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Quantum Dynamics under Gravitational Potential; Insights from Gravitational-Redshift

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Abstract: This short paper briefly explores the interplay between quantum mechanics and gravity, drawing from the principle of gravitational redshift and bringing it into quantum mechanical frameworks. The exploration basically modifies quantum mechanical relations and equations such as Planck-Einstein relation, De-Broglie's relation and the relativistic wave equations, accounting for the presence of gravitational fields through gravitational redshifts and the effect of gravitational potential on the quantum states of fields and particles. The general idea presented in this paper is conceptually similar to what we know as QFT in Curved Space-time, but is not entirely on based the same formulation. This paper also presents a reformulation of the mass-energy equivalence in the context of gravitational potential and redshift.

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Introduction

The gravitational potential is a scalar quantity defined as the gravitational potential per unit mass of a body. It defines the strength of the gravitational field.

From Newtonian gravity, the energy of a gravitational field is given as $U = GMm/r$, dividing by mass leads to the field potential $\Phi = GM/r$, alternatively the field potential is written as $\Phi = gr$ for a gravitational potential energy $\mathrm{U} = mgr$. The current description of gravity departs from Newtonian ideologies that considers gravity as a force of attraction between massive bodies according to newton's inverse square law, and is now involving the geometry of a 4-dimensional space-time, where gravity is said to be the effect of the curvature of space-time. This theory is the general theory of relativity [14,15,19] developed by Albert-Einstein.

Quantum mechanics [7–9,13] is another popular branch of physics studying the quantization and behavior of particles, and other unique phenomena and properties that these particles possess and experiences.

The dynamics of these particles and quantum fields are studied using frameworks such as relativistic quantum mechanics, canonical quantization, and path integral. Mathematically, Relativistic quantum mechanics uses wave-equations such as the Klein-Gordon's equation for spin-0 scalar fields, and Dirac's equation for spin-1/2 spinor fields.

While these frameworks and equations have proven to be useful tools in the study of particles and quantum fields, fitting in the concept of gravity or general relativity into these frameworks still remains an ongoing area of research referred to as quantum gravity [6].

There are several attempts at developing theories of quantum gravity. One such attempt is loop-quantum gravity [4,5] which is a canonical theory of quantum gravity that takes the space-time continuum to be a complex geometric structure known as spin-networks. A similar attempt in developing a quantum theory of gravity is the Causal Dynamical triangulation [20,21] which discretizes space-time in a lattice structure. CDT employs a path integral style formulation but replaces the integrals with discrete sum which is an approximation to integrals hence leading to the lattice structure. Another prominent attempt is strings theory [3] though this is a much wider theory that defines quantum fields and particles as vibrational modes of a string, where a particular vibrational mode exists that leads to the existence of graviton, a hypothetical particle expected to be the carrier of the gravitational force.

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While the theories mentioned above are fully within quantum domains, another theory proposed as a semi-classical approximation to quantum gravity is QFT in Curved Space-time [1,2]. This theory defines quantum field theories in curved background space-time, unlike the usual flat background. It is considered semi-classical because we're only treating quantum field quantum mechanically while we treat space-time classically.

Gravitational redshift [16–18] is a theory proposed within the domains of general relativity, it predicts a shift in the wavelength or frequency of light as it goes further away from the source of the gravitational field. This shift would then be noticed as the change in color of the light. According to general relativity this shift in wavelength is attributed to gravitational time dilation effects experienced by the light as it moves further away from the gravitational field.

The quantum and relativistic nature of light as well as its experience of redshift motivates this paper, discussing how the quantum states of light can be influenced by gravitational redshifts, and presents a modification to common relativistic wave equations such that they account for gravitational effects (particularly the gravitational red-shift).

