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A Relationship between the Subatomic Particles' Mass and the Square of the Magnetic Flux Quantum Provides a Novel Significance of the Mass and the Wave Function

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A Relationship between the Subatomic Particles' Mass and the Square of the Magnetic Flux Quantum Provides a Novel Significance of the Mass and the Wave Function.

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Abstract: The motivation for investigating the issues presented in this article stemmed from a need to understand better the relationship in long-established physics formulas and extract the information they provided using conventional methods. Based on the definition of magnetic flux quantization, which is based on a combination of fundamental physical constants, the Planck constant, and the particle's elementary charge, this work explores the results that arise when this equation is combin/ed with the equation of the Bohr-level electrostatic force acting on the electron, which leads to a new mathematical relationship between the combined expressions. The new relationship yields novel theoretical findings related to the known universal constants. Following this insight, the formalism developed in this paper indicates that the mass of the electron and other subatomic particles is associated with the magnitude of the square of the magnetic flux quantum, which makes up the particle. This conclusion is not as strange as it may seem, as the square of the magnetic flux quantum appears in the context of magnetic energy in a current loop and, according to Einstein's special theory of relativity, energy is equivalent to mass. Another aspect described at the end of this paper demonstrates the connection between the square of the magnetic flux quantum through the Bohr radius and the significance of the wave function in the atom. The theoretical results are in full accordance with experimental results published by NIST CODATA 2018 that I've used, which validates the existence of the relationship.

Keywords: a novel significance of the mass; a novel significance of the wave function

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Introduction.

The formalism developed in this paper introduces a relationship between the masses of the electron, proton, and neutron and the square of the magnetic flux quantum. This relation was never known or probed. The Method used today to calculate the proton and neutron masses theoretically is based on the quantum chromodynamics theory of binding energy, which combines the kinetic energy of the quarks and the energy of the gluons within these particles, which requires a complicated calculation. The theory presented here calculates the electron, proton, and neutron masses in straightforward, nearly identical formulas, whose main component is the square of the magnetic flux quantum; the only difference between them is their Compton wavelength component, which is responsible for their different masses. Another formalism yields the proton's (hydrogen nucleus) and the neutron's radii directly from theory. The proton charge radius is a part of the actual proton radius. This theory presents a different approach to finding the proton's and the neutron's radii. It involves using universal constants such as Planck's mass and the universal gravitational constant, which ultimately yields a novel physical constant unfamiliar to science, in which the strong coupling constant in QCD is its derivative. Using this constant, it is possible to obtain the proton's radius. Combining the novel constant and the proton radius makes it possible to accurately calculate the Compton wavelength constants of the proton and the neutron, which are used to calculate their masses later. The theory also presents a novel way to describe the Planck mass and length and the gravitation constant through them. The gravitational constant is identified with Newton's law of universal gravitation. The new formula for the gravitational constant developed in this paper contains elements from the atomic domain (proton's mass and radius), which represent the quantum reality environment; in this way, they demonstrate the integration of the quantum and gravity levels.

Method of Analysis.

This paper presents a method of analyzing long-established physics formulas to extract hidden information that reveals new variables combined with known variables. This method led to the discovery of the relationship of the particle's mass to the magnetic flux quantum. The technique is simple in light of the mathematical means and practical to obtain results

Results and Discussion.

1. Relationships between the electron mass and the square of the magnetic flux quantum and between the Bohr radius and the vacuum permittivity.

The magnetic flux quantum Φ_0 [1] is defined according to the following

$$e = h/2\Phi_0 \rightarrow e^2 = h^2/4\Phi_0^2 \qquad (1)$$

where e is the elementary charge of an electron and h is Planck's constant. The electrostatic force acting on the electron at the Bohr level is

$$e^2/4\pi a_0^2 \varepsilon_0 = m_e v_e^2/a_0$$

$$e^2 = m_e v_e^2 4\pi a_0 \varepsilon_0 \qquad (2)$$

where a_0 is the Bohr radius, ε_0 is the vacuum permittivity, m_e is the electron mass, and v_e is the electron velocity at the Bohr radius. The electron's angular momentum at the Bohr radius is

$$\frac{h}{2\pi} = m_e v_e a_0 \rightarrow h = 2\pi m_e v_e a_0 \qquad (3)$$

By substituting Eq. (3) in Eq. (1) (the squared term), we obtain

$$e^2 = h^2/4\Phi_0^2$$

$$e^2 = m_e v_e 2\pi a_0 \times m_e v_e 2\pi a_0 / 4\Phi_0^2 \qquad (4)$$

We can then rewrite Eq. (4) as follows:

$$e^2 = m_e v_e^2 4\pi a_0 \times (\pi a_0 m_e / 4\Phi_0^2) \qquad (5)$$

By multiplying both the numerator and denominator of Eq. (5) by ε_0, we have

$$e^2 = m_e v_e^2 4\pi a_0 \varepsilon_0 (\pi a_0 m_e / 4\varepsilon_0 \Phi_0^2) \qquad (6)$$

