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Hybridized Shock Filtering and High Order Partial Differential Equation for Preserving the Legacy of Historical Document from Degradation and Distortion

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Abstract: Image restoration plays a vital role in image processing and pattern recognition in producing better recognition accuracy. It was observed that, most of the traditional restoration algorithms were meant for removing a single type of noise and efforts have been made towards the use of partial differential equation model (PDE), to restore different types of images such as super-resolution, bleeding through and deblurring. This research work deployed hybridized shock filter, shock diffusion of higher order PDE to remove degraded images that have been affected by atmospheric turbulence and water smear images. Degraded images of different types were acquired from various sources. The degraded images were subjected to shock filter algorithm, shock diffusion of higher order PDE. The shock filter was combined with PDE, likewise the shock diffusion was also combined with PDE, and the three algorithms were also combined to test the efficiency of the hybridized method on different degraded images. The result was validated using Mean Square Error, Structural Similarity Index Measure (SSIM), Peak-to-Noise-Signal -Ratio (PNSR) and universal quality index (UQI). From the result, it was observed that the combination of many image restoration algorithms does not give better results whereas, the combination of two of the algorithms has really proved effective.

Keywords: document restoration; shock filter; shock diffusion; partial differential equation

1. Introduction

Image restoration plays a very vital role in further enhancing the usability of degraded images for recognition systems to perform very well (Jinjin et al., 2020). Extracting useful information from images of historical documents is a challenging problem because these images usually suffer from various degradations, such as noise, spots, bleed- through, or low-contrast ink strokes Liu et al., (2010). However, the major cause of image degradation is the atmosphere since it serves as the transmission medium. If the exposure time is too long, the refractive index along the optical transmission line can significantly degrade the performance of recognition system, leading to geometric distortion, de-focused blur, additive noise and multiplicative noise (Katake, 2006). The atmosphere is the medium of transmission for any optical system. Therefore, removing the impact of atmospheric turbulence from images is crucial for recognition systems to perform better (Zhang et al., 2014).

Image restoration is the process of recovering an image that has been degraded by some knowledge of degradation function and the additive noise term (Banham & Katsaggelos, 1997) Thus,

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in restoration, degradation is modeled, and its inverse process is applied to recover the original image.

The degradation process can be modeled as a degradation function together with additive noise, operating on an input image f (x, y) to produce a degraded image g(x, y). Mathematically, the observed degraded image is defined as:

$$\text{image } g(x,y)= f(x,y)*h(x,y)+ e(x,y), \tag{1}$$

where * denotes convolution, f(x,y) is the noiseless image, h(x, y) is the degradation function (assumed to be linear), and e(x, y) is the noisy perturbations of each pixel value.

Restoration attempts to reconstruct or recover an image that has been degraded by using prior knowledge of the degradation phenomenon. Restoration techniques are oriented towards modeling the degradation and applying the inverse process to recover the original image.

PDE-based methods for image restoration are based on propagating the information (typically, intensity values and gradients) at the boundaries of the missing region inwards. The propagation is performed by solving a partial differential equation with specified boundary conditions. The simplest case of PDE-based image restoration would be to simply use the Laplace equation for inward propagation of intensities (Schönlieb, 2015).

$$\Delta I(x)=0, \tag{2}$$

Here, x = (xy) labels of different pixels in the image, I(x) is the intensities at these pixels.

$$\Delta= {\partial^2}/{\partial x^2} + {\partial^2}/{\partial y^2} \tag{3}$$

Δ is the known intensities at the boundary of the missing region. Intuitively, it might be helpful to recall that the equation above describes the long-time behavior of the concentration of a set of independent random walkers (solution of the diffusion equation), where the concentration of random walkers at the boundaries is kept fixed. Using the solution to Laplace equation is the simplest way of restoring an image and is referred to as harmonic inpainting by scientists in the field. Most of the document that has been kept for several decades have degraded in one form or the other such as broken document, bleed through document, distorted and document affected by noise (Renardy & Rogers, 2006; Sulaiman et al., 2019).efforts have been made in restoring some of these degraded documents using the traditional methods such as image enhancement techniques, median filter, mode filter and average filter. Though, successes have been achieved with some of these traditional methods,

Over the years many partial differential equation models have been used alongside some of the traditional approaches for enhancing image degradation. Bilateral and Gaussian filters are both commonly used image denoising algorithms. Bilateral filters are generally considered to be more effective at preserving image features, while Gaussian filters are more efficient to compute. The enhanced version of the shock filter model with partial differential equations of higher order 4 is a more complex denoising algorithm that can be used to remove noise from images while preserving sharp edges and fine details. However, it is also more computationally expensive than bilateral and Gaussian filters, but the introduction of PDEs has provided tremendous, improved restoration of degraded documents.

