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Article

Sustainability of Automated Manufacturing Systems with Resources by Means of Their Deadlock Prevention

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Abstract: This paper is devoted to Petri net (PN) based models of Automated Manufacturing Systems (AMS) with resources in order to prevent deadlocks in them. The sustainability can be seen as the process of deadlock-freeness leading to correct and fluent production, because AMS with deadlocks work neither correctly nor fluently, need reconstruction and cause downtime in production. A paradigm S^3PR (Systems of Simple Sequential Processes with Resources) of such PN models is well known as to the deadlock prevention point of view. Here, Extended S^3PR (ES 3PR) will be explored as to modelling and deadlock prevention. While in case of S^3PR Ordinary Petri Nets (OPN) were used for these aims, here, for ES 3PR Generalized Petri Nets (GPN) are used. The reason for such a procedure is a possible presence of multiplex directed arcs in the structure of PN models of AMS. The significant alternation is that while in former case the elementary siphons and dependent ones were sufficient for the supervisor synthesis, here, in later case, the GPN and their siphons have to satisfy the max-cs property.

Keywords: automated manufacturing systems; deadlock prevention; Petri nets; resource allocation systems; siphons; supervisor; sustainability; traps

1. Introduction

It is very important to deal with deadlocks in industrial MAS (Multi Agent Systems). AMS (Automated Manufacturing Systems) where two or more production lines share robots, machine tools, conveyors, automatically controlled vehicles, etc., are a typical example of such systems. These shared entities represent resources. Such production systems are very important in social practice. They should fluently produce expected products without any obstacles. However, deadlocks occurring in them prevent this. In general, deadlock is a status when two (or more) processes are waiting to continue their activity, but are preventing each other from doing so. Namely, they are competing for the same resources. A deadlock is an undesirable state in the global system, because the system itself, or a part of it, stagnates. Sustainability of the manufacturing process continuity is strongly disturbed. Therefore, the original intention of such a system cannot be achieved. In order to sustain processes in AMS fluently work it is necessary to prevent deadlocks.

S^3PR , i.e., Systems of Simple Sequential Processes with Resources, modeling the Resource Allocation Systems (RAS) on the base of PN were defined, analyzed and controlled in [1]. S^3PR were modeled there by Ordinary PN (OPN) and controlled by siphons. Here, more complicated Extended S^3PR (ES 3PR) models of RAS are analyzed by Generalized PN (GPN) and controlled by their siphons.

1.1. Basic PN Terminology

A Petri net is a quadruplet $N = (P, T, F, W)$ with P and T being finite nonempty sets, a set of places $P = \{p_1, p_2, \dots, p_n\}$ ($|P| = n$) and a set of transitions $T = \{t_1, t_2, \dots, t_m\}$ ($|T| = m$), where $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$, and \emptyset means an empty set. The set $F = (P \times T) \cup (T \times P)$ is a flow relation of N . Such flow

consists of directed arcs from places to transitions and vice versa. The mapping $W: (P \times T) \cup (T \times P) \rightarrow \mathcal{N}$ assigns a weight to an arc: $W(f) > 0$ if $f \in F$ and $W(f) = 0$ otherwise. Here $\mathcal{N} = \{0, 1, 2, \dots\}$ is the set of natural numbers plus zero. The net N is ordinary net if $\forall f \in F, W(f) = 1$. Such net is denoted as $N = (P, T, F)$. If $\exists f \in F, W(f) > 1, N = (P, T, F, W)$ is named as a generalized net GPN.

The set M expressing states of marking of particular PN places, simply said the PN marking M , is an $(n \times 1)$ vector named also as the state vector. Its evolution is performed by means of the matrix/vector equation $M_{k+1} = M_k + [N] \cdot \sigma_k, k \in \mathcal{N}$, where M_0 is an initial marking. This equation represents the mathematical model of the net N . The matrix $[N] = [Post]^T - [Pre]$ is the $(n \times m)$ incidence matrix. It represents the matrix notation of the set F . It can also be expressed as $[N](p, t) = W(t, p) - W(p, t)$. After all, σ_k is a $(m \times 1)$ vector expressing firing of transitions named also as the control vector. Namely, it expresses the state of transitions (a transition may be firable—it is indicated by 1, or not—it is indicated by 0).

For a place $p, M(p)$ is an integer indicating the number of tokens placed inside of this place. We say that the place p is marked by M iff $M(p) > 0$. The abbreviation iff means *if and only if* in the whole paper.

A subset $D \subseteq P$ is marked by M iff at least one place $p \in D$ is marked by M . Thus, $M(D) = \sum_{p \in D} M(p)$ denotes the sum of tokens in all places in D .