According to Eq. (2), the expression in parentheses in Eq. (6) should equal unity; thus, we obtain

$$(\pi a_0 m_e / 4\varepsilon_0 \Phi_0^2) = 1 \qquad (7)$$

We can then substitute the following values published by the National Institute of Standards and Technology Committee on Data for Science and Technology in 2018 (NIST CODATA 2018) [2] in Eq. (7) (SI units):

$a_0 = 5.291772109 \times 10^{-11}$ m

$\varepsilon_0 = 8.8541878128 \times 10^{-12}$ $\text{A}^2\ \text{s}^4\ \text{kg}^{-1}\text{m}^{-3}$

$\Phi_0 = 2.067833848 \times 10^{-15}$ $\text{kg m}^2\ \text{s}^{-2}\ \text{A}^{-1}$

$\Phi_0^2 = 4.275936823 \times 10^{-30}$ $\text{kg}^2\ \text{m}^4\ \text{s}^{-4}\ \text{A}^{-2}$

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$m_e = 9.1093837015\times10^{-31}\text{kg}$

These substitutions give us the following relationship:

$$\frac{\pi}{4}\times\frac{5.291\,772\,109\times10^{-11}\ \text{m}}{8.854\,187\,812\,8\times10^{-12}\ \text{A}^2\text{s}^4\text{kg}^{-1}\text{m}^{-3}}\times\frac{9.109\,383\,7015\times10^{-31}\ \text{kg}}{4.275\,936\,823\times10^{-30}\ \text{kg}^2\text{m}^4\text{s}^{-4}\text{A}^{-2}}=1 \quad (8)$$

Finally, we can consider the multiplication of the vacuum permittivity and the square of the magnetic flux in the denominator of Eq. (8). We can multiply their units:

$$(\text{A}^2\text{s}^4\text{kg}^{-1}\text{m}^{-3})(\text{kg}^2\text{m}^4\text{s}^{-4}\text{A}^{-2}) = \text{kg m} \quad (9)$$

We consider this reduction further in the next section.

2. Analysis of Equation (8).

a. After reducing the units in the denominator of Eq. (8), we obtained units corresponding to those in the numerator in Eq. (9). The result in Eq. (9) implies two options: Either Φ_0^2 has units of mass $[\text{kg}]$ or units of length $[\text{m}]$ or, vice versa, ε_0 has units of mass $[\text{kg}]$ or has units of length $[\text{m}]$.

b. The orders of magnitude in Eq. (8) indicate the scale of magnitude of these parameters**.** The Bohr radius scale is 10^{-11}m, and the vacuum permittivity scale is 10^{-12}F m^{-1}. The scale of the electron mass is 10^{-31}kg, and the scale of the square of the magnetic flux quantum is $10^{-30}[\text{Wb}]^2$. From this consideration and the two options presented in (a), it is clearly nonsensical for ε_0 to have units of mass or Φ_0^2 to have units of length. No particles with a mass on the scale of 10^{-12}kg or a length on the scale

c. of 10^{-30}m exist in the atom domain.From these considerations, we can conclude with a high degree of certainty that the Bohr radius a_0 and the vacuum permittivity ε_0 are the same entity:

$$\varepsilon_0 \equiv a_0 \quad (10)$$

Moreover, the electron mass m_e and the square of the magnetic flux quantum Φ_0^2 are the same entity:

$$m_e \equiv \Phi_0^2 \quad (11)$$

d. The last conclusion is not as strange as it may seem, as the square of the magnetic flux quantum appears in the context of magnetic energy in a current loop and according to Albert Einstein's special theory of relativity, energy is equivalent to mass.