Consequently, this research work combines some of the traditional image restoration techniques and the PDE to have a better restored image that will improve the performance of the recognition system. the traditional denoising algorithm and hybridized shock filter or shock diffusion algorithm to test the efficacy of the restoration algorithms.

2. Related Works

Image denoising is the process of removing noise from an image while preserving its important features. It is a challenging task, as different types of noise can manifest in different ways and can be difficult to distinguish from image features. There are many different types of noise that can affect images, such as Gaussian noise, Poisson noise, and speckle noise. Each type of noise has its own

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characteristics, which can make it difficult to remove without also blurring the image (Gunturk, & Li, 2012 ; Liang, et al., 2021; Lehtinen, et al., 2018;). In many cases, noise can be difficult to distinguish from image features, such as edges and textures. This can make it difficult to remove the noise without also removing important image features (Helstrom, 1967Many denoising algorithms are computationally expensive, especially for large images or images with a lot of noise. This can make it difficult to use denoising algorithms in real-time applications (Buades, et al. 2005). Despite the challenges, there are several promising directions for future work in image denoising research. Some of these directions include the following related works:

Zhou et al. (2009) proposed a nonlocal structured beta process for image denoising and inpainting. This model introduces nonlocal self-similarity as a structure prior, which enables it to effectively remove noise while preserving image features. A key feature of the proposed technique is that it does not require fore knowledge of the noise variance. Experimental result using the PSNR and SSIM evaluation metrics revealed the effectiveness of the proposed technique for suppressing noise and preserving the texture of the denoised image.

Genggeng et al. (2022) proposed an unsupervised denoising feature learning method for the classification of corrupted images. This method uses an autoencoder to denoise the images and extract robust low-dimensional representations. The extracted features are then fed to different classifiers to achieve high classification accuracy.

Jemni et al. (2021) proposed a multi-task adversarial network for handwritten document image enhancement. This network combines a handwritten text recognizer (HTR) with a discriminator to enhance handwritten images while preserving their readability. The proposed technique was evaluated on synthetitically degraded handwritten images using five metrics: PSNR, FM, Fps, DRD and Avg. Experimental results demonstrated major improvements at PSNR and FM scores of recovered degraded images while preserving the quality and readability of the degraded images.

Khadija et al. (2016) proposed a hybrid wavelet and bilateral filtering system for the restoration of old stained manuscripts. This system combines three denoising algorithms to achieve better performance than any of the individual algorithms.

Gongping et al. (2020) proposed a CNN-based blind deconvolution method for the deblurring of images degraded by atmospheric turbulence. This method used a stacked FENSB asymmetric U-net to extract features from the input image and suppress noise. Furthermore, the authors replaced the traditional convolution layer for U-Net with a deblurring noise depression block to suppress noise before deblurring in order to achieve rich feature maps. Experimental results of the proposed technique on both real and simulated data outperformed other algorithms in terms of noise suppression, details of restoration and sharpened edges.

Huang et al. (2016) compared nine classic denoising algorithms for Chinese calligraphy images. These algorithms are anisotropic diffusion filter, Wiener filter, total variation minimization, non-local means, bilateral filtering, hard visuShrink thresholding, soft visuShrink thresholding, sureShrink thresholding and bayesShrink thresholding. The performance of the algorithms on different rubbing images were evaluated based on PSNR, MSE, SNR, UQI and the SSIM metrics. The experimental results revealed that bilateral filtering achieved the best significant improvement in most of the images followed closely by bayesShrink with no obvious difference in the results of MSE and SSIM. On the other hand, bayesShrink outperformed other algorithms on most of the denoised images using SNR, UQI and SSIM. Thus, no single algorithm outperformed all others on all evaluation metrics as such, there is need to further investigate how to improve existing models.

This research work will deploy the traditional method such as bilateral and Gaussian filter with the enhanced version of shock filter model with partial differential equations of higher order 4 on the degraded document to see the efficiency of our proposed denoising algorithm techniques. The considerations for choosing the denoising algorithm for restoration of degraded documents are:

1. The type of noise present in the document- Bilateral filters are generally good at removing additive noise, such as Gaussian noise and Poisson noise. Gaussian filters are also good at removing additive noise, but they may not be as effective at removing multiplicative noise, such

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as speckle noise. The enhanced shock filter model is effective at removing both additive and multiplicative noise.

