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The Muon $g-2$ in a Regularized Electrodynamics

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Abstract: The present paper reports exact results for the muon self-energy and anomalous g-factor. A unique to the muon cut-off terms that remove the radial singularity in the system and regularize the corresponding electrodynamics are presented. Equations of motion and a transcendental equation satisfied by the muon anomalous g-factor are derived, with solution $a_\mu = 0.001165920162(198)$. The obtained value matches the latest experimental one found in the literature to about 0.43 ppb ruling out a possible tension between theory and experiment.

Keywords: muon; self-energy; anomalous magnetic dipole moment; regularization; electrodynamics

1. Introduction

The study of the electron's anomalous magnetic dipole moment have played an essential role in the development of quantum theory [1–4]. After its successful theoretical description it was expected that the followed calculation of the anomalous component in the muon's magnetic dipole moment will strengthen and crystallize the established knowledge [5–8]. Over the decades, however, with the improvement of the experimental setup and the consequent highly precise measurements the gap between the measured and calculated values not only remained but thickened [9–13]. The difference between the most recent experimental result [14] and the most recent consensus for the theoretical value [15] is about 2.49 ppb, a value that is greater than the relevant uncertainty. In the light of the seeming discrepancy many efforts to revise and improve the hadronic vacuum polarization corrections [16–20] and the hadronic light-by-light scattering one [19–23] have been considered. It is believed that these contributions have the prospect to reduce the obtained tension. On the other hand, the regularization procedure [24–30] within the classical and semi-classical methods have recently demonstrated great potential in quantifying the self-interaction and electron's anomalous magnetic moment.

The present study implements the regularization procedure proposed in Ref. [30] (see also Ref. [31]) to quantify the anomalous component in the muon's magnetic dipole moment and to study the seeming tension between theory and experiment. The used approach represents a regularized electrodynamics that is tightly bound to the quantum theory beyond the corresponding principle. Accordingly, exact results for the muon's self-energy, anomalous g-factor and all intrinsic characteristics underlying the dynamics of its self-interaction are reported. Improved accuracy in the calculation of the muon anomalous g-factor is obtained (0.43 ppb), overcoming the existing gap between the average value predicted by the quantum theory [15] and the measured one [14]. An essential outcome of the obtained accuracy is an exact bound on the sum of the muon and electron neutrinos' rest masses.

2. Theoretical Framework

Thereunder, the used mathematical framework follows closely the one introduced in Ref. [30].

2.1. Generalities

Let $\mathbf{R}$ be the rest frame of reference of a free muon, with rest mass and electric charge denoted by m_μ and $\bar{e} = -e$, where e is the elementary charge. Let $r_{c\mu} = \alpha \bar{\lambda}_{c\mu}$ be the muon's electromagnetic radius in $\mathbf{R}$, where α and $\bar{\lambda}_{c\mu}$ are the fine structure constant and associated reduced Compton wavelength, respectively. Let $\mathbf{r}_\mu$ be the intrinsic field vector of the muon, with magnitude r_μ, and $\tilde{u}_\mu$ be the magnitude of the tangential velocity $\tilde{\mathbf{u}}_\mu$ related to its rotation about the origin of $\mathbf{R}$ in the plane perpendicular to the muon's relative velocity $\mathbf{u}_\mu = u_\mu \boldsymbol{\kappa}$, where $\boldsymbol{\kappa}$ is the respective unit vector. The corresponding oscillation is characterized by an angular velocity $\boldsymbol{\omega}_\mu$, with magnitude $\omega_\mu = \tilde{u}_\mu r_\mu^{-1}$ representing the muon's intrinsic angular frequency. For more details the reader may consult Ref. [30]. The quantities r_μ and $\tilde{u}_\mu$ are conjugate and satisfy

$$r_\mu \tilde{u}_\mu = \bar{\lambda}_{c\mu} c, \tag{1}$$

where c is the light speed in vacuum.

The charge ρ_e and mass ρ_{m_μ} densities are defined within the spherically symmetric spatial domain $\Omega_{c\mu} \in \mathbb{R}^3$, with radius $r_{c\mu}$, boundary $\partial\Omega_{c\mu}$ and volume $V_{c\mu}$. They satisfy the relation $\rho_e \rho_{m_\mu}^{-1} = e m_\mu^{-1}$. Moreover, ρ_{M_μ} is the muon's effective mass density and $M_\mu = m_\mu(1 + a_\mu)$ is the corresponding effective rest mass defined within $\partial\Omega_{c\mu}$, where a_μ is the muon's anomalous g-factor. Here, $\rho_{M_\mu} = \rho_{M_\mu}(r)$ is a smooth function of the radial parameter r, with $r \in (0, +\infty)$ and $\rho_{M_\mu} > \rho_{m_\mu}$ for all r. Note that if $\nexists\, e$, then $\rho_{M_\mu} = \rho_{m_\mu}$.