3. Using the conclusions from Sections 2(c) and 2(d) for the electron mass and vacuum permittivity.

By rearranging Eq. (7) for the electron mass calculation and substituting the values of $\varepsilon_0, a_0, \Phi_0^2$ from NIST CODATA 2018, we obtain

$$m_e = \frac{4}{\pi}\times\frac{\varepsilon_0}{a_0}\times\Phi_0^2 = \frac{4}{\pi}\times\frac{8.8541878128\times10^{-12}\text{m}}{5.291772109\times10^{-11}\text{m}}\times4.275936823\times10^{-30}\text{kg} = 9.109383697\times10^{-31}\text{kg}$$

(12)

This value compares well with the NIST value of $m_e = 9.1093837015\times10^{-31}\text{kg}$

We can obtain another expression for m_e from Eq. (3) with the electron Compton wavelength $\lambda_e\ (v_e = c\alpha)$

$$\lambda_e = h/m_e c = \alpha 2\pi a_0 \quad (13)$$

where c is the speed of light in vacuum and α is the fine structure constant. Here, we can rearrange Eq. (12) as $a_0 = 4\varepsilon_0 \Phi_0^2 \pi^{-1} m_e^{-1}$ and substitute this term in Eq. (13) for another expression of m_e:

$$m_e = \left(\frac{8\ \alpha\ \varepsilon_0}{\lambda_e} \right) \Phi_0^2 \qquad (14)$$

To calculate the value of the vacuum permittivity ε_0, we rewrite Eq. (7) and substitute the values of Φ_0^2, m_e, a_0 from NIST CODATA 2018, which yields

$$\varepsilon_0 = \frac{\pi}{4} \times \frac{m_e}{\Phi_0^2} \times a_0 = \frac{\pi}{4} \times \frac{9.1093837015\times10^{-31}\,\mathrm{kg}}{4.275936823\times10^{-30}\,\mathrm{kg}} \times 5.291772109\times10^{-11}\mathrm{m} = 8.854187816\times10^{-12}\,\mathrm{m}$$

(15)

This numerical value compares well with the NIST value of $\varepsilon_0 = 8.8541878128\times10^{-12}\,\mathrm{A^2\,s^4\,kg^{-1}m^{-3}}$.

4. Using the conclusions from Sections 2(c) and 2(d) for elementary charge e and the Planck constant h.

In this subsection, we derive a new expression for the elementary charge e with $m_e = 4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1}$ from Eq. (12) and with the fine structure constant α and the speed of light in vacuum c.

Here, we substitute m_e in Eq. (2) with $v_e^2 = c^2\alpha^2$ to obtain an expression of the elementary charge e:

$$e^2 = m_e c^2 \alpha^2 4\pi\varepsilon_0 a_0 = (4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1})(c^2\alpha^2 4\pi\varepsilon_0 a_0)$$

$$e = 4c\alpha\varepsilon_0\Phi_0 \qquad (16)$$

In Eq. (16), we substitute the values of ε_0, Φ_0, the speed of light in vacuum c, and the fine structure constant α from NIST CODATA 2018:

$c = 2.99792458\times10^{8}\ \mathrm{m\,s^{-1}}$

$\alpha = 7.2973525684\times10^{-3}$

This substitution yields

$$e = 1.602176633\times10^{-19}\,\mathrm{C} \qquad (17)$$

This value compares well with the NIST value of $e = 1.602176634\times10^{-19}\,\mathrm{C}$ .We can substitute $c^2\varepsilon_0 = \mu_0^{-1}$ (where μ_0 is the vacuum permeability) in Eq. (16) to obtain another expression for the elementary charge e:

$$e = 4\alpha\Phi_0(\varepsilon_0/\mu_0)^{1/2} \qquad (18)$$

By substituting the expression of e from Eq. (16) in Eq. (1), we obtain a new expression for h :

$$h = 8c\alpha\varepsilon_0\Phi_0^2 \qquad (19)$$

By applying the values of $\varepsilon_0, \Phi_0, c, \alpha$ from NIST CODATA 2018 in Eq. (19), we find

$$h = 6.626070146\times10^{-34}\,\mathrm{kg\,m^2 s^{-1}}$$

This value compares well with the NIST value of $h = 6.626070146 \times 10^{-34}\ \mathrm{kg\ m^2 s^{-1}}$. Moreover, we obtain another expression for the Planck constant h by utilizing $c^2\varepsilon_0 = \mu_0^{-1}$:

$$h = 8\alpha(\varepsilon_0/\mu_0)^{1/2}\Phi_0^2 \tag{20}$$

5. Radius of the proton (hydrogen nucleus).

For further development, it is necessary to find the proton's radius. The nucleus of a hydrogen atom (proton) revolves around the center of mass shared with the electron. The rotation of both the electron and nucleus arises from considerations of momentum conservation in an isolated system and is taken into account by a computational correction called the reduced mass of the electron. The center of mass is very close to the axis of the nucleus because of its larger mass; thus, we can assume that the trajectory depicted by the nucleus while revolving around the center of mass lies at a distance almost equivalent to the nucleus radius. We will denote this radius as the proton radius, validated in the final result. As a side note, this radius is not equivalent to the proton's charge radius; however, there is a connection between these two parameters, which will be clarified in Section 5b. To find the proton's radius, we will use known formulas generated for the natural units of the Stoney [3] and Planck [4] scales. We will start with the Stoney scale, from which we will move to the Planck scale.