2. The desired level of denoising- If a high level of denoising is required, then a more complex algorithm, such as the enhanced shock filter model, may be necessary. However, if a lower level of denoising is acceptable, then a simpler algorithm, such as a bilateral filter or Gaussian filter, may be sufficient.
3. The computational resources available- If computational efficiency is a primary concern, then a simpler algorithm, such as a bilateral filter or Gaussian filter, may be a better choice. The enhanced shock filter model is more computationally expensive than bilateral and Gaussian filters, but it may be necessary for achieving a high level of denoising

3. Materials and Methods

The method deployed for the implementation of the hybridized image restoration techniques involved collection of degraded documents, digitization of the degraded documents, deployment of shock-filter diffusion algorithms on the degraded documents and the evaluation of the efficacy of each of these algorithms using Mean Square Error, Structural Similarity Index, Universal Quality Index and Peak Signal to Noise Ratio. The phases of the implementation are discussed in the subsections.

3.1. Data Acquisition

Dataset was acquired from available documents retrieved from Library shelves; indigenous museums that contain degraded documents; off shelve newspapers and from stores. The degraded documents were affected by different environmental conditions that makes it not suitable for the training of our model. These images were classified as images affected by moisture, water smear, oil spillage, several years of being on the shelves, mutilated and blurred images as a result of transmission from offline mode to digital mode. Samples of the degraded images collected are shown in Figure 1.

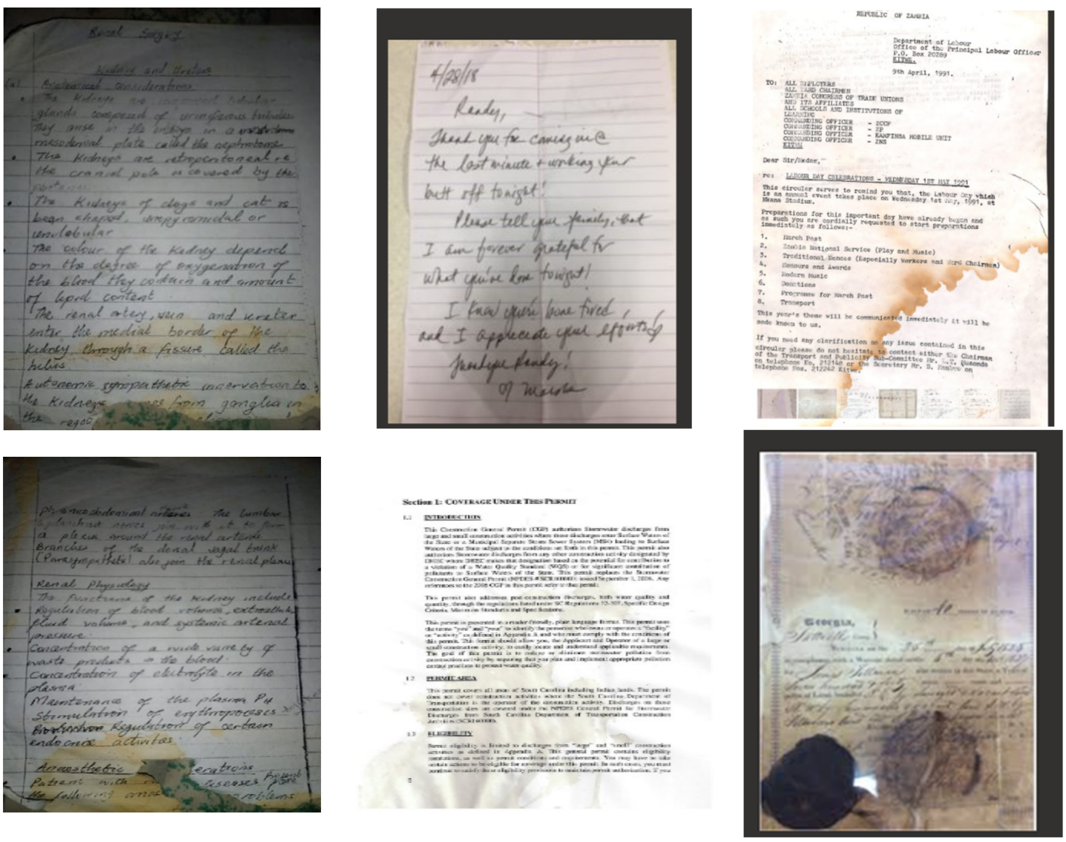

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Figure 1. Samples of degraded documents collected from Library shelves, indigenous museums, and stores.