$\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$ is the preset of a node $x \in P \cup T$, while $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$ is the postset of a node $x \in P \cup T$.

A transition t such that $|\bullet t| > 1$ is named as a fork. A transition t such that $|t^\bullet| > 1$ is named as a join. A place p such that $|p^\bullet| > 1$ is named as a choice. A place p such that $|\bullet p| > 1$ is named as an attribution.

$SP(x_1, x_n)$ means a simple path from x_1 to x_n . It is a path whose all nodes are different.

We say that a transition $t \in T$ is enabled at a marking M iff $\forall p \in \bullet t, M(p) \geq W(p, t)$. After firing t a new marking M' is evolved such that $\forall p \in P, M'(p) = M(p) - W(p, t) + W(t, p)$. However, enabled transition t may be fired, but it may not be fired.

A non-empty subset of places $S \subseteq P$ is a *siphon* iff $\bullet S \subseteq S^\bullet$. A non-empty subset of places $S \subseteq P$ is a *trap* iff $S^\bullet \subseteq \bullet S$. Siphon S is *minimal* iff it contains no other siphons (as its proper subset). If a minimal siphon S does not contain a marked trap, it is named *strict*.

A column vector $I: P \rightarrow Z$ indexed by P , where Z is the set of integers, is named the P -vector. A column vector $J: T \rightarrow Z$ indexed by T is named the T -vector. For the economy of space, a P -vector I is denoted by $\sum_{p \in P} I(p)p$ and T -vector J is denoted by $\sum_{t \in T} J(t)t$.

Let I is a P -invariant of (N, M_0) . Then $\forall M \in R(N, M_0), I^\bullet M = I^\bullet M_0$. Here $R(N, M_0)$ is the state space of N . In other words, it is a set of all reachable markings of N .

As we can see, the invariant I is a P -vector. $||I|| = \{p \in P \mid I(p) \neq 0\}$ means a support of P . $||I||^+ = \{p \in P \mid I(p) > 0\}$ is the positive support of P and $||I||^- = \{p \in P \mid I(p) < 0\}$ is the negative support of P .

P -invariant of (N, M_0) is a natural solution of the equation $I^\bullet [N] = 0$. If all elements of P -invariant I are nonnegative this invariant is named as P -semiflow.

T -invariant of (N, M_0) is a natural solution of the equation $[N] \cdot J = 0$. It gives information about a possible loop in the net, i.e., about a sequence of transitions which leads back to the marking it starts in. If all elements of T -invariant J are nonnegative this invariant is named as T -semiflow.

When a place p is given, then $\max_{t \in p^\bullet} \{W(p, t)\}$ is denoted by \max_{p^\bullet} .

More details about PN are introduced in [1–4] and in basic papers about PN [17–20].

1.2. Structure of the Article

In the next section preliminaries concerning definitions of particular PN models of AMS are introduced.

The third section is devoted to control of ES³PR paradigm of AMS model. It represents one of two substantive parts of this paper. Since ES³PR model belongs to a subclass of S⁴PR paradigm of models, accordingly such paradigm is defined there. The basic influence of siphons to the control of PN models of AMS is emphasized there too. Finally, definitions concerning the siphon-based control

as well as the procedure making possible to find parameters of the supervisor, ensuring the deadlock-freeness of the ES³PR model, are introduced. To illustrate ES³PR model, simple example is presented in Figure 1 and its siphons are computed. Unfortunately, these siphons, satisfying for S³PR models cannot be used for the supervisor synthesis of ES³PR model, Namely, they do not ensure the deadlock-freeness.

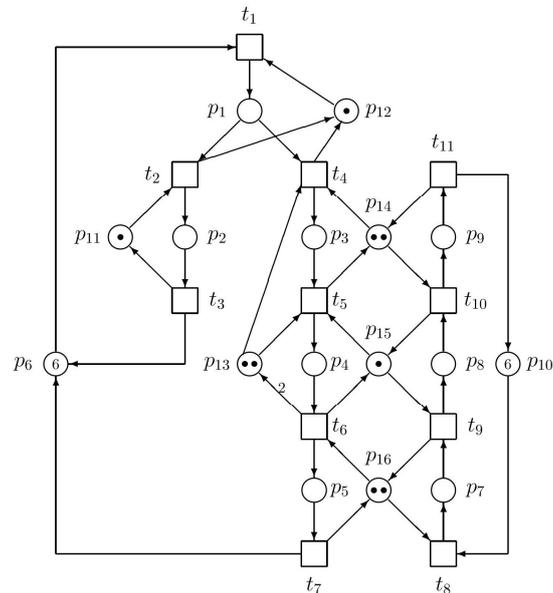


Figure 1. The GPN model of AMS, where $W(t_6, p_{13}) = 2$.

