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Article

Perturbed Flrw Metric Explains the Difference in Measurements of the Hubble Constant

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Abstract: A cosmological model described by the perturbation metric is proposed, in which space is entrained by matter expanding as a result of the Big Bang. This model explains the difference in measurements of the Hubble constant using gravitational lensing and a distance ladder.

Keywords: perturbed FLRW metric; Einstein equations; particle Lagrangian; Hubble constant; gravitational lensing; distance ladder

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1. Introduction

In the ΛCDM (Lambda Cold Dark Matter) cosmological model, based on the Friedmann-Lemaître-Robertson-Walker metric, matter moves synchronously with the expanding space. We will consider a model in which space is carried away by matter dispersing as a result of the Big Bang. It is described by a perturbed metric [1–3], which is a generalization of the FLRW metric.

Determination of the Hubble constant using the cosmic distance ladder [4,5] and gravitational lensing [6,7] yields a difference of about 1/10 of its value. We consider the dependence of the change rate of photon energy on the direction of its motion. It is determined using the principle of the extremum of the photon's energy integral [8–12] and the relationship between it and the energy of a material particle obtained using Lagrange mechanics [8].

2. The Solution of the Einstein Equations for the Perturbed Metric

A perturbed metric [1–3] is a generalization of the FLRW metric. Here we consider spacetime with the following line element:

$$ds^2 = c^2dt^2 - 2cf(t)rdrdt - (a(t)^2 - f(t)^2r^2)dr^2 - a(t)^2r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where $f(t)$ is an additional metric function. For small r, this metric is approximated by the metric of flat expanding spacetime. The components of the Einstein tensor [13] are given by

$$G^{11} = \frac{1}{a^6}\{3a^4\dot{a}^2 - 6a^3f\dot{a} + r^2[2(a^2f^2 - f^3\dot{a})\dot{q} + 2af^3\dot{a} - 4a^2f^2\dot{a}^2] + r^4[f^4\dot{a}^2 + 2af^3\dot{a}\dot{f}]\}, \tag{2}$$

$$G^{12} = G^{21} = \frac{fr}{a^6}[(2a\dot{f}\dot{a} + f\dot{a}^2)r + (2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2)a], \tag{3}$$

$$G^{22} = \frac{1}{a^6}\{-2a^3\ddot{a} - a^2\dot{a}^2 + 2a^2\dot{f} + 2af\dot{a} + r^2(2af\dot{a}\dot{f} + f^2\dot{a}^2)\}, \tag{4}$$

$$G^{33} = G^{44}\sin^2\theta = \frac{1}{a^6r^2}\{-2a^3\ddot{a} - a^2\dot{a}^2 + 2a^2\dot{f} + 2af\dot{a} + r^2(a\ddot{a}f^2 - f^2\dot{a}^2 + fa^2\ddot{f} + a^2\dot{f}^2 + af\dot{a}\dot{f})\}, \tag{5}$$

where ($\dot{}$) denotes differentiation with respect to $x^1 = ct$.

The components of the contravariant energy-momentum tensor correspond to the energy density, momentum density, energy flux through unit surfaces per unit time, and stresses [14]. The energy-momentum tensor of matter in contravariant form is given by

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$$T^{ij} = (c^2\rho + p)u^i u^j - g^{ij}p. \tag{6}$$

The non-zero contravariant coefficients for the metric (1) will be as follows:

$$g^{11} = \frac{a^2 - f^2r^2}{a^2}, g^{12} = g^{21} = -\frac{fr}{a^2}, g^{22} = -\frac{1}{a^2}, g^{33} = -\frac{1}{a^2r^2}, g^{44} = -\frac{1}{a^2r^2 sin^2\theta}. \tag{7}$$

In the absence of angular momentum of matter ($u^3 = u^4 = 0$), the non-zero components of the energy-momentum tensor (6) are given by

$$T^{11} = (c^2\rho + p)(u^1)^2 \text{-} \left(1 - \frac{f^2r^2}{a^2}\right)p, \tag{8}$$

$$T^{12} = T^{21} = (c^2\rho + p)u^1u^2 + \frac{fr}{a^2}p, \tag{9}$$

$$T^{22} = (c^2\rho + p)(u^2)^2 + \frac{1}{a^2}p, \tag{10}$$

$$T^{33} = T^{44}\sin^2\theta = \frac{1}{a^2r^2}p \tag{11}$$

Let's consider a model with a cosmological constant $\Lambda = 0$. We will seek a solution for a region where the radial velocity u^2 is small, and the term of component T^{22} containing its square is negligible and can be disregarded. Next, we will justify this assumption. Also, we assume that the terms of the Einstein tensor components (2)-(5) containing r^2 in the numerator are small and can be neglected. Given the satisfaction of this condition and for g^{11}, Einstein equations $G^{ij} = \chi T^{ij} + g^{ij}\Lambda$ with the energy-momentum tensor components (8)-(11), yield