3.2. Framework for the Hybridized Shock Diffusion and PDE Restoration System

The proposed system framework for hybridized image restoration has preprocessing, shock filtering and high-order PDE components/phases (choose either component/phases). The other two of five components/phases framework as shown in Figure 2 are shock diffusion and postprocessing.

At the Preprocessing phase, the input image is preprocessed to remove any gross noise or artifacts. This may involve steps such as denoising, contrast enhancement, and edge detection.

Shock filtering: The image is shock filtered to remove high-frequency noise and preserve edges. High-order PDE: The image is denoised using a high-order PDE. The PDE can be tailored to the specific type of noise that needs to be removed.

Shock diffusion: The image is shock diffused to further reduce noise and smooth edges.

Postprocessing: The image is postprocessed to improve its overall appearance. This may involve steps such as contrast enhancement and sharpening. The system framework of the new proposed model is shown in Figure 2.

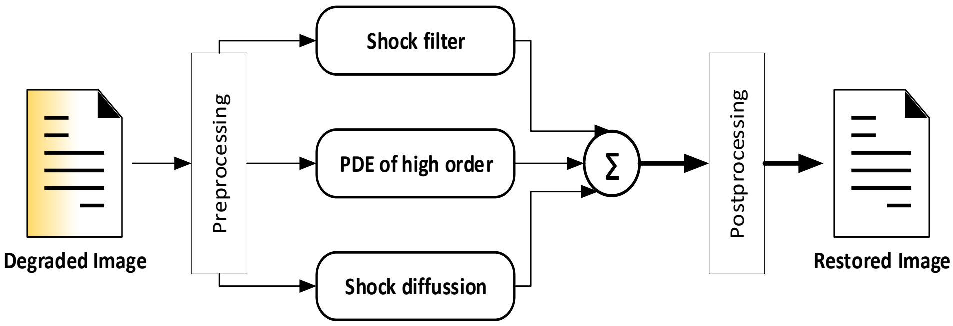

Figure 2. Description of Framework for the restoration system.

3.3. The Shock Filter with Shock Diffusion and Higher Order PDE

The mathematical model for the hybridization of Shock filter, shock diffusion and higher order PDE for restoration of degraded images can be formulated as follows:

3.3.1. Shock Filter

The shock filter is a non-linear filter used for edge enhancement and denoising. A shock filter is a technique used in image processing to reduce or eliminate the effects of shock, or sharp intensity transitions, in an image. The shock filter is often applied using partial differential equations (PDEs). It is based on the partial differential equation (PDE):

$$\frac{\partial u}{\partial t} = -sgn(\Delta u)|\nabla u| \tag{4}$$

Where u is the image, t is the time, Δu is the Laplacian of u. ∇u is the gradient of u, and sgn is the sign function.

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One common PDE used for this purpose is the Perona-Malik equation. The Perona-Malik equation is a diffusion-based PDE that aims to reduce image noise and enhance edges while preserving the overall structure of the image. The basic form of the equation is as follows:

$$\partial I/\partial t = \nabla \cdot (c(x, y, t)\ \nabla I)$$

where: $\partial I/\partial t$ is the change in intensity with respect to time. ∇ represents the gradient operator. c(x, y, t) is a function that depends on the image and time; while "I" is the image.

The shock filter operates by evolving the image over time to reduce the impact of sharp transitions (shocks).

3.3.2. Shock Diffusion

Shock diffusion is a process that combines the shock filter with a diffusion process to avoid the creation of new edges. It can be modeled by the following PDE:

$$\frac{\partial u}{\partial t} = div\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda \nabla u \tag{5}$$

Where div is the divergence operator, and λ is a parameter controlling amount of diffusion.

3.3.3. Higher Order PDE

Higher order PDEs are used to further improve the restoration of degraded images. A common choice is the fourth order PDE:

$$\frac{\partial u}{\partial t} = -\Delta^2 u \tag{6}$$

Where Δ^2 u is the Laplacian of the Laplacian of u.