In the fourth section, representing the second of two substantive parts of the paper, controllability of siphons in GPN is analyzed and elementary and dependent siphons are checked as to max-cs property. Then the supervisor computed in the sense of such a procedure is set. The Figure 2 illustrates the interconnections between the original uncontrolled model and the supervisor ensuring the deadlock-freeness.

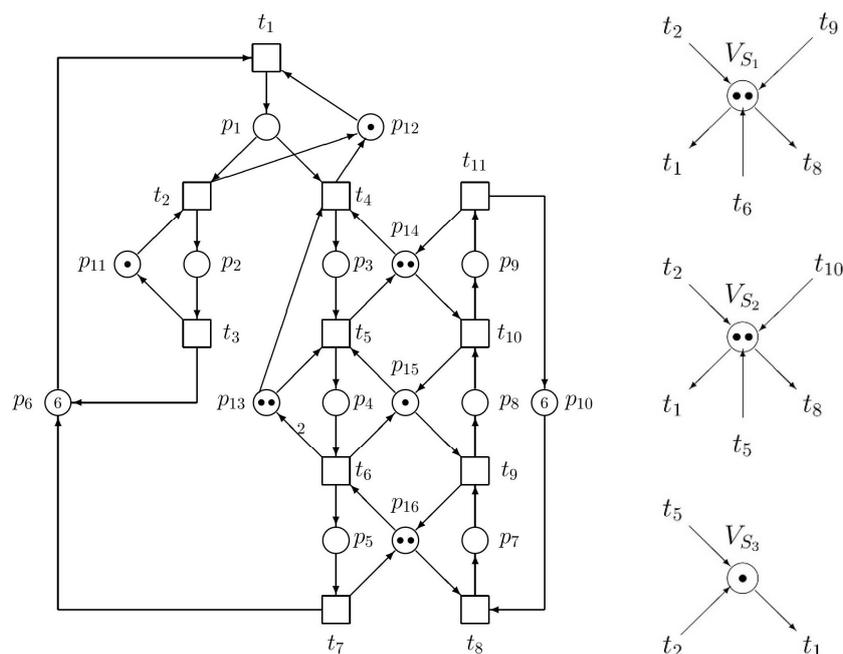


Figure 2. The ES³PR paradigm of the original GPN model of AMS (left) and the supervisor (right) consisting of three monitors.

Fifth section represents conclusions, where the topic and contribution of the paper is evaluated and the further work in future is pointed out to.

2. Preliminaries

As to nomenclature of sets in PN used below, P_S is a set of operation places, P_R is a set of resource places, P_0 is a set of idle places.

Definition 1 [6]: An ordinary Petri net $N = (P_S \cup \{p_0\}, T, F)$ is a Simple Sequential Process (S²P) when the following hold true:

1. $P_S \neq \emptyset$, $p_0 \notin P_S$.
2. N is a strongly connected state machine (i.e., OPN where each transition has only one input place and only one output place).
3. Every circuit in N contains place p_0 .

Definition 2 [6]: An extended simple sequential process with resources—ES²PR is a generalized pure Petri net $N = (P_S \cup \{p_0\} \cup P_R, T, F, W)$, such that:

1. The subnet generated by the set $X = P_S \cup \{p_0\} \times T$ is an S²P.
2. $P_R \neq \emptyset$ and $(P_S \cup \{p_0\}) \cap P_R = \emptyset$.
3. $\forall t \in T, \forall p \in \bullet t, W(p, t) = 1$.
4. $\forall r \in P_R, \exists$ a minimal P-semiflow I_r such that $\{r\} = ||I_r|| \cap P_R$, $p_0 \notin ||I_r||$, $||I_r|| \cap P_S \neq \emptyset$ and $I_r(r) = 1$.

Definition 3 [6]: Let $N = (P_S \cup \{p_0\} \cup P_R, T, F, W)$ be an ES²PR. An initial marking M_0 is acceptable for N iff:

1. $M_0(p_0) > 0$.
2. $\forall p \in P_S, M_0(p) = 0$.
3. $\forall r \in P_R, M_0(r) > 0$
4. Elements in the support of each minimal T -semiflow can be fired sequentially.

Definition 4 [6]: An ES³PR is the composition of a finite set of ES²PR via the fusion of resource places.

