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Article

Thermodynamical Metrics in General Relativity Theory

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Abstract: We demonstrate that multiple well-known metrics, such as the Schwarzschild metric and the extremal solution of the Reissner-Nordström metric, can be expressed in what we will call thermodynamic form. These formulations appear to be valid for all $R_s = ct$ growing black holes and, therefore, also within the framework of $R_h = ct$ growing black hole cosmology. However, the metrics can also be applied to non-black hole gravitational objects as long as one calculates the CMB and Hawking temperature of the hypothetical black hole from the gravitational mass.

Keywords: thermodynamics; metrics; General relativity; Hawking temperature; CMB temperature; Planck scale; black holes

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1. Background on the Hawking Temperature and the New CMB Temperature Formula

In 1974, Hawking [1,2] introduced what is now known as the Hawking temperature of a black hole:

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi GM}. \quad (1)$$

Where k_b is the Boltzmann constant, and $\hbar$ is the reduced Planck constant, also known as the Dirac constant ($\hbar = \frac{h}{2\pi}$). The Hawking temperature formula plays a central role in many papers analyzing black holes. Additionally, Tatum et. al [3] introduced a similar formula for the CMB temperature heuristically in 2015:

$$T_{cmb} = \frac{\hbar c^3}{k_b 8\pi G\sqrt{Mm_p}} = \frac{\hbar c}{k_b 4\pi \sqrt{R_h 2l_p}} = 2.7276^{+0.0723}_{-0.0743}K \quad (2)$$

where m_p is the Planck mass and l_p is the Planck length, M is the black hole mass (critical Friedmann mass), and $R_h = \frac{c}{H_0}$ is the Hubble radius. The CMB temperature value given is calculated based on using the Sneppen et al. [4], which reports $H_0 = 67 \pm 3.6$, km/s/Mpc. Additionally, we have included the NIST CODATA (2018) uncertainty in the Planck length. The other constants are exact according to the NIST CODATA (2018) standard.

This formula, in more general terms for a black hole, is given by:

$$T_{cmb} = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_{BH} m_p}} = \frac{\hbar c}{k_b 4\pi \sqrt{R_s 2l_p}} \quad (3)$$

where M_{BH} is the mass of a Schwarzschild black hole, $M_{BH} = \frac{c^2 R_s}{2G}$ where R_s is the Schwarzschild radius $R_s = \frac{2GM_{BH}}{c^2}$. Recently, Haug and Wojnow have demonstrated that the formula can be derived from the Stefan-Boltzmann law [5,6]. Haug and Tatum [7] have also demonstrated that the formula can be derived as a geometric mean temperature of the maximum and minimum temperatures related to a black hole. CMB, or cosmic microwave background, temperature is a bit of a misnomer, as it is only for a Hubble sphere-sized black hole that the geometric mean internal temperature in the black hole correspond to microwave wavelength, for smaller black holes this temperature increase and the corresponding wavelength is shortened. A perhaps better word for this temperature would simply be the geometric mean internal black hole temperature.

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To measure the CMB temperature (black hole geometric mean internal temperature) one would need to be inside a black hole. According to black hole cosmology we are inside a black hole. This idea that the Hubble sphere could be a black hole was likely first suggested by Pathria [8] in 1972 and the idea that the universe we live in could be a black hole is actively discussed to this day, see for example [9–17].

Haug [5] has also demonstrated that we must have:

$$\frac{T_{cmb}^2}{T_{Hw}^2} = \frac{l_p}{\bar{\lambda}} \tag{4}$$

Furthermore, Haug [5,18] has demonstrated that Einstein's [19] field equation can be rewritten as:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{8\pi G}{c^4}T_{\mu\nu} \\ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{8\pi l_p^2}{c\hbar}T_{\mu\nu} \end{aligned} \tag{5}$$