3.3.4. Hybrid Model

The hybridization of these three methods can be achieved by combining the three PDEs into a single PDE. This can be done by taking a weighted sum of the right-hand sides of the three PDEs

$$\frac{\partial u}{\partial t} = \alpha(-sgn(\Delta u)|\nabla u|) + \beta\left(div\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda \Delta u\right) + \gamma(-\Delta^2 u) \tag{7}$$

where $\propto$,β and γ are weights that control the contribution of each method to the result. The weights can be chosen based on the specific requirements of the image restoration task.

3.4. Performance Evaluation Metrics

The metrics used for the evaluation of the proposed image restoration techniques as follows:

- Peak Signal-to-Noise Ratio (PSNR)

PSNR is a fundamental metric for assessing image quality, quantifying the ratio between the maximum possible power of an image and the power of corrupting noise. It is expressed in decibels (dB) and calculated as:

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{\mathrm{MAX}^2}{\mathrm{MSE}}\right) \tag{8}$$

where "MAX" is the maximum possible pixel value of the image (255 for 8-bit images). Higher PSNR values indicate better image restoration quality, signifying lower levels of noise and distortions.

- Mean Squared Error (MSE)

MSE is a widely used metric that calculates the average squared difference between the original and restored images. It is given by:

$$\text{MSE} = \frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}[g(x,y) - f(x,y)]^2 \quad (9)$$

where g(x,y) and f(x,y) represent the pixel values of the restored and original images, respectively. Lower MSE values indicate higher accuracy in the restoration process

- Structural Similarity Index (SSIM)

SSIM is designed to improve upon traditional metrics by considering perceptual image quality. It evaluates the structural similarity between two images, accounting for changes in luminance, contrast, and structure. The SSIM is computed as:

$$\text{SSIM}(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (10)$$

where μ and σ represent the mean and variance, respectively, and σ_{xy} is the covariance between the images. SSIM values range from -1 to 1, with higher values indicating greater similarity and thus better image quality.

- Universal Quality Index (UQI)

UQI measures visual quality by evaluating the loss of correlation, luminance distortion, and contrast distortion between the original and restored images. The formula for UQI is:

$$\text{UQI} = \frac{4\sigma_{xy}\mu_x\mu_y}{(\sigma_x^2 + \sigma_y^2)(\mu_x^2 + \mu_y^2)} \quad (11)$$

Similar to SSIM, UQI values range from -1 to 1, where 1 indicates perfect quality. UQI provides a comprehensive assessment by combining multiple factors affecting image quality.

4. Results and Discussion

The implemented restoration system was rigorously tested on a set of degraded images to evaluate its effectiveness. Four sample images were chosen for the experiments, each subjected to different restoration techniques: shock filtering, shock diffusion, and a fourth-order partial differential equation (PDE). Additionally, the restoration algorithms were hybridized to investigate the complementary effects of these combinations. Specifically, we combined shock filtering with shock diffusion, shock diffusion with the fourth-order PDE, and shock filtering with the fourth-order PDE. The restored output images are presented in Figures 3 and 4. The results of the restoration process are summarized in Table 1.

4.1. Parameter Settings

Consistent parameter settings were used across all experiments to ensure a fair comparison. The parameters for each restoration technique were as follows:

- Shock Filtering: Conducted with a smoothing parameter of λ=0.1 and a time step Δt=0.05. The number of iterations was set to 50;
- Shock Diffusion: Implemented with a diffusion coefficient α=0.2, a time step Δt=0.05, and 50 iterations;
- Fourth-Order PDE: Utilized with a parameter β=0.1 and a time step Δt=0.01, iterated 100 times.
- Hybrid Techniques: For the hybrid algorithms, the parameters were combined, and iterative processes were alternated to ensure the effective integration of the techniques. For instance, the combination of shock filtering and diffusion used the parameters specified for each technique, with iterations alternating every 10 steps for 100 iterations

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Table 1. Image Restoration algorithm on different degraded images.