$$\frac{3a\dot{a}^2 - 6f\dot{a}}{a^3} = \chi c^2\rho, \tag{12}$$

$$\frac{2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2}{a^3} fr = \chi a^2(c^2\rho + p)u^1u^2 + \chi frp, \tag{13}$$

$$\frac{-2a^2\ddot{a} - a\dot{a}^2 + 2a\dot{f} + 2f\dot{a}}{a^3} = \chi p. \tag{14}$$

The absence of additional energy and momentum influx implies the equality of the corresponding densities T^{12}, T^{21} to zero, i.e., the right-hand side of equation (13). From its left-hand side we obtain

$$2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2 = 0. \tag{15}$$

Solving this equation for f under the condition $a(t_0) = 1$ we find

$$f(t) = \frac{1}{a}\left(f_0 + \frac{3}{2}c\int_{ct_0}^{ct} \dot{a}^2 a dt\right) \tag{16}$$

with $f(t_0) = f_0$. Substituting this into equation (12) under the condition

$$\rho = \frac{\rho_0}{a(t)^3}, \tag{17}$$

with average density of the Universe at the present time $\rho_0 = \varrho(t_0)$ yields

$$3a\dot{a}^2 - \frac{6f_0\dot{a}}{a} - \frac{9\dot{a}}{a}c\int_{ct_0}^{ct} \dot{a}^2 a dt = \chi c^2\rho_0. \tag{18}$$

Expressing the integral from here and differentiating it with respect to x^1, from the obtained equality we find

$$\chi c^2\rho_0\left(1 - \frac{a\ddot{a}}{\dot{a}^2}\right) = -3\dot{a}^2 a + 3\ddot{a}a^2. \tag{19}$$

This equation has a solution

$$a(t) = e^{cA_1(t-t_0)}\,, \tag{20}$$

where A_1 is a constant.

From equations (14) and (15), we obtain the pressure

$$p = -\frac{2}{c\chi}\frac{d}{dt}\left(\frac{\dot{a}}{a}\right), \tag{21}$$

which, due to (20), turns out to be $p = 0$. In this case, from (13) under the condition of the absence of energy and momentum influx from the outside (15), we find the radial component of the matter velocity in the comoving reference frame.

$$u^2 = 0. \tag{22}$$

Thus, it turns out to be a valid assumption that the radial velocity in the region under consideration is small and may not be taken into account in the component of the energy-momentum tensor (10).

The function (16), with the scale factor of space (20), takes the form

$$f(t) = e^{-cA_1(t-t_0)}f_0 + \frac{1}{2}A_1\left(e^{2cA_1(t-t_0)} - e^{-cA_1(t-t_0)}\right) \tag{23}$$

and equation (18) yields

$$3{A_1}^2 - 6f_0A_1 = \chi c^2\rho_0\,. \tag{24}$$

3. Change in Photon Energy

Let's determine how the energy of a photon changes in the considered space-time using the principle of the extremum of the photon's energy integral [8–12]. The Euler-Lagrange equations for the covariant components of the energy-momentum vector of a light-like particle of unit energy

$$\frac{dp_\lambda}{d\mu} - F_\lambda = 0 \tag{25}$$

with

$$F_\lambda = \frac{1}{2u_1u^1}\frac{\partial g_{ij}}{\partial x^\lambda}u^iu^j, \tag{26}$$

corresponding to the energy, take the form

$$\frac{dp_1}{d\mu} = -\frac{1}{u_{1ph}u^1_{ph}}\left[\dot{f}ru^1_{ph}u^2_{ph} + (\dot{a}a - \dot{f}fr^2)(u^2_{ph})^2 + \dot{a}a\Omega^2\right] \tag{27}$$

under the designation

$$\Omega^2 = r^2((u^3_{ph})^2 + \sin^2\theta(u^4_{ph})^2). \tag{28}$$

From the equality of interval (1) to zero, we obtain

$$u^2_{ph} = \frac{-fru^1_{ph} + \sigma\sqrt{a^2(u^1_{ph})^2 - (a^2 - f^2r^2)\Omega^2}}{a^2 - f^2r^2}, \tag{29}$$

where σ takes on values of ±1 depending on the direction of the light. For small deviations from radial motion:

$$\Omega^2 << (u^1_{ph})^2 \tag{30}$$

we find

$$u^2_{ph} = \frac{1}{fr + \sigma a}u^1_{ph} - \frac{\sigma\Omega^2}{2au^1_{ph}}. \tag{31}$$