Gravitational Redshift [16–18]

Gravitational redshift as stated in the introduction is one of the various phenomena predicted by general relativity, sometimes referred to as Einstein shift. It is the increase in wavelength or decrease in frequency of light or electromagnetic waves as they travel further away from the gravitational field, this is attributed to the fact that the wave loses energy as it travels further away from the gravitational well. The opposite effect referred to as blue shift involves an energy gain as the wave travels into the gravitational well, leading to increase in the frequency and decrease in the wavelength. Before proceeding to the main ideas of this brief paper, we review some of the mathematical expressions of gravitational redshift which will be instrumental in communicating the subject of this article.

The gravitational red-shift is expressed as;

$$z_f = \frac{\Delta f}{f_0} = -\frac{\Delta\phi}{c^2}, \quad z_\lambda = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta\phi}{c^2}$$

z_f *is the frequency shift, and* z_λ *is the shift in the wavelength* From which we can state the following;

$$f = f_0(1 - \frac{\Delta\phi}{c^2}) \qquad \lambda = \lambda_0(1 + \frac{\Delta\phi}{c^2})$$

Also the gravitational time dilation according to general relativity is given by $\tau = \frac{t}{\sqrt{1-\frac{\Delta\phi}{c^2}}}$

These expressions marks the starting point of the subject to be explored in this paper.

Planck-Einstein Relation

The Planck-Einstein relation is one of the foundational equations of quantum mechanics, relating the energy of a photon to its frequency. The relation implies a direct proportionality between the energy of the photon and its frequency with its proportionality constant given to be the Planck's constant.

This is the equation $E = hf_0$ *where* f_0 *is the natural or actual frequency of light*

However upon entering a gravitational well, the frequency becomes shifted to;

$$f = f_0(1 - \frac{\Delta\phi}{c^2})$$

Expressing Planck's relation in terms of its observed/shifted frequency$E = hf$, the expression yields;

$$E = h[f_0\left(1 - \frac{\Delta\phi}{c^2}\right)]$$

Which we alternatively write as $E = h\left(f_0 - \frac{\Delta\phi}{c^2} f_0\right)$…now implying an energy shift.

By doing this the relation accounts for the effect of gravity on the energy of the photon.

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However we understand that the Planck-Einstein relation is not limited to photons only, and that it applies to any particle as long as they exhibits wave properties. And if particles such as electrons have wave-properties it can be theorized that they may also be capable of experiencing shifts in their energy, frequency and wavelength when climbing out of a gravitational well.

If this be the case, then the modified Planck's relation given above may apply to all matter waves.

De-Broglie'S Relation

The De-Broglie's relation developed by Louis De-Broglie is based on the wave-nature of matter, the relation describes the natural/actual wavelength of matter waves as;

$$\lambda_0 = \frac{h}{p}$$

Taking momentum to be the subject of the relation leaves us with $p = \frac{h}{\lambda_0}$.

Just like we did for the planck-einstein relation, considering gravitational red shift of the natural frequency to the observed frequency, gives us the momentum shift;

$$p = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

From both the Planck's relation and de-Broglie's relations we have seen that gravitational redshifts can be accounted for in a particle's energy and momentum states through the shifting of their wavelengths and frequencies.

Mass-Energy Equivalence

Now that we've considered the shifts in energy and momentum of particles based on their shifts in wavelength and frequency.

We also consider the mass-energy equivalence principle [10] known by the popular equation

$$E = MC^2$$

Referring back to the expressions for the shifts

$$z_f = \frac{\Delta f}{f_0} = -\frac{\Delta\phi}{c^2} \qquad z_\lambda = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta\phi}{c^2}$$

Let c^2 be the subject of the relation

$$c^2 = \frac{\Delta\phi}{z_\lambda} \quad where \ \ z_\lambda = \frac{\Delta\lambda}{\lambda_0}$$

Substituting for c^2 in the mass-energy equation, the equation takes a different form

$$E = M\Delta\phi {z_\lambda}^{-1} \qquad z_\lambda = \frac{\Delta\lambda}{\lambda_0}$$

The gravitational redshift is thus accounted for in the mass energy equivalence. Hence the following statements can be made that the energy of a particle is directly proportional to the mass of the particle and the change in gravitational potential acting on it, with a constant of proportionality given by the inverse of the shift.