The Stoney length l_s in natural units is

$$l_s = \sqrt{\frac{e^2 G}{4\pi\varepsilon_0 c^4}} \tag{21}$$

where G is the gravitational constant. The Stoney mass m_s from the natural units is

$$m_s = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \tag{22}$$

Or rewrite Eq. (22) for the gravitational constant G:

$$G = \frac{e^2}{4\pi\varepsilon_0 m_s^2} \tag{23}$$

By substituting the relation $e^2 = 2hc\alpha\varepsilon_0$ from the relation introduced by the Physicist Arnold Sommerfeld $\alpha = e^2/2hc\varepsilon_0$ [5], in Eq. (23), we obtain

$$G = \frac{2hc\alpha\varepsilon_0}{4\pi\varepsilon_0 m_s^2} \tag{24}$$

The orbital angular momentum of the proton at the trajectory around the center of mass should be expressed by the reduced Planck constant.

The proton's velocity at this trajectory is denoted here as v_p. An initial estimation of the velocity v_p yields approximately one fifth of the speed of light in vacuum. Hence, it is necessary to add a relativistic element $(1-v_p^2/c^2)^{1/2}$ with $v_p = c\beta$:

$$\frac{h}{2\pi} = m_p v_p r_p \sqrt{1-v_p^2/c^2}$$
$$h = m_p c\beta 2\pi r_p \sqrt{1-\beta^2} \tag{25}$$

where m_p is the proton mass, β is the ratio of v_p to c, and r_p is the proton radius. By substituting the expression of h from Eq. (25) in Eq. (24) and reducing the expression, we obtain

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$$G = \frac{m_p c^2 \alpha \beta r_p \sqrt{1-\beta^2}}{m_s^2} \tag{26}$$

The β is similar to the fine structure constant α, also known as the electromagnetic coupling constant, and it appears in the electron's velocity expression at the Bohr radius as $v_e = c\alpha$.

We can divide Eq. (21) by Eq. (22) ($4\pi\varepsilon_0$ is reduced, and the elementary charge e is partially reduced):

$$\frac{l_s}{m_s} = \sqrt{\frac{e^2}{e^2} \times \frac{G^2}{c^4}} = \frac{e}{e} \times \frac{G}{c^2} \tag{27}$$

Then rearrange Eq. (27) to obtain an expression for G:

$$G = \frac{e}{e} \times \frac{l_s}{m_s} c^2 \tag{28}$$

By setting the expressions in Eq. (28) and Eq. (26) equal to each other, we have

$$\frac{e}{e} \times \frac{l_s}{m_s} c^2 = \frac{m_p c^2 \alpha \beta r_p \sqrt{1-\beta^2}}{m_s^2} \tag{29}$$

We then divide both sides of Eq. (29) by $\left(\frac{e}{e}\right)$, multiply both sides by m_s^2, reduce, and rearrange:

$$m_s \times l_s \times c^2 = \left[\left(\frac{1}{e} m_p \beta\right)\left(\frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right)\right] \times c^2 \tag{30}$$

Eq. (30) presents a similarity between the right and left flanks (mass component and length component). The expression is split into two parts on the right-hand side of the equation because it contains the solutions corresponding to actual experimental results in the final analysis.

The following new expressions are proposed solutions for the Stoney units.

New expression of Stoney mass m_s:

$$m_s = \frac{1}{e} m_p \beta$$

New expression of Stoney length l_s:

$$l_s = \left(\frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right)$$

Note that the $(1/e)$ expression in Eq. (30) represents a dimensionless number, for instance, the number of charged particles in one Coulomb $[\mathrm{C}]$ unit.

$$\frac{1}{e} = \frac{1\not{C}}{1.602176634 \times 10^{-19} \not{C}} = 6.2415090744 \times 10^{18} \tag{31}$$

This number, as a multiplier, creates a quantity of charged particles (in our case, the number of protons contained within the Planck mass, which corresponds to a quintillion protons) or, as a divisor, creates the smallest length (in our case, a contracted radius of the proton within the Planck mass under internal attraction forces, which corresponds to a quintillionth of the proton radius that represents the Planck length). This expression is displayed in the following equations, as in Eq. (31), to indicate that this value is dimensionless.