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The expression for the time component of the covariant velocity vector is as follows:

$$u_{1ph} = u^1_{ph} - fru^2_{ph} = \frac{a}{a+\sigma fr}u^1_{ph} + \frac{\sigma fr\Omega^2}{2au^1_{ph}}. \tag{32}$$

Further we will consider $fr, \dot{a}/a, \dot{f}/f$ to be small quantities. After substituting the components u^2_{ph} и u_{1ph} into equation (27) without accounting for higher order small quantities, it is rewritten in the form

$$\frac{dp_1}{d\mu} = -\frac{\sigma\dot{a}+\dot{f}r}{fr+\sigma a}. \tag{33}$$

Since the covariant and contravariant temporal components of the generalized momenta of a light-like particle

$$p_\lambda = \frac{u_\lambda}{u^1 u_1}, \tag{34}$$

$$p^\lambda = \frac{u^\lambda}{u^1 u_1} \tag{35}$$

are related by

$$p^1 = \frac{u^1_{ph}}{u_{1ph}}p_1 = \left(1+\sigma\frac{fr}{a}\right)p_1, \tag{36}$$

the rate of energy change can be written in the form

$$\frac{dp^1}{d\mu} = \left(1+\sigma\frac{fr}{a}\right)\frac{dp_1}{d\mu} + \sigma\frac{1}{a^2u^1_{ph}}\left[r(\dot{f}a - \dot{a}f)u^1_{ph} + fau^2_{ph}\right]. \tag{37}$$

Substituting u^2_{ph} from equation (31) and $dp_1/d\mu$ from equation (33) into this expression, we find

$$\frac{dp^1}{d\mu} = -\frac{\dot{f}r+\sigma\dot{a}}{\sigma a} + \frac{1}{\sigma a^2}\left[r(\dot{f}a - \dot{a}f) + \frac{fa}{fr+\sigma a} - \frac{\sigma f\Omega^2}{2(u^1_{ph})^2}\right]. \tag{38}$$

Assuming that the terms in this equation containing r are small in the region under consideration, we can write

$$\frac{dp^1}{d\mu} = -\frac{\dot{a}}{a} + \frac{f}{\sigma a^2} - \frac{f\Omega^2}{2a^2(u^1_{ph})^2}. \tag{39}$$

In the case of the motion of light emitted towards the observer, we choose $\sigma = -1$. Then, upon substitution of (20) and (23), this expression is transformed to become

$$\frac{dp^1}{d\mu} = -A_1\left(\frac{3}{2} - \frac{1}{2}e^{-3cA_1(t-t_0)}\right) - f_0e^{-3cA_1(t-t_0)} - \frac{\Omega^2}{2(u^1_{ph})^2}\left[\frac{1}{2}A_1\left(1-e^{-3cA_1(t-t_0)}\right) + f_0e^{-3cA_1(t-t_0)}\right]. \tag{40}$$

In the region under consideration, the energy of a radially moving photon is given by

$$E_{ph} = h\nu_0\left(1 - r\left.\frac{dp^1}{d\mu}\right|_{t=t_0}\right), \tag{41}$$

where ν_0 - is the value of the photon frequency at the observation point. Due to the fact that in the cosmological model described by an orthogonal metric, the Hubble constant is expressed as

$$\breve{H} = c\frac{\dot{a}(t_0)}{a(t_0)}, \tag{42}$$

from equation (24), when comparing it with the Friedman equation

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$$\chi c^2 \rho = \frac{3}{a^2}[k + (\dot{a})^2] - \Lambda, \tag{43}$$

it follows that A_1 can be a quantity of the same order as the Hubble parameter. After substituting the function f (23) without small quantities, equation (40) takes the form

$$\left.\frac{dp^1}{d\mu}\right|_{t=t_0} = -A_1 - f_0 - \frac{f_0 \Omega^2}{2(u^1_{ph})^2}. \tag{44}$$

Since in the region under consideration the spacetime metric (1) approximates the Minkowski metric, the affine parameter will be close to $\mu = ct$.