A more suitable equation for the energy of massless particles in the presence of gravity would then be

$$E = p(\sqrt{\Delta\phi {z_\lambda}^{-1}})$$

where p is the momentum of the particle and $\sqrt{\Delta\phi {z_\lambda}^{-1}}$ is equivalent to the speed of light "c"

Relativistic Energy

Since the equation $E = M\Delta\phi {z_\lambda}^{-1}$ has been derived for particles with mass and $E = p(\sqrt{\Delta\phi {z_\lambda}^{-1}})$ for particles without mass. An analog for the relativistic energy given by $E = \sqrt{p^2c^2 + m^2c^4}$ can now be assumed in the context of gravitational shifts as

$$E = \sqrt{p^2\Delta\phi {z_\lambda}^{-1} + m^2(\Delta\phi {z_\lambda}^{-1})^2}$$

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Quantization via promotion of the momentum and energy in the relativistic energy equation to momentum and energy operators respectively, gives rise to the relativistic wave equation [11,12] one of which is known as the Klein Gordon's equation. The same can be done with the modified energy equation, yielding a form of the Klein Gordon's equation that includes the effect of gravitational red-shift.

Klein Gordon's Equation

Klein Gordon's relativistic wave-equation used in studying scalar field's or spin-0 fields, klein-gordon's equation has the following form

$$\frac{ih^2}{c^2}\frac{\partial^2}{\partial t^2}\psi = i\hbar^2\nabla^2\psi + m^2c^2\psi$$

On the left hand side of this equation is the energy operator acting on scalar field ψ and on the right hand side of the equation we have the momentum operator multiplied by the speed of light also acting on the same scalar field and we also have the additional term mc^2.

Introducing gravitational red-shift into this equation would involve minimal substitutions for the speed of light "c" as we did in other sections of this paper.

Taking the substitutions we arrive at the following

$$\frac{ih^2}{\Delta\phi {z_\lambda}^{-1}}\frac{\partial^2}{\partial t^2}\psi = i\hbar^2\nabla^2\psi + m^2\psi\Delta\phi {z_\lambda}^{-1}$$

This equation treats the dynamics of a scalar field under the Gravitational potentials via. The influence of Gravitational redshift on the scalar field ψ and Gravitational interactions becomes inherent in the framework of relativistic quantum mechanics through minimal substitutions.

Dirac's Equation

The relativistic equation put forward by Paul-Dirac describes the dynamics of spinor field, taking the spins of the particle into consideration. Although initially developed as a theory for studying the dynamics of electrons, Dirac's equation also applies to fermions with ½ spins.

Dirac's equation which is typically given by

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

The spin of the particle is accounted for through the gamma-matrices satisfying the relation $\{\gamma^\mu, \gamma^v\} = 2\eta^{\mu v} I$

And this draws a major distinction between Dirac's equation and klein-gordon's equation, where ψ is a spinor field.

When gravitational redshift is considered Dirac's equation becomes

$$[i\hbar\gamma^\mu\partial_\mu - m(\Delta\phi {z_\lambda}^{-1})^{\frac{1}{2}}]\psi = 0$$

Now satisfying the relation $\{\gamma^\mu, \gamma^v\} = 2g^{\mu v} I$ for a curved spacetime.

Hence the spinor field is said to be experiencing changes in its energy as moves further away from the region where the gravitational potential is at its greatest.