The inherent dynamics of $\mathbf{r}_\mu$ underpin the occurrence of intrinsic magnetic moment $\boldsymbol{\mu}_\mu = -\frac{1}{2} g_\mu \mu_B \boldsymbol{\kappa}$, where $g_\mu = 2(1 + a_\mu)$ is the g-factor of the muon and μ_B is the Bohr magneton. We further have

$$g_\mu = \frac{2}{V_{c\mu}} \int_{\Omega_{c\mu}} G_\mu \mathrm{dv}, \qquad G_\mu = \frac{e\rho_{M_\mu}}{m_\mu \rho_e}. \tag{2}$$

2.2. *Electromagnetic Field Scalar Potential and Energy*

Within the considered approach the electromagnetic field related to the self-interacting muon is time independent and does not propagate in space independently from the particle. The Lorenz gauge is trivially satisfied and the radial singularity is removed by a regularization. For the corresponding electromagnetic field scalar potential, we have

$$\varphi_\mu(r) = \gamma_\mu \eta_\mu \psi_\mu(r), \qquad \eta_\mu = 1 + \frac{u_\mu^2}{c^2}, \tag{3}$$

where γ_μ is the Lorentz factor,

$$\psi_\mu(r) = \frac{\bar{e}}{4\pi\varepsilon_o r}\left(2 - e^{-\chi_\mu r} - e^{-\tilde{\chi}_\mu r}\right), \tag{4}$$

ε_o is the electric constant. Here, the scaling constants of both cutoff terms in eq. (4) read

$$\chi_\mu = \frac{\gamma_\mu m_\mu c}{(1 + a_e)h}, \qquad \tilde{\chi}_\mu = \frac{\gamma_\mu (m_e + \delta m_\nu)c}{(1 + a_\mu)h},$$

where a_e and m_e denote the electron's anomalous g-factor and rest mass, respectively. Furthermore, we have $\delta m_\nu = m_{\nu_\mu} + m_{\bar{\nu}_e}$, where m_{ν_μ} is the muon neutrino rest mass and $m_{\bar{\nu}_e}$ is the electron anti-neutrino rest mass. Both neutrinos emerge from a decay of the self-interacting muon, with $\delta m_\nu > 0$ for all $r \in (0, +\infty)$. We would like to point out that the two Yukawa terms in eq. (4) do not represent an on shell massive photon [32]. The photons are massless and from the perspective of quantum physics χ_μ and $\tilde{\chi}_\mu$ will represent the wave numbers of effectively massive off shell photons, with total effective mass $m_\mu(1 + a_e)^{-1}$ and $(m_e + \delta m_\nu)(1 + a_\mu)^{-1}$, respectively.

The field equation satisfied by the function in Equation (4) reads

$$\Delta_r \psi_\mu(r) - \chi_\mu^2 \phi_\mu(r) - \tilde{\chi}_\mu^2 \tilde{\phi}_\mu(r) = 0,$$

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where Δ_r represents the radial Laplace operator in spherical symmetry, ϕ_μ and $\tilde{\phi}_\mu$ are the cut-off terms implying the following boundary conditions

$$\psi_\mu(r) = \begin{cases} 0, & r \to \infty, \quad u_\mu < c \\ (\chi_\mu + \tilde{\chi}_\mu)\frac{\bar{e}}{4\pi\varepsilon_o}, & r \to 0, \quad u_\mu < c \end{cases}.$$

For the corresponding vector potential representation the reader is encouraged to consult Ref. [30].