4. Change in the Energy of a Material Particle in the Co-Moving Frame

The Lagrangian of a material particle [8] in space-time (1) is given by

$$L = cm\{(u^1)^2 - 2f(t)ru^1u^2 - [a(t)^2 - f(t)^2r^2](u^2)^2 - a(t)^2r^2((u^3)^2 + \sin^2\theta(u^4)^2)\}^{1/2}. \tag{45}$$

The motion of a material particle in a gravitational field, like that of a photon, in accordance with equation (25) at $\mu = s$ can be represented [6,10,12] as the result of the action of generalized forces

$$F_\lambda = \frac{\partial L}{\partial x^\lambda} = \frac{1}{2} cm \frac{\partial g_{ij}}{\partial x^\lambda} u^i u^j, \tag{46}$$

which, however, do not generally form a first-order tensor. The components of the vector of generalized forces associated with F_λ,

$$F^k = \frac{1}{2} cm g^{k\lambda} \frac{\partial g_{ij}}{\partial x^\lambda} u^i u^j \tag{47}$$

are related to gravitational forces $\mathrm{F}^k = cF^k$. The equations of motion (25) for the contravariant momentum are transformed into the form

$$\frac{d\bar{p}^k}{d\mu} = \frac{dp^k}{d\mu} + \frac{d\vec{p}^k}{d\mu} = F^k, \tag{48}$$

where

$$\frac{d\overleftrightarrow{p}^k}{d\mu} = g^{k\lambda} \frac{\partial g_{\lambda i}}{\partial x^j} u^j p^i \tag{49}$$

represents the energy imparted to the gravitational field.

Let's consider a particle of matter in a comoving frame, in which it is motionless (22). The component of the force vector (47) corresponding to energy, F^1_m, becomes zero. The rate of change of energy transferred to the gravitational field (49) will be

$$\frac{d\overleftrightarrow{p}^1_m}{ds} = cm \frac{f\dot{f}r^2}{a^2}. \tag{50}$$

Due to (48), the energy of the material particle changes at a rate of

$$\frac{dp^1_m}{ds} = -cm \frac{f\dot{f}r^2}{a^2}. \tag{51}$$

The derivative of the function $f(t)$ has the form

$$\dot{f} = -A_1 f_0 e^{-cA_1(t-t_0)} + A_1^2 \left(e^{2cA_1(t-t_0)} + \frac{1}{2} e^{-cA_1(t-t_0)}\right). \tag{52}$$

Now we can express the change in energy of the material particle at the present time as

$$\left.\frac{dp^1_m}{ds}\right|_{t=t_0} = cmA_1 \left(f_0 + \frac{3}{2}A_1\right) f_0 r^2. \tag{53}$$

In the considered region, the energy of the material particle will be

$$E_m = c\left(cm - r\frac{dp_m^1}{ds}\bigg|_{t=t_0}\right). \quad (54)$$

5. Cosmological Redshift in the Radial Motion of Photons

The equation (24) is rewritten in the form

$$3(A_1 + f_0)^2 - 12f_0A_1 - 3{f_0}^2 = \chi c^2\rho_0 . \quad (55)$$

We assume that f_0 is of the same order of magnitude as A_1 and, consequently, with the Hubble constant. Since in the considered region the quantity fr is small, taking into account the expressions for the rate of change of particle energies (44) and (53), the change in the energy of a material particle will be small in comparison with the change in the equivalent energy of a photon (41) because of

$$\left(\frac{dp_m^1}{ds}\bigg|_{t=t_0}\right)\Big/\left(h\nu_0\frac{dp^1}{d\mu}\bigg|_{t=t_0}\right) = o(fr), \quad (56)$$

where the differentiation parameters are

$$s \approx \mu \approx ct. \quad (57)$$

The observed redshift of photon energy is considered as a result of the change in the ratio of their energy (41), (44) to the energy of atoms (54). In a small neighborhood without small higher-order quantities due to (56), this will be

$$\frac{E_{ph}}{E_m} = \frac{h\nu_0}{c^2m}[1 - r(A_1 + f_0)], \quad (58)$$

from which the value of the Hubble constant at present time

$$H_0 = c(A_1{+}f_0), \quad (59)$$

can be deduced, obtained as a result of registering radially moving photons.

In this case, equation (55) takes the form

$$3\left(\frac{H_0}{c}\right)^2 - 12\frac{H_0}{c}f_0 + 9{f_0}^2 = \chi c^2\rho_0 . \quad (60)$$

The length element

$$dl^2 = \left(\frac{g_{1p}g_{1q}}{g_{11}} - g_{pq}\right)dx^p dx^q \quad (61)$$

with $p, q = 1,2,3$ in the space-time (1) is given by

$$dl = a\Omega d\mu. \quad (62)$$

Due to (20), the relative rate of its change does not depend on time and on the coefficient f_0, which makes it impossible to determine it directly from the value of the Hubble constant.