Method	Image	SSIM	MSE	PSNR	UQI
Shock_Filter_Diffusion	Image 1	0.928	1.8286	28.8754	0.0385
Shock_Diffusion_PDE		0.5587	10.9042	19.9752	0.1071
Shock_Filter_PDE		0.5053	10.8504	18.8561	0.1219
Shock Filter		0.9285	0.8639	28.7709	0.0389
Shock Diffusion		0.9974	0.5056	51.0929	0.003
PDE Filter		0.5598	8.9316	19.7963	0.1094
Shock_Filter_Diffusion	Image 2	0.9448	1.7688	31.9328	0.0397
Shock_Diffusion_PDE		0.5836	10.1967	21.9471	0.1252
Shock_Filter_PDE		0.5711	9.8854	21.5039	0.1318
Shock Filter		0.9478	0.7873	31.8686	0.04
Shock Diffusion		0.9975	0.5154	51.0094	0.0044
PDE Filter		0.5883	8.3653	21.8901	0.126
Shock_Filter_Diffusion	Image 3	0.9952	1.1771	41.0501	0.0134
Shock_Diffusion_PDE		0.9483	2.1175	30.5185	0.045
Shock_Filter_PDE		0.9484	1.491	30.3703	0.0457
Shock Filter		0.9962	0.3261	41.6984	0.0124
Shock Diffusion		0.9994	0.6258	50.1667	0.0047
PDE Filter		0.9534	1.2271	30.9377	0.0428
Shock_Filter_Diffusion	Image 4	0.9904	1.2	39.3703	0.0168
Shock_Diffusion_PDE		0.8908	2.7894	28.4964	0.0588
Shock_Filter_PDE		0.8931	2.2755	28.443	0.0592
Shock Filter		0.9918	0.3807	39.725	0.0162
Shock Diffusion		0.9992	0.5498	50.7287	0.0046
PDE Filter		0.8994	1.8231	28.8684	0.0564

The text continues here (Figure 2 and Table 2).

Figure 3. Comparison of SSIM metric.

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The results indicate that hybridized algorithms generally outperform individual techniques across all metrics. Visually, images restored using hybridized algorithms appear to be of higher quality, effectively removing noise while preserving edges and textures. This visual improvement is corroborated by quantitative metrics.

For instance, in Image 1, the hybrid combination of shock diffusion and PDE achieved a significantly lower MSE (10.9042) and higher PSNR (19.9752 dB), SSIM (0.5587), and UQI (0.1071) compared to individual algorithms. Similar trends are observed across other images. The hybrid approach of shock filter and diffusion consistently achieved high SSIM and PSNR values, indicating effective noise removal and feature preservation. For example, in Image 3, it achieved a PSNR of 41.0501 dB and SSIM of 0.9952. The hybrid of shock diffusion and PDE showed notable improvements in noise reduction and structural preservation, achieving an MSE of 2.7894 and PSNR of 28.4964 dB for Image 4. The combination of shock filter and PDE demonstrated balanced performance, reducing noise effectively while preserving structural details, with a UQI of 0.1318 for Image 2, indicating better visual quality.

Individual techniques such as shock filter and shock diffusion performed well in preserving features but had limitations in noise removal. The individual PDE filter also showed strong performance in preserving structural details but benefited significantly when hybridized. The complementary effects of hybrid algorithms arise from leveraging the strengths and compensating for the weaknesses of individual techniques. Shock filtering excels at preserving features but struggles with noise removal, while shock diffusion effectively reduces noise but may compromise structural details. The fourth-order PDE balances noise removal and feature preservation but is computationally intensive. By hybridizing these techniques, the strengths of each are utilized, leading to superior restoration performance.

The restored images and their respective evaluation metrics—MSE, PSNR, SSIM, and UQI—are shown in Figures 4 and 5. Visually, hybridized algorithms produce higher quality images compared to individual algorithms, as they remove more noise and preserve more image features such as edges and textures. The quantitative metrics support this, showing that hybrid algorithms achieve lower MSE and higher PSNR, SSIM, and UQI values, indicating more accurate restoration of the degraded images.