Feynman Propagators

With the modified klein-gordon's equation already given, a propagator accounting for the gravitational redshift would be necessary

The propagator may be given by;

$$G(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2\Delta\phi {z_\lambda}^{-1} - m^2(\Delta\phi {z_\lambda}^{-1})^2 + i\epsilon}$$

The propagator takes this form due to the fact that the constant speed of light "c" retains its presence in the equations, unlike common forms of the Klein Gordon equation and Energy-momentum relation typically given by

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$$E^2 = p^2 + m^2 \quad and \quad i\hbar^2 \frac{\partial^2}{\partial t^2}\psi = i\hbar^2\nabla^2\psi + m^2\psi$$

Whether we the "c" is present or not, both forms are still applicable and preserves the mathematical relationship given by the equation. Where we have already stated that $c = \sqrt{\Delta\phi z_\lambda{}^{-1}}$.

QFT in Curved Spacetime

The equations and relations discussed in this paper are quite similar in concepts but different in formulation to QFT in Curved Space-time [1,2]. As QFT in CS studies quantum field theory with the effect of space-time geometry, while the concepts studied in this paper does not explicitly accounts for the geometry of space-time. However they both involve the interplay between quantum fields and gravity.

In summary the fundamental difference between the QFT in CS and the subject of our study is that the first incorporates effects of gravity in terms of geometry and the latter incorporates effects of gravity through gravitational red-shift. However both geometry and Gravitational red-shifts are both concepts revolving around general relativity.

Recasting Divergences

The following can be said of a propagator; given the difference between the two UV divergent propagators with one lower limit at 0 and the other has a non-zero lower limit where both are corrected. When the difference is taken, one would find that the resulting value is equivalent to the value of a corrected divergent IR propagator whose upper limit and lower limits carries the corrections to the momentum, as seen below:

$$\int_{0+\Delta k}^{k2+\Delta k} \frac{d^4k}{(2\pi)^4} \frac{i}{k^2\Delta\phi z_\lambda{}^{-1} - m^2(\Delta\phi z_\lambda{}^{-1})^2 + i\epsilon}$$

$$= \int_{k2+\Delta k}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{i}{k^2\Delta\phi z_\lambda{}^{-1} - m^2(\Delta\phi z_\lambda{}^{-1})^2 + i\epsilon} - \int_{0+\Delta k}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{i}{k^2\Delta\phi z_\lambda{}^{-1} - m^2(\Delta\phi z_\lambda{}^{-1})^2 + i\epsilon}$$

With Δk essentially being the small correction to the momentum at the upper and lower limit of the integral, Correcting the propagator this essentially avoids IR divergence, by ensuring that momentum at all scales are corrected rather than just cut-off.

Conclusion

Gravitational redshift is a concept that lies in between general relativity and the wave nature of light, revealing how the properties of light waves such as its wavelength, energy and frequency are altered as it climbs out of the gravitational well.

However this wave nature of light can be studied within the domains of the fundamental principle of quantum mechanics, fundamental principle such as the planck-einstein relation and the de-Broglie's relation, where the redshifts of the wavelength and frequencies is considered in the relations. Due to the wave properties of matter it can also be theorized that matter-waves apart from just photons may also be able to experience gravitational red-shifts.

The reformulation of mass-energy equivalence can also be made in the context of gravitational redshift, by substituting the speed of light with terms from the gravitational redshift relation. From these the relativistic energy equation can be expressed in terms of gravitational red-shift. Upon quantization the gravitational red-shift is carried over to the equations of relativistic quantum mechanics.

The approaches taken so far in this theoretical exploration shows that gravitational red-shift is a relevant concept in both quantum mechanics and general relativity, when the particles are in a region where gravitational potentials is present. This bears similarities with QFT in CS in that quantum fields are studied in the presence of gravity, with the distinction being that QFT in CS treats gravity geometrically, while this study is based on gravitational redshifts

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Declarations

I hereby declare that this article, titled; "Quantum Dynamics under Gravitational Potential: Insights from gravitational redshift".

- Is written with no conflicting interest, neither is there any existing or pre-existing affiliation with any institution.
- No prior funds is received by the author from any organization, individual or institution.
- The content of the article is written with respect to ethics.
- The content of the article does not involve experimentation with human and/or animal subjects.
- Data-availability; no data, table or software prepared by an external body or institution is directly applicable to this article.

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