6. Difference in Hubble Constant Measurements

Let's consider how the energy of a photon will change in the presence of gravitational lensing. We denote the change in the energy of the photon when moving along the path L_l, which includes gravitational lensing, as

$$\delta p_l^1 = \int^{L_l} \frac{dp^1}{d\mu}d\mu \quad (63)$$

and along the radial path as

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$$\delta p_r^1 = \int^{L_r} \frac{dp^1}{d\mu} d\mu. \tag{64}$$

We assume that the coordinate distance Δr from the beginning of the paths to the observer is the same. Under the gravitational lensing, with the above assumptions and neglecting small quantities, the Hubble constant is determined as follows:

$$H_0 = -\frac{c(\delta p_l^1 - \delta p_r^1)}{\delta\mu_l - \delta\mu_r}, \tag{65}$$

where $\delta\mu_l$ and $\delta\mu_r$ are the change in the affine parameter along each of the paths.

The coordinate system is chosen such that when the photon moves along the path L_l the condition $\theta = \frac{\pi}{2}$ is satisfied. If we take into account only the influence of the deviation from radial motion, then the difference in the time taken for the photon to travel along both paths will be

$$\delta t = \frac{1}{c}\int_0^{\Delta r} (1 - \cos(\alpha(r))dr, \tag{66}$$

where in the first approximation we consider

$$\cos(\alpha(r)) = -\frac{u_{ph}^2}{\sqrt{(u_{ph}^2)^2 + (ru_{ph}^4)^2}}. \tag{67}$$

Due to condition (30), the integral (66) transforms into the form

$$\delta t = \frac{1}{c}\int_0^{\Delta r} \frac{(ru_{ph}^4)^2}{2(u_{ph}^2)^2} dr. \tag{68}$$

In the considered region, we have

$$\delta\mu_l - \delta\mu_r = c\delta t. \tag{69}$$

By substituting the corresponding value from (44) into (63) and (64), we find the additional difference in energy change caused by the discrepancy in the rate of its during radial and angular motion

$$\Delta(\delta p_l^1 - \delta p_r^1) = -\int_0^{\Delta r} \frac{f_0(ru_{ph}^4)^2}{2(u_{ph}^1)^2} dr. \tag{70}$$

The change in the value of the Hubble constant when light moves along a curved path will be

$$\Delta H_0 = -\frac{c\Delta(\delta p_l^1 - \delta p_r^1)}{\delta\mu_l - \delta\mu_r}. \tag{71}$$

Since for the considered motion of the photon, the relations

$$u_{ph}^1 \approx -au_{ph}^2 \approx -u_{ph}^2 \tag{72}$$

hold, by substituting the values (69) and (70) into (71), we obtain

$$\Delta H_0 = cf_0. \tag{73}$$

Let's estimate the values of A_1 and f_0, using the results of determining the Hubble constant with and without gravitational lensing. The Hubble constant was determined using gravitational lensing of the cosmic microwave background: 68.3±1.5 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [6] and radiation from the supernovae: $66.6^{+4.1}_{-3.3}$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [7]. The value obtained using the extragalactic distance ladder to supernovae is 73.5±1.1 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [4] and $75.4^{+3.8}_{-3.7}$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [5]. These results yield the value $\Delta H_0 = cf_0 = -7^{+2.4}_{-2.6}$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and due to relation (59) which corresponds to the radial motion of the photon, we obtain $cA_1 = 81.5 \pm 2.4$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

When the value of f_0 is negative, light moving from the stars to the observer experiences additional deceleration due to the dragging of space by the expanding matter compared to the motion tangential to the circle from the center of observation or in the opposite direction. This leads to additional energy loss for the photon. The dependence of the metric properties of spacetime on the observer's position is similar to the manifestation of the curvature of spacetime in the FLRW cosmological model with non-zero curvature.

7. Conclusions

Unlike the ΛCDM cosmological model, in which space moves synchronously with matter, in this case space is carried away by matter flying apart as a result of the Big Bang. This effect is similar to what occurs in Kerr space-time and can be considered as a manifestation of Mach's principle as interpreted by Einstein [15].

In the considered model of the local part of the Universe, the absence of additional energy and momentum corresponds to the exponential expansion of space and zero pressure. An equation is obtained that relates the constant of relative expansion of space and the acceleration parameter of matter expansion with the energy density within the limits of known cosmological parameters for a model with a zero cosmological constant. The considered metric can also be applied to a cosmological model with a nonzero Λ -term.

The redshift in the spectra of galaxies is determined using Lagrange mechanics in its application to the principle of the extremal integral of photon energy. This made it possible to establish a connection between the parameters of the metric and the Hubble constant based on the difference in its value obtained using gravitational lensing and distance ladder.

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