The energy of the electromagnetic field in the considered system, $W_\mu(r)$, is also regularized. Integrating the corresponding energy density $\varepsilon_o|\nabla_r\varphi_\mu(r)|^2$ over $\mathbb{R}^3$, we obtain

$$W_\mu(r) = C_\mu \frac{r_{c\mu}}{2r}\Bigg(8\big(e^{2(\chi_\mu+\tilde{\chi}_\mu)r} - e^{(2\chi_\mu+\tilde{\chi}_\mu)r} - e^{(\chi_\mu+2\tilde{\chi}_\mu)r}\big) + (2+\chi_\mu r)e^{2\tilde{\chi}_\mu r}$$
$$+ (2+\tilde{\chi}_\mu r)e^{2\chi_\mu r} + 4\frac{(\chi_\mu + \tilde{\chi}_\mu + \chi_\mu\tilde{\chi}_\mu r)}{\chi_\mu + \tilde{\chi}_\mu}\Bigg)e^{-2(\chi_\mu+\tilde{\chi}_\mu)r},$$

where $C_\mu = \gamma_\mu^2\eta_\mu^2 m_\mu c^2$. At the origin of $\mathbf{R}$ and for $u_\mu < c$ the electromagnetic field energy is finite. Thus, we have

$$\lim_{r\to 0} W_\mu(r) = C_\mu r_{c\mu}\frac{\chi_\mu^2 + 6\chi_\mu\tilde{\chi}_\mu + \tilde{\chi}_\mu^2}{2(\chi_\mu + \tilde{\chi}_\mu)}.$$

It is worth noting that in contrast to the non-regularized electrodynamics, here for all values of r the electromagnetic field constrained to the particle vanish when the particle's rest mass equals zero. Thus, hypothetically, a particle of zero rest mass will appear as electrically neutral even in the case it possess electric charge.

3. Self-Energy

To represent the self-energy of the muon we take into account the Hamiltonian formalism.

3.1. The Hamiltonian

By analogy to the case of self-interacting electron (see Ref. [30]), the energy of a self-interacting muon is spatially independent and the corresponding Hamiltonian do not depend explicitly on time. We have

$$H_\mu = \gamma_\mu m_\mu c^2 + \Sigma_\mu, \tag{5}$$

where Σ_μ is the self-energy term. The latter is not a potential energy of a gradient field and equals the spatial average over the domain $\Omega_{c\mu}$ of the interaction energy $\bar{e}\varphi_\mu$. In particular, with respect to Equation (3), we have the representation

$$\Sigma_\mu = \gamma_\mu c^2 \int_{\Omega_{c\mu}} (\rho_{M_\mu} - \rho_{m_\mu})\mathrm{dv}, \tag{6}$$

where the effective mass density reads

$$\rho_{M_\mu} = \rho_{m_\mu}\Big(1 + \eta_\mu\frac{r_{c\mu}}{r}(2 - e^{-\chi_\mu r} - e^{-\tilde{\chi}_\mu r})\Big). \tag{7}$$

3.2. The Hamiltonian Density

The information about the muon's intrinsic dynamics is embedded in the Hamiltonian density $\mathcal{H}_\mu$ associated to Equation (5). Taking into account Equation (1) for $r \equiv r_\mu$ we get

$$\mathcal{H}_\mu = c^2\rho_{m_\mu}\Big(\gamma_\mu + \frac{\alpha}{m_\mu c}\mathcal{P}_\mu\Big),$$

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where

$$\mathcal{P}_\mu = \gamma_\mu \eta_\mu m_\mu \tilde{u}_\mu \left(2 - e^{-\chi_\mu r_\mu} - e^{-\tilde{\chi}_\mu r_\mu}\right)$$

is the corresponding generalized momentum. Accordingly, we have the equations of motion

$$\tilde{u}_\mu = \int_{\Omega_{c\mu}} \frac{\partial \mathcal{H}_\mu}{\partial \mathcal{P}_\mu} \mathrm{dv}, \qquad \dot{\mathcal{P}}_\mu = 0$$

and subsequently the exact values

$$\tilde{u}_\mu = \alpha c, \quad \eta_\mu = 1 + \alpha^2, \quad \gamma_\mu^{-1} = \sqrt{1 - \alpha^2}. \tag{8}$$

According to the applied formalism the magnitude of the tangential velocity is invariant with respect to the particles mass and hence $\tilde{u}_\mu = \tilde{u}_e$, where $\tilde{u}_e$ is associated to the electron.

3.3. Effective Mass-Energy Equivalence

Accounting for Equation (2), from Equations (6) and (7) we obtain

$$\Sigma_\mu = a_\mu \gamma_\mu m_\mu c^2.$$

As a result, from Equation (5) we get the effective rest energy

$$\mathcal{E}_\mu = \gamma_\mu M_\mu c^2. \tag{9}$$

Therefore, it appears that as a result of the self-interaction the muon's energy would be $\gamma_\mu(1 + a_\mu)$ times higher that its rest energy.