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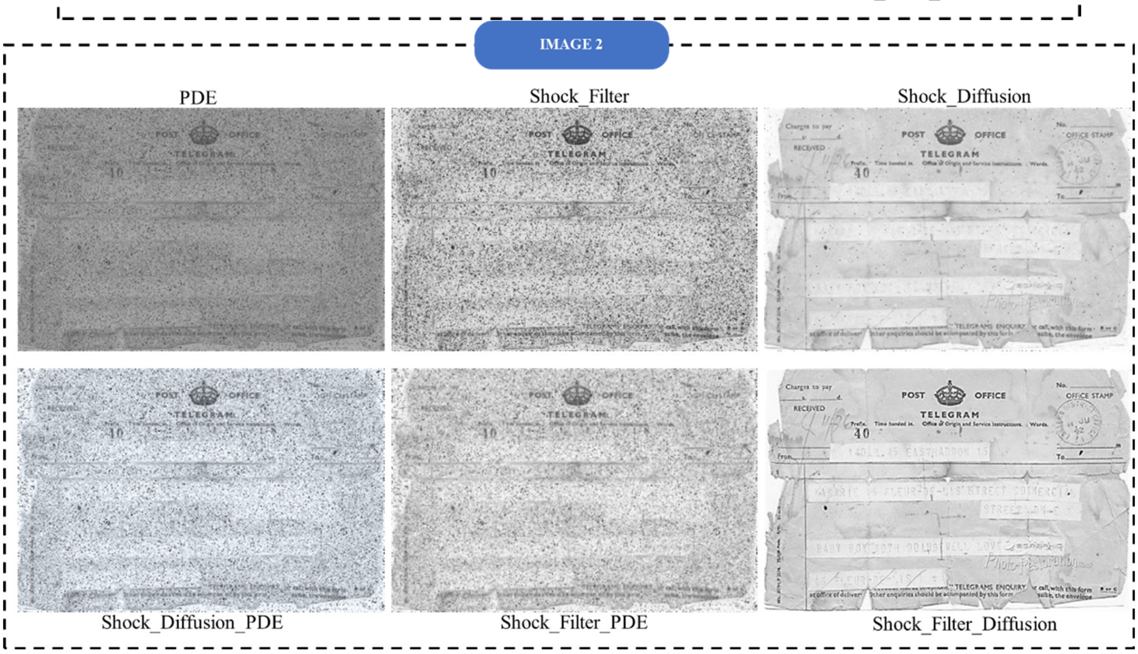

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Figure 4. Restored image from the different filtering method.

4. Discussion

The results of the restoration of a degraded image using shock filtering, shock diffusion, partial differential equation of order 4, and hybridized algorithms are shown in Figures 4 and 5.

Visually, the restored images using the hybridized algorithms appear to be of higher quality than the restored images using the individual algorithms. The hybridized algorithms are able to remove more noise and preserve more image features, such as edges and textures.

The quantitative evaluation metrics also show that the hybridized algorithms outperform the individual algorithms. The hybridized algorithms have lower MSE and higher PSNR, SSIM, and UQI values. This indicates that the hybridized algorithms is effective in restoring degraded image more accurately.

The complementary effect of the hybridized algorithms is due to the different strengths and weaknesses of the individual algorithms. Shock filtering is effective at preserving image features, but it can be less effective at removing noise. Shock diffusion is more effective at removing noise, but it can be less effective at preserving image features. The partial differential equation of order 4 removed noise while preserving image features, but it can be computationally expensive.

The hybridized algorithms combine the strengths of the individual algorithms to achieve better performance. For example, the hybrid algorithm that combines shock filtering with shock diffusion can remove more noise while preserving more image features than either shock filtering or shock diffusion alone.

5. Conclusions

The results of the restoration of a degraded image using shock filtering, shock diffusion, partial differential equation of order 4, and hybridized algorithms show that the hybridized algorithms outperform the individual algorithms in terms of both visual quality and quantitative evaluation metrics. This is due to the complementary effect of the hybridized algorithms.

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References

1. Zhou, L., Huang, R., & Zhang, D. (2009). Image restoration via Bayesian dictionary learning with nonlocal structured beta process. IEEE Transactions on Image Processing, 18(10), 2383-2394.
2. Genggeng, Z., & Zhang, L. (2022). Unsupervised denoising feature learning for classification of corrupted images. IEEE Transactions on Image Processing, 31, 6391-6401.
3. Jemni, M., Boujlel, M., Alimi, A., & Glatard, T. (2021). Multi-task adversarial network for handwritten document image enhancement. IEEE Transactions on Image Processing, 30(10), 6026-6039.
4. Khadija, A., Yousra, B., & Mohamed, D. (2016). Restoration of old stained manuscripts via a hybrid wavelet and bilateral filtering system. Journal of Real-Time Image Processing, 15(5), 901-915.
5. Gongping, L., Yichao, L., & Zili, L. (2020). Blind de-convolution of images degraded by atmospheric turbulence. IEEE Access, 8, 229912-229922.
6. uang, X., Wang, Z., Zhang, Y., & Lu, Y. (2016). Comparison of different image denoising algorithms for Chinese calligraphy images. IEEE Access, 4, 8793-8802.
7. Jinjin, G., Haoming, C., Haoyu, C., Xiaoxing, Y., Ren, J. S., & Chao, D. (2020). Pipal: a large-scale image quality assessment dataset for perceptual image restoration. In Computer Vision–ECCV 2020: 16th European Conference, Glasgow, UK, August 23–28, 2020, Proceedings, Part XI 16 (pp. 633-651). Springer International Publishing.