The gravitational constant G in Stoney units from Eq. (28) with the proposed new expressions is:

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$$G=\frac{\left(\dfrac{\alpha\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right)}{\left(\dfrac{1}{e}m_p\beta\right)}\times c^2 \qquad (32)$$

We can then set Eq. (23) and Eq. (32) equal to each other and substitute the square of the Stoney mass term $m_s^2=\left(\frac{1}{e}m_p\beta\right)^2$ in the denominator of Eq. (23) as:

$$\frac{e^2}{4\pi\varepsilon_0\left(\dfrac{1}{e}m_p\beta\right)^2}=\frac{\alpha\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}\left(\dfrac{1}{e}m_p\beta\right)}\times c^2 \qquad (33)$$

Multiplying both sides of Eq. (33) by $4\pi\varepsilon_0 m_p\beta$, reducing, and rearranging yields

$$e^2=2\left[m_p c\beta 2\pi r_p\sqrt{1-\beta^2}\right]c\alpha\varepsilon_0 \qquad (34)$$

The expression of Eq. (34) shows the equivalence of $e^2=2hc\alpha\varepsilon_0$, where the right-hand side (in brackets) contains the expression of the Planck constant h with the proton parameters introduced in Eq. (25). This result confirms the choice of the proposed solutions for the Stoney units of mass and length from Eq. (30). Although this option was based on a logical consideration, there are additional combinations that could be chosen that yield incorrect results.

We multiply the numerator and denominator of Eq. (32) by $\alpha^{-1/2}$ to obtain the gravitational constant G at the Planck scale

$$G=\frac{\left(\dfrac{\alpha^{1/2}\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right)}{\dfrac{1}{\alpha^{1/2}}\left(\dfrac{1}{e}m_p\beta\right)}\times c^2 \qquad (35)$$

Note: The difference between the Stoney and Planck units arises from the need to multiply Planck units by the square root of the fine structure constant, $\alpha^{1/2}$. Consequently, we obtain the following expressions.

New expression of Planck mass m_{pl}:

$$m_{\text{pl}}=\frac{1}{\alpha^{1/2}}\left(\frac{1}{e}m_p\beta\right)$$

New expression of Planck length l_{pl}:

$$l_{\text{pl}}=\left(\frac{\alpha^{1/2}\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right)$$

By using the Planck mass in natural units and the new expression of Planck mass, we can derive the expression and value of β.

The Planck mass, defined by natural units, is

$$m_{\text{pl}}=\sqrt{\frac{hc}{2\pi G}} \qquad (36)$$

The new expression for the Planck mass from Eq. (35) is

$$m_{\mathrm{pl}} = \frac{1}{\alpha^{1/2}}\left(\frac{1}{e} m_p \beta\right) \quad (37)$$

We set Eq. (36) and Eq. (37) as equal:

$$\sqrt{\frac{hc}{2\pi G}} = \frac{1}{\alpha^{1/2}}\left(\frac{1}{e} m_p \beta\right) \quad (38)$$

Then rearrange Eq. (38) to obtain an expression for β:

$$\beta = \sqrt{\frac{hc\alpha}{2\pi G}} \times \frac{1}{1/e \times m_p} \quad (39)$$

In Eq. (39), we substitute the values of h, c, α, m_p and the following values from NIST CODATA 2018:

$1/e = 6.2415090744 \times 10^{18}$

$G = 6.6743 \times 10^{-11} \mathrm{m}^3\,\mathrm{kg}^{-1}\mathrm{s}^{-2}$

We obtain $\beta = 0.178090537$

As a side note, the relationship of β to nuclear research is through the strong coupling constant in QCD, which is a β derivative.

$\alpha_S = 2/3\beta \rightarrow \alpha_S = 2/3 \times 0.178090537 = 0.118727$

This value compares well with the value obtained experimentally [6], [7], $\alpha_S \approx 0.1187$.

Using the Planck length from natural units and the new expression for the Planck length, we can derive the expression and value of the proton radius r_p.

$$l_{\mathrm{pl}} = \sqrt{\frac{hG}{2\pi c^3}} \quad (40)$$

The new expression for the Planck length from Eq. (35) is

$$l_{\mathrm{pl}} = \left(\frac{\alpha^{1/2}\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right) \quad (41)$$

By setting the expressions in Eq. (40) and Eq. (41) as equal, we obtain

$$\sqrt{\frac{hG}{2\pi c^3}} = \left(\frac{\alpha^{1/2}\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right) \quad (42)$$

We then rearrange Eq. (42) for the proton radius r_p:

$$r_p = \sqrt{\frac{hG}{2\pi c^3 \alpha}} \times \frac{1}{e} \times \frac{1}{\sqrt{1-\beta^2}} \quad (43)$$

Substituting $\beta = 0.178090537$ and the values of h, G, c, α and $(1/e)$ from NIST CODATA 2018 in Eq. (43) yields $r_p = 1.200094665 \times 10^{-15}\,\mathrm{m}$

This value compares well with the value obtained experimentally [8], explained below:

a. The proton radius obtained in Eq. (43) complies with the experimental formulation that assumes a spherical nucleus with radius expressed by the Fermi equation for the nuclear radius R_{n}: $R_{\mathrm{n}} = R_0 \mathrm{A}^{1/3}$, where R_0 is an essamtion made from experimental results to be $R_0 = 1.2 \times 10^{-15}\,\mathrm{m}$, and A is the atomic number. For hydrogen, A=1 and $R = 1.2 \times 10^{-15}\,\mathrm{m}$.