4. The Anomalous g-Factor

In addition to the quantum field theory approach [11,15] in calculating the muon's anomalous g-factor, we have a classical one yielding a unique transcendental equation for its calculation. In particular, accounting for Equations (2), (7) and (8), we obtain

$$a_\mu = a_e + 3\eta_\mu \left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\alpha\gamma_\mu\xi_\mu}{2\pi(1+a_\mu)}} \left(1 + \frac{\alpha\gamma_\mu\xi_\mu}{2\pi(1+a_\mu)}\right)}{\left(\frac{\alpha\gamma_\mu\xi_\mu}{2\pi(1+a_\mu)}\right)^2} \right) \right), \tag{10}$$

where

$$\xi_\mu = \frac{m_e + \delta m_\nu}{m_\mu}. \tag{11}$$

The value of a_μ calculated from Equation (10) is given in the second row of Table 1, where the values of a_e and α are taken from Ref. [30]. The obtained accuracy regarding the most recent experimental measurements [14] is about 0.43 ppb, see Figure 1. On the same figure, a comparison with the latest known prediction of the quantum theory is also depicted. The value of mass ratio in Equation (11) is given in the third column of Table 1. We have $\delta m_\nu = 3.57763 \times 10^{-38}$ kg, where the values of electron's and muon's rest masses are taken from NIST [33]. This result suggest that the sum of the muon neutrino and electron anti-neutrino rest energies is approximately 0.02013 eV, which is consistent with the reported upper bound on the sum of the three flavor neutrino rest energies of about 0.120 eV (see Ref. [34]). Moreover, it suggests that the electron anti-neutrino mass satisfies the inequality $m_{\bar{\nu}_e} < 3.57763 \times 10^{-38}$ kg. This bound is approximately 40 times lower than the one set by KATRIN collaboration [35], see also Ref. [36].

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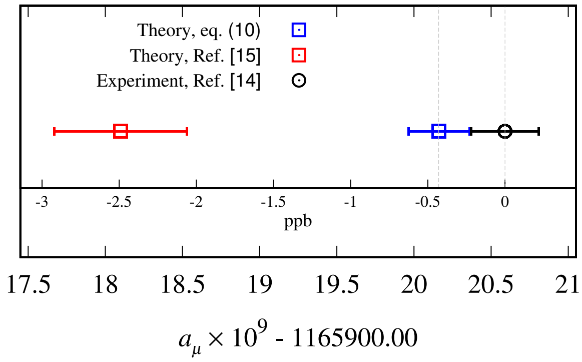

Figure 1. Comparison between the most recent experimental result (black circle) for the muon's anomalous g-factor and its value obtained from Equation (10) (blue square). In addition the latest result predicted by the quantum theory (red square) is also shown. The depicted data is further provided in Table 1.

Table 1. Theoretical and experimental data for the muon's anomalous g-factor (second column). The classical electrodynamics (CED) and quantum field theory (QFT) results are given in the second and third rows, respectively. Fourth row represents the summary of the most recent measurements. The value of mass ratio ξ_μ follows from Equation (11). For additional details see Figure 1.

	a_μ	ξ_μ	Ref.
CED	0.001165920162(198)	0.00483633188245(30)	Eq. (10)
QFT	0.00116591810(43)	–	[15]
Experiment	0.00116592059(22)	–	[14]

5. Summary and Conclusions

The present paper demonstrates the potential of regularized classical theory in describing qualitatively and quantitatively the self-energy, spin and anomalous magnetic moments of the muon. It also uncovers a key aspects from the elusive interrelationship between the classical and quantum theory that may reshape our understanding of their frontiers.

In particular, all observables characterizing the muon's self-interaction are calculated exactly including those underlying its wave-like behavior. The muon's anomalous g-factor is computed with high accuracy, improving the one obtained from the latest quantum theory calculations (see Figure 1 and Table 1) suggesting a possible ground for weakening the tension between the quantum theory and experiment. The obtained accuracy implies that the muon and electron neutrinos have a non-zero rest mass contributing to the elector-muon mass ratio, with upper bound of their sum equal to 3.57763×10^{-38} kg, see Equations (10) and (11).

The used approach can be further applied to quantify the intrinsic dynamics of the tau lepton and fix the range of values taken by the corresponding anomalous g-factor determined by the multiplicity of branching fractions.

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