 doi:10.20944/preprints202407.1313.v1

8. Liu, R., Lin, Z., Zhang, W., & Su, Z. (2010). Learning PDEs for image restoration via optimal control. In Computer Vision–ECCV 2010: 11th European Conference on Computer Vision, Heraklion, Crete, Greece, September 5-11, 2010, Proceedings, Part I 11 (pp. 115-128). Springer Berlin Heidelberg.
9. Katake, A. B. (2006). Modeling, image processing and attitude estimation of high-speed star sensors (Vol. 69, No. 01).
10. Banham, M. R., & Katsaggelos, A. K. (1997). Digital image restoration. IEEE signal processing magazine, 14(2), 24-41.
11. hang, J., Zhao, D., Xiong, R., Ma, S., & Gao, W. (2014). Image restoration using joint statistical modeling in a space-transform domain. IEEE Transactions on Circuits and Systems for Video Technology, 24(6), 915-928.
12. Schönlieb, C. B. (2015). Partial differential equation methods for image inpainting (Vol. 29). Cambridge University Press.
13. Renardy, M., & Rogers, R. C. (2006). An introduction to partial differential equations (Vol. 13). Springer Science & Business Media.
14. Sulaiman, A., Omar, K., & Nasrudin, M. F. (2019). Degraded historical document binarization: A review on issues, challenges, techniques, and future directions. Journal of imaging, 5(4), 48.
15. Ito, K., & Xiong, K. (2000). Gaussian filters for nonlinear filtering problems. IEEE transactions on automatic control, 45(5), 910-927.
16. Cabello, F., León, J., Iano, Y., & Arthur, R. (2015, September). Implementation of a fixed-point 2D Gaussian Filter for Image Processing based on FPGA. In 2015 Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA) (pp. 28-33). IEEE.
17. Zhang, B., & Allebach, J. P. (2008). Adaptive bilateral filter for sharpness enhancement and noise removal. IEEE transactions on Image Processing, 17(5), 664-678.
18. Qiao, T., Yang, Z., Ren, J., Yuen, P., Zhao, H., Sun, G., ... & Benediktsson, J. A. (2018). Joint bilateral filtering and spectral similarity-based sparse representation: A generic framework for effective feature extraction and data classification in hyperspectral imaging. Pattern Recognition, 77, 316-328.
19. Singh, M., Mandal, M. K., & Basu, A. (2005). Gaussian and Laplacian of Gaussian weighting functions for robust feature based tracking. Pattern Recognition Letters, 26(13), 1995-2005.
20. Singh, P., & Shree, R. (2016). A comparative study to noise models and image restoration techniques. International Journal of Computer Applications, 149(1), 18-27.
21. Weickert, J. (2003, September). Coherence-enhancing shock filters. In Joint Pattern Recognition Symposium (pp. 1-8). Berlin, Heidelberg: Springer Berlin Heidelberg.
22. Rudin, L. I. (1987). Images, numerical analysis of singularities and shock filters (Doctoral dissertation, California Institute of Technology).
23. Lyra, M., & Ploussi, A. (2011). Filtering in SPECT image reconstruction. Journal of Biomedical Imaging, 2011, 1-14.
24. Liang, J., Cao, J., Sun, G., Zhang, K., Van Gool, L., & Timofte, R. (2021). Swinir: Image restoration using swin transformer. In Proceedings of the IEEE/CVF international conference on computer vision (pp. 1833-1844).
25. Lehtinen, J., Munkberg, J., Hasselgren, J., Laine, S., Karras, T., Aittala, M., & Aila, T. (2018). Noise2Noise: Learning image restoration without clean data. arXiv preprint arXiv:1803.04189.
26. Gunturk, B. K., & Li, X. (Eds.). (2012). Image restoration: fundamentals and advances. CRC Press.
27. Helstrom, C. W. (1967). Image restoration by the method of least squares. Josa, 57(3), 297-303.
28. Buades, A., Coll, B., & Morel, J. M. (2005). A review of image denoising algorithms, with a new one. Multiscale modeling & simulation, 4(2), 490-530.