where l_p is the Planck length [20,21]: $l_p = \sqrt{\frac{G\hbar}{c^3}}$. That is, we have simply solved the Planck length formula for G, which gives $G = \frac{l_p^2 c^3}{\hbar}$ and replaced G in Einstein's field equation with this. Solving the Planck units for G was suggested at least as early as 1984 by Cahill [22,23]. However, Cohen [24] in 1987 correctly pointed out that this leads to a circular problem, as no one had found a way to determine the Planck length independent of first knowing G. This view has been reiterated at least up to 2016; see, for example, [25]. However, in 2017, Haug [26] was able to demonstrate that one could find the Planck length independently of knowing G, and further, without knowledge of G and $\hbar$; see [27,28]. Recently, it has also been demonstrated how one can find the Planck length from the Hubble constant and the observed cosmological redshift by the formula:

$$l_p = \frac{H_0}{T_{cmb}^2}\frac{\hbar^2 c}{k_b 32\pi^2} \tag{6}$$

Haug [29] has recently demonstrated how one can also extract the Planck length from the observed redshift of 580 supernovas in the Union2 database. Additionally, it is important to note that any kilogram mass can be expressed by simply solving the Compton [30] wavelength formula: $\bar{\lambda} = \frac{\hbar}{mc}$. With respect to M, this gives:

$$M = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{7}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. Some may claim that such a formula can only be used for electrons; however, this is not correct. The formula can be applied to any kilogram mass, and the reduced Compton wavelength can be determined for any mass. What is true is that likely only elementary particles have a physical Compton wavelength. The Compton wavelength of composite masses, however, can be seen as an aggregate of the Compton wavelengths of all the elementary particles making up the composite mass, as discussed, for example, in [31], see also [32,33].

By simply replacing G with $G = \frac{l_p^2 c^3}{\hbar}$ and M with $M = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ the Schwarzschild [34] metric can be rewritten as:

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 - r^2d\Omega^2 \\ ds^2 &= -\left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)c^2dt^2 + \left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}dr^2 - r^2d\Omega^2 \end{aligned} \tag{8}$$

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where the term $\frac{l_p}{\bar{\lambda}_M}$ represents the reduced Compton frequency per Planck time. This, in our view, is the genuine quantization of matter and gravity, a notion supported by recent research indicating that matter indeed 'ticks' at the Compton frequency [35,36]. .

Since we have $\frac{T_{cmb}^2}{T_{Hw}^2} = \frac{l_p}{\bar{\lambda}}$ this implies that the Schwarzschild metric can also be expressed as:

$$ds^2 = -\left(1 - \frac{2l_p}{r}\frac{T_{cmb}^2}{T_{Hw}^2}\right)c^2dt^2 + \left(1 - \frac{2l_p}{r}\frac{T_{cmb}^2}{T_{Hw}^2}\right)^{-1}dr^2 - r^2d\Omega^2 \quad (9)$$

This is what we will call the thermodynamical version of the Schwarzschild metric. In the extreme solution of the Reissner-Nordström [37,38] (RN), as well as in the minimal solution of the Haug-Spavieri [39] metric, which is mathematically identical to the RN extremal solution, one can rewrite the metric as:

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2GM}{c^2r} + \frac{G^2M^2}{c^4r^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4r^2}\right)^{-1}dr^2 - r^2d\Omega^2 \\ ds^2 &= -\left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M} + \frac{l_p^2}{r^2}\frac{l_p^2}{\bar{\lambda}_M^2}\right)c^2dt^2 + \left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M} + \frac{l_p^2}{r^2}\frac{l_p^2}{\bar{\lambda}_M^2}\right)^{-1}dr^2 - r^2d\Omega^2 \end{aligned} \quad (10)$$

This suggests that the extremal solution of the Reissner-Nordström metric and the minimal solution of the Haug-Spavieri metric can be expressed in the following thermodynamic form:

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2l_p}{r}\frac{T_{cmb}^2}{T_{Hw}^2} + \frac{l_p^2}{r^2}\frac{T_{cmb}^4}{T_{Hw}^4}\right)c^2dt^2 \\ &\quad + \left(1 - \frac{2l_p}{r}\frac{T_{cmb}^2}{T_{Hw}^2} + \frac{l_p^2}{r^2}\frac{T_{cmb}^4}{T_{HW}^4}\right)^{-1}dr^2 - r^2d\Omega^2 \end{aligned} \quad (11)$$

Table 1 shows predictions derived from the Schwarzschild metric written in their standard form as well as in their thermodynamical form. The thermodynamical form is valid as long as the Hawking temperature and CMB temperature are calculated using the hypothetical CMB and Hawking temperature of the equivalent black hole of the gravitational mass, so in this way it can be used for any gravitational mass.

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Table 1. The table displays a series of gravity predictions derived from general relativity theory, presented in both their conventional form and their thermodynamic form. The thermodynamic form is applicable when the gravitational object is a Schwarzschild black hole.

Prediction	Formula:
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2 l_p}{R^2} \frac{T_{cmb}^2}{T_{Hw}^2}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c \frac{T_{cmb}}{T_{Hw}} \sqrt{\frac{l_p}{R}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{\frac{T_{cmb}}{T_{Hw}} c \sqrt{\frac{l_p}{R}}}$
Gravitational red shift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p}{R_1} \frac{T_{cmb}^2}{T_{Hw}^2}}}{\sqrt{1 - \frac{2l_p}{R_2} \frac{T_{cmb}^2}{T_{Hw}^2}}} - 1$
Time dilation	$T_R = T_f \sqrt{1 - \frac{2GM}{Rc^2}} = T_f \sqrt{1 - \frac{2l_p}{R} \frac{T_{cmb}^2}{T_{Hw}^2}}$
Gravitational deflection	$\theta = \frac{4GM}{c^2 R} = 4 \frac{l_p}{R} \frac{T_{cmb}^2}{T_{Hw}^2}$
Schwarzschild radius	$R_s = \frac{2GM}{c^2} = 2 l_p \frac{T_{cmb}^2}{T_{Hw}^2}$

Table 2 provides a deeper level of the gravity formulas in their quantized form. Once more, the term $\frac{l_p}{\lambda}$ represents the reduced Compton frequency per Planck time.

Table 2. The table displays a series of gravity predictions derived from general relativity theory, presented in their conventional form, as well as in the new quantized formulation of the field equation. Each formula includes the term $\frac{l_p}{\lambda}$, which represents the reduced Compton frequency in the mass M per Planck time.

Prediction	Formula:
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2 l_p}{R^2} \frac{l_p}{\lambda_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c \sqrt{\frac{l_p}{R} \frac{l_p}{\lambda_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{c \sqrt{\frac{l_p}{R} \frac{l_p}{\lambda_M}}}$
Gravitational red shift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p}{R_1} \frac{l_p}{\lambda_M}}}{\sqrt{1 - \frac{2l_p}{R_2} \frac{l_p}{\lambda_M}}} - 1$
Time dilation	$T_R = T_f \sqrt{1 - \frac{2GM}{Rc^2}} = T_f \sqrt{1 - \frac{2l_p}{R} \frac{l_p}{\lambda_M}}$
Gravitational deflection	$\theta = \frac{4GM}{c^2 R} = 4 \frac{l_p}{R} \frac{l_p}{\lambda_M}$
Schwarzschild radius	$R_s = \frac{2GM}{c^2} = 2 l_p \frac{l_p}{\lambda}$

2. Conclusion

We have demonstrated that metrics such as the Schwarzschild metric, the extremal solution of the Reissner-Nordström metric, and the minimal solution of the Haug-Spavieri metric can be expressed in what we can call thermodynamic forms. The term $\frac{T_{\mathrm{CMB}}^2}{T_{\mathrm{Hw}}^2}$, which represents the ratio of the squared CMB temperature to the squared Hawking temperature (or the geometric mean internal temperature of a black hole), gives the reduced Compton frequency per Planck time in the gravitational object. This, in our view, is a central element for quantization in gravity.

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