* The example for the proton charge radius in the following paragraph is presented without overall proof,

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which requires a separate article.

b. The proton charge radius r_{pcr} represents the maximum distance from the proton axis that the electron or muon reaches in their penetration to the proton due to interactions with up quarks. This radius is expressed as

$r_{pcr} = 4\beta \times r_p(1-\beta^2)^{1/2}$ Substituting the following in the expression for r_{pcr} ; $r_p = 1.200094665\times10^{-15}\,\mathrm{m}$ and $\beta = 0.178090537$, we obtain $r_{pcr} = 8.4123564\times10^{-16}\,\mathrm{m}$. This value is similar to the NIST value of $r_{pcr} = 8.414\times10^{-16}\,\mathrm{m}$.

The proton's Compton wavelength from Eq. (25) is

$$\lambda_p = \frac{h}{m_p c} = 2\pi\beta r_p\sqrt{1-\beta^2} \tag{44}$$

Substituting the values of β and r_p in Eq. (44) yields

$$\lambda_p = 2\pi\times0.178090537\times1.200094665\times10^{-15}\,\mathrm{m}\times0.984014106 = 1.3214098539\times10^{-15}\,\mathrm{m}$$

This values compares well with the NIST value of

$$\lambda_p = 1.32140985539\times10^{-15}\,\mathrm{m}$$

The last result shows that β combined with the proton radius r_p obtained from Eq. (43) and used in Eq. (44) is entirely consistent with the value of the proton's Compton wavelength λ_p from NIST CODATA 2018, confirming the validity of our approach.

To obtain the gravitational constant G, we utilize Eq. (32) and substitute the m_p, c, α from NIST CODATA 2018 and also $1/e = 6.2415090744\times10^{18}$, β, and r_p :

$$G = \frac{\left(\dfrac{r_p\sqrt{1-\beta^2}}{6.2415090744\times10^{18}}\right)\alpha}{\left(6.2415090744\times10^{18}\times m_p\right)\beta}\times c^2 \tag{45}$$

It yields $G = 6.6743\times10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

Which compares well with the NIST value of

$$G = 6.6743\times10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$$

6. Radius of the neutron.

The ratio between the proton mass and neutron mass is the same as the ratio between the neutron Compton wavelength and proton Compton wavelength. The values of $m_p, m_n, \lambda_n, \lambda_p$ are substituted from the NIST CODATA 2018 in the following ratios:

$$\frac{m_p}{m_n} = \frac{1.67262192369\times10^{-27}\,\mathrm{Kg}}{1.67492749804\times10^{-27}\,\mathrm{Kg}} = 0.998623478$$

$$\frac{\lambda_n}{\lambda_p} = \frac{1.31959090581\times10^{-15}\,\mathrm{m}}{1.32140985539\times10^{-15}\,\mathrm{m}} = 0.998623478$$

This result indicates that the ratio is also appropriate for the ratio between the neutron and proton radii.

$$\frac{m_p}{m_n} = \frac{r_n}{r_p} \rightarrow r_n = \frac{m_p}{m_n}\times r_p \tag{46}$$

By substituting the values of m_p, m_n from NIST CODATA 2018 and the radius r_p from Eq. 43 in Eq. (46) for the neutron radius, we obtain

$$r_n = \frac{1.67262192369\times10^{-27}\,\text{kg}}{1.67492749804\times10^{-27}\,\text{kg}}\times1.200094667\times10^{-15}\,\text{m}$$

$$r_n = 1.19844271\times10^{-15}\,\text{m}$$

The proton and neutron are almost identical in size, and the β constant is found to be related to both radii. Consequently, $v_n = v_p = c\beta$. It is validated by the neutron Compton wavelength λ_n with β and r_n, as follows

$$\lambda_n = \frac{h}{m_n c} = 2\pi\beta r_n\sqrt{1-\beta^2} \tag{47}$$

Substituting the values of β and r_n in Eq. (47) for λ_n, we obtain

$$\lambda_n = 2\pi\times0.178090537\times1.19844271\times10^{-15}\,\text{m}\times0.984014105 = 1.31959090471\times10^{-15}\,\text{m}$$

This value matches well with the NIST value of

$$\lambda_n = 1.31959090581\times10^{-15}\,\text{m}$$

The result obtained by combining β with the neutron radius r_n given by Eq. (46) and substituting in Eq. (47) is entirely consistent with the NIST CODATA 2018 value of the neutron Compton wavelength λ_n, confirming the validity of our approach.

