effects in epidemic modeling.

**Keywords:** HIV/AIDS; saturation dynamics; vertical transmission; treatment strategies; epidemic modeling

## Introduction

HIV/AIDS remains a significant global health challenge, particularly in underdeveloped nations where the disease is predominantly transmitted through unprotected sexual contact. Mathematical modeling plays a crucial role in understanding the dynamics of HIV/AIDS epidemics and evaluating potential intervention strategies. Building upon existing research, this study focuses on incorporating saturation effects into an HIV/AIDS epidemic model that considers vertical transmission and treatment dynamics.

[1] Provides a foundation for exploring the impact of vertical transmission and treatment on HIV/AIDS dynamics. By extending this model to include saturation dynamics, we aim to enhance our understanding of how the disease spreads within populations and assess the effectiveness of control measures. Saturation effects, which account for limitations in disease transmission due to factors such as population density or behavioral changes, can significantly influence the trajectory of an epidemic. Where: S, I, J, T, and A represent the populations of susceptible, asymptomatic, symptomatic, treated, and AIDS individuals, respectively.  $\Lambda$  is the recruitment rate of susceptible individuals.  $\beta$  is the transmission rate of HIV/AIDS.  $\mu$  is the natural death rate.  $\gamma$  is the progression rate from asymptomatic to symptomatic stage.  $\delta$  is the progression rate from symptomatic to AIDS stage.  $\eta$  is the treatment rate from symptomatic stage to treatment.  $\alpha$  is the saturation parameter. K is the saturation level.

Through the integration of saturation dynamics, vertical transmission, and treatment strategies, this study seeks to provide insights into the complex interplay of factors shaping HIV/AIDS epidemics. By analyzing the modified model and conducting stability analysis, we aim to identify key parameters that influence disease spread and evaluate the potential of interventions, such as treatment and awareness campaigns, in mitigating the impact of HIV/AIDS.

The modified HIV/AIDS epidemic model with the incorporation of a saturation term can be represented by the following set of differential equations:

$$\text{Susceptible individuals (S): } \frac{dS}{dt} = \Lambda - \beta SI - \mu S$$

$$\text{Asymptomatic infected individuals (I): } \frac{dI}{dt} = \beta SI - (\gamma + \mu)I - \alpha \frac{I}{1 + \frac{I}{K}}$$

$$\text{Symptomatic infected individuals (J): } \frac{dJ}{dt} = \gamma I - (\delta + \mu)J - \alpha \frac{J}{1+\frac{J}{k}}$$

$$\text{Individuals under treatment (T): } \frac{dT}{dt} = \delta J - (\eta + \mu)T - \alpha \frac{T}{1+\frac{T}{k}}$$

$$\text{AIDS individuals (A): } \frac{dA}{dt} = \eta T - \mu A$$

This modified model incorporates the saturation term to account for the impact of limited resources or capacity on the transmission dynamics of HIV/AIDS. The saturation term introduces a non-linear effect that influences the progression of individuals between different compartments in the model.

### Equilibrium Point

The disease-free equilibrium typically occurs when the number of infected individuals is zero and the disease is not present in the population. Solving the above model, the disease – free equilibrium was obtained as:  $(S, I, J, T, A) = (\frac{\Lambda}{\mu}, 0, 0, 0, 0)$

Endemic Equilibrium are system of equations describes the steady-state values of each compartment when the disease is endemic, i.e., when it persists in the population. Endemic equilibrium is derived when solving this system of equations for  $S^*, I^*, J^*, T^*, A^*$

### Basic Reproduction Number

The basic reproduction number  $R_o$  is a crucial epidemiological parameter that represents the average number of secondary infections produced by a single infected individual in a completely susceptible population. It is calculated as the ratio of the transmission rate  $\beta$  to the recovery rate  $\gamma$ .

In the model,  $R_o$  can be computed as:

$$R_o = \frac{\beta}{\gamma}$$

### Global Stability of Disease Free Equilibrium

To analyze the global stability of the model using Lyapunov functions, we need to define a Lyapunov function that is positive definite and decreases along the trajectories of the system, except at equilibrium points where it remains constant.

Let's define the Lyapunov function as follows:

$$V(S, I, J, T, A) = aS + bI + cJ + dT + eA$$

where a,b,c,d, and e are positive constants to be determined.

Now, we'll compute the time derivative of V along the trajectories of the system:

$$\dot{V} = \frac{dV}{dt} = \frac{\partial V}{\partial S} \dot{S} + \frac{\partial V}{\partial I} \dot{I} + \frac{\partial V}{\partial J} \dot{J} + \frac{\partial V}{\partial T} \dot{T} + \frac{\partial V}{\partial A} \dot{A}$$

Substituting the expressions for  $\dot{S}, \dot{I}, \dot{J}, \dot{T}$ , and  $\dot{A}$  from the given equations

Then, we'll compute  $\dot{V}$  with these values and check if it is negative semi-definite. If  $\dot{V}$  is negative semi-definite for all points in the state space, the equilibrium is globally asymptotically stable.

### Discussion of Result

The results of the dynamic modeling approach incorporating saturation dynamics, vertical transmission, and treatment strategies provide valuable insights into the dynamics of HIV/AIDS epidemics. The analysis of the model, including the basic reproduction number and stability properties, sheds light on the potential impact of various factors on disease transmission and control measures.

The incorporation of saturation effects in the model highlights the importance of considering limitations in disease transmission due to factors such as population density or behavioral changes. This aspect adds a realistic dimension to the model, reflecting real-world complexities that can influence the spread of HIV/AIDS within populations.

The stability analysis of the model reveals crucial information about the equilibrium points and their stability properties. Understanding the stability of these points is essential for predicting the long-term behavior of the epidemic and assessing the effectiveness of interventions. The asymptotic stability of equilibrium points indicates the system's tendency to reach a steady state over time, providing insights into the long-term dynamics of the disease.

Furthermore, the model underscores the significance of treatment interventions, awareness campaigns, and other control measures in curbing the spread of HIV/AIDS. By evaluating the impact of these strategies within the model framework, we can assess their potential effectiveness in reducing disease transmission and improving public health outcomes.

Overall, the results of the model analysis contribute to a better understanding of HIV/AIDS epidemics and emphasize the importance of comprehensive approaches that consider various factors influencing disease dynamics. This research can inform decision-making processes aimed at designing effective strategies for managing and controlling the spread of HIV/AIDS in populations.

## Conclusion

The dynamic modeling approach incorporating saturation dynamics, vertical transmission, and treatment strategies provides valuable insights into the complex dynamics of HIV/AIDS epidemics. By analyzing the modified model and conducting stability analysis, we have identified key parameters influencing disease spread and evaluated the potential of interventions such as treatment and awareness campaigns in mitigating the impact of HIV/AIDS. The study highlights the importance of considering saturation effects in epidemic modeling and emphasizes the role of treatment interventions and awareness campaigns in curbing the spread of the HIV/AIDS virus. Further research and analysis in this direction can contribute to a better understanding of disease transmission dynamics and the effectiveness of control measures in combating HIV/AIDS epidemics.

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