7. Additional expressions for the proton and neutron masses and radii.

We divide Eq. (3) (with $(v_e = c\alpha)$ by Eq. (25) as follows:

$$\frac{m_e\alpha 2\pi a_0}{m_p\beta 2\pi r_p\sqrt{1-\beta^2}} = 1 \tag{48}$$

By rearranging Eq. (48) and solving for the proton mass, we obtain

$$m_p = \frac{m_e\alpha a_0}{\beta\times r_p\sqrt{1-\beta^2}} \tag{49}$$

Substituting $m_e = 4\varepsilon_0\Phi_0^2\pi^{-1}a_0^{-1}$ from Eq. (12) in Eq. (49) yields the following expression for the proton m_p:

$$m_p = \frac{4}{\pi}\times\frac{\alpha}{\beta}\times\frac{\varepsilon_0}{r_p\sqrt{1-\beta^2}}\times\Phi_0^2 \tag{50}$$

Substituting $\lambda_e = \alpha 2\pi a_0$ and $\lambda_p = \beta 2\pi r_p\sqrt{1-\beta^2}$ in Eq. (48) yields

$$\frac{m_e}{m_p} = \frac{\lambda_p}{\lambda_e} \quad \text{Or} \rightarrow m_p = \frac{m_e\lambda_e}{\lambda_p} \tag{51}$$

We substitute $\lambda_e = \alpha 2\pi a_0$ and m_e from Eq. (12) in Eq. (49) to obtain another expression for m_p:

$$m_p = \left(\frac{8\,\alpha\,\varepsilon_0}{\lambda_p}\right)\Phi_0^2 \tag{52}$$

Substituting the values of $\alpha, \varepsilon_0, \lambda_p, \Phi_0^2$ from NIST CODATA 2018 in Eq. (52) for the proton mass m_p, it gives

$$m_p = \frac{8\times 7.297352569\times 10^{-3}\times 8.8541878128\times 10^{-12}\,\mathrm{m}}{1.321409\,85539\times 10^{-15}\,\mathrm{m}}\times 4.27593682\times 10^{-30}\,\mathrm{kg} = 1.6726219217\times 10^{-27}\,\mathrm{kg} \quad (53)$$

This value matches well with the NIST value of $m_p = 1.67262192369\times 10^{-27}\,\mathrm{kg}$.

We can rearrange Eq. (47) to obtain the orbital angular momentum of the neutron:

$$\frac{h}{2\pi} = m_n c\beta r_n\sqrt{1-\beta^2} \text{ Or } h = m_n c\beta 2\pi r_n\sqrt{1-\beta^2} \quad (54)$$

We then divide Eq. (3) by Eq. (54), as follows:

$$\frac{m_e c\alpha 2\pi a_0}{m_n c\beta\, 2\pi r_n\sqrt{1-\beta^2}} = 1 \quad (55)$$

Rearranging Eq. (55) for the neutron mass, gives

$$m_n = \frac{m_e \alpha a_0}{\beta\, r_n\sqrt{1-\beta^2}} \quad (56)$$

Substituting m_e from Eq. (12) in Eq. (56) yields an expression for the neutron mass m_n:

$$m_n = \frac{4}{\pi}\times\frac{\alpha}{\beta}\times\frac{\varepsilon_0}{r_n\sqrt{1-\beta^2}}\times\Phi_0^2 \quad (57)$$

Substituting $\lambda_e = \alpha 2\pi a_0$ from Eq. (13) and the neutron $\lambda_n = \beta 2\pi r_n\sqrt{1-\beta^2}$ from Eq. (47) in Eq. (55) yields

$$\frac{m_e}{m_n} = \frac{\lambda_n}{\lambda_e} \text{ Or } \rightarrow m_n = \frac{m_e\lambda_e}{\lambda_n} \quad (58)$$

We substitute $\lambda_e = \alpha 2\pi a_0$ and $m_e = 4\varepsilon_0\Phi_0^2\pi^{-1}a_0^{-1}$ from Eq. (12) in Eq. (58) for another expression of m_n:

$$m_n = \left(\frac{8\ \alpha\ \varepsilon_0}{\lambda_n}\right)\Phi_0^2 \quad (59)$$

Substituting the values of $\alpha, \varepsilon_0, \lambda_n, \Phi_0^2$ from NIST CODATA 2018 in Eq. (59) for the neutron mass m_n:

$$m_n = \frac{8\times 7.2973525693\times 10^{-3}\times 8.8541878128\times 10^{-12}\,\mathrm{m}}{1.319\,59090581\times 10^{-15}\,\mathrm{m}}\times 4.275936823\times 10^{-30}\,\mathrm{kg} = 1.6749274973\times 10^{-27}\,\mathrm{kg}$$

(60)

This value compares well with the NIST value of $m_n = 1.67492749804\times 10^{-27}\,\mathrm{kg}$.

The ratio of proton mass to electron mass is obtained from Eq. (49). After rearranging and substituting the NIST CODATA 2018 values for α, a_0 and β and r_p, it gives

$$m_p/m_e = \alpha/\beta\times a_0/r_p\sqrt{1-\beta^2} = 1836.152675 \quad (61)$$

This ratio can also be obtained from Eq. (51). This value matches well with the NIST ratio of

$$m_p/m_e = 1836.1526734$$

The ratio of neutron mass to electron mass is obtained from Eq. (56). After rearranging and substituting NIST CODATA 2018 values for α, a_0 and β and r_n, it gives

$$m_n/m_e = \alpha/\beta\times a_0/r_n\sqrt{1-\beta^2} = 1838.683661 \quad (62)$$

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This ratio can also be obtained from Eq. (58). This value matches well with the NIST ratio of

$$m_n / m_e = 1838.683661 .$$

8. **The squared values of the magnetic flux quantum used in the wave function, yield solutions which depict the flow pattern of the magnetic flux surrounding electrons at a given energy level.**

Using the Normalized Wave Function of the Hydrogen like Atom equation [9] , here

$$R_{nl}(r) = \sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-1-l)!}{2n[(n+l)!]^3}} \left(\frac{2Zr}{na_0}\right)^l e^{-zr/na_0} L_{n+l}^{2l+1}\left(\frac{2Zr}{na_0}\right) \quad (63)$$

Substituting $a_0 = 4\varepsilon_0 \Phi_0^2 \pi^{-1} m_e^{-1}$ a rewrite of Eq. (7) in Eq. (63), and after rearranging, it gives

$$R_{nl}(r) = \left(\frac{\pi}{2\varepsilon_0} \times \frac{zm_e}{n\Phi_0^2}\right)^{3/2} \left(\frac{(n-1-l)!}{2n[(n+l)!]^3}\right)^{1/2} \times \left(\frac{\pi r}{2\varepsilon_0} \times \frac{zm_e}{n\Phi_0^2}\right)^l e^{-zr/na_0} L_{n+l}^{2l+1}\left(\frac{\pi r}{2\varepsilon_0} \times \frac{zm_e}{n\Phi_0^2}\right)$$

(64)

Now using the third term in Eq. (64) as following

$$\left(\frac{\pi r}{2\varepsilon_0} \times \frac{zm_e}{n\Phi_0^2}\right)^l \quad (65)$$

As an example it provides the value of the derivative r at the Electron Wave Function of Hydrogen like Atom equation, as following

$$r = \frac{2\varepsilon_0}{\pi zm_e} \times n\Phi_0^2 \quad \text{When } l = 0 \to \left(\frac{\pi r}{2\varepsilon_0} \times \frac{zm_e}{n\Phi_0^2}\right)^0 \quad (66)$$

Conclusions.

a. The conclusions from Eq. (66) for the electron wave function of Hydrogen like Atom $R_{nl}(r)$: It is mainly a function of the changing value of the square of the magnetic flux quantum $n\Phi_0^2$ and the number of electrons zm_e at the relevant level (Please notice that the number of electrons in a neutral atom is equal to the number of protons in the nucleus of the atom, the atomic number Z).

b. The electron wave function of a Hydrogenlike Atom depends on the radius r of the atomic level expressed by several Bohr radii, and since the Bohr radius is a function of the square of the magnetic flux quantum Φ_0^2, the electron wave function describes the flow pattern of the magnetic flux surrounding electrons at the energy levels in the atom.

c. The mass of the electron and other subatomic particles is related to the magnitude of the square of the magnetic flux quantum, which makes up the particles. This relationship results in a novel expression of universal constants. The formalism developed in this paper yields the radii of the proton and the neutron from theory.

d. The Gravitational constant is identified based on Newton's law of universal gravitation. The new formula for the Gravitational constant developed in this paper contains elements from the atomic domain (proton's mass and radius), which represent the quantum reality environment; in this way, they demonstrate the integration of the quantum and gravity levels.

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