Definition 5 [6]: Let $N_i = (P_i, T_i, F_i, W_i)$ ($i = 1, 2$) be two generalized Petri nets. Then:

1. N_1 and N_2 are composable (via fusion of places) iff $T_1 \cap T_2 = \emptyset$, and $P_C = P_1 \cap P_2 \neq \emptyset$.
2. Net $N = (P, T, F, W)$ is called the composed net of N_1 and N_2 , denoted by $N_1 \circ N_2$, where $P = P_1 \cup P_2$, $T = T_1 \cup T_2$, $W(p, t) =$ if $t \in T_1$ then $W_1(p, t)$ else $W_2(p, t)$.

Definition 6 [6]: Let I_m be a set of indices, $N_i = (P_i, T_i, F_i, W_i)$ ($i \in I_m$) be a set of generalized Petri nets. We denote by $N = \bigcirc_{i \in I_m} N_i$ the net obtained by the following operation: if $\text{card}(I_m) = 1$ then $N = N_1$ else $N = N_k \circ \bigcirc_{i \in I_m \setminus \{k\}} N_i$.

Definition 7 [6]: An ES³PR is the composition of a finite set of ES²PR via the fusion of resource places.

Definition 8 [6]: (N, M_0) , $\bigcirc_{i \in I_m} N_i$, $N_i = (P_i, T_i, F_i, W_i)$, and (N_i, M_{0i}) being initially acceptably marked ES²PR, is an acceptably marked ES³PR iff:

1. $\forall i \in I_m, \forall p \in P_{Si} \cup \{p_{0i}\}, M_0(p) = M_{0i}(p)$.
2. $\forall i \in I_m, \forall r \in P_R, M_0(r) =$ if $\text{card}(I_m) = 1$ then $M_{0i}(r)$ else $\max_{i \in I_m} M_{0i}(r)$.

In [10] the following property was proved:

Let $N = (P_S \cup P_0 \cup P, T, F, W)$ be an ES³PR, where $P_0 = \bigcup_{i \in I_m} \{p_{0i}\}$. Then, $\text{rank}([N]) = |P_S|$.

In [11] the following Theorem was proved:

Theorem 1 [11]: Let S be a strict minimal siphon in an ES³PR $N = (P_S \cup P_0 \cup P, T, F, W)$. Then $S = S_R \cup S_S$ satisfies $S \cap P_0 = \emptyset$, $S \cap P_R = S_R \neq \emptyset$, and $S \cap P_S = S_S \neq \emptyset$.

3. Control of ES³PR

The paradigm ES³PR models a class of concurrently cyclic sequential processes sharing common resources. An approach to the deadlock prevention by controlling siphons in ES³PR will be presented here. Namely,

1. the control of elementary siphons
2. finding the relevant approach to finding conditions for control of dependent siphons;
3. finding the way of making the ES³PR deadlock-free.

With respect to [6] ES³PR is a subclass of S⁴R (Systems of Sequential Systems with Shared Resources) defined by the next definition.

Definition 8 [6]: An S⁴R is a generalized pure net $N = \bigcirc_{i \in I_m} N_i = (P, T, F, W)$, where

1. $N_i = (P_{Si} \cup \{p_{0i}\} \cup P_{Ri}, T_i, F_i, W_i)$, $i \in I_m$
2. $P = P_S \cup P_0 \cup P_R$ is a partition such that
 - $P_S = \bigcup_{i \in I_m} P_{Si}$, $P_{Si} \neq \emptyset$ and $P_{Si} \cap P_{Sj} = \emptyset$, $\forall i \neq j$ ($i, j \in I_m$)
 - $P_R = \bigcup_{i \in I_m} P_{Ri} = \{r_1, r_2, \dots, r_n\}$, $n > 0$
 - $P_0 = \bigcup_{i \in I_m} \{p_{0i}\}$
 - The elements in P_0 , P_S , and P_R are called idle, operation, and resource places, respectively
 - The output transitions of an idle place are called source transitions.
3. $T = \bigcup_{i \in I_m} T_i$, $T_i \neq \emptyset$, $T_i \cap T_j = \emptyset$ for all $i \neq j$.
4. $\forall i \in I_m$ the subset N_i generated by $P_{Si} \cup \{p_{0i}\} \cup T_i$ is a strongly connected state machine such that every cycle contains p_{0i}
5. $\forall r \in P_R$, there exists a unique minimal P -semiflow $I_r \in IN^{|P|}$ (here $IN = \{0, 1, 2, \dots\}$) such that $\{r\} = \|\|I_r\| \cap P_R, P_0 \cap \|\|I_r\| = \emptyset, P_S \cap \|\|I_r\| \neq \emptyset$ and $I_r(r) = 1$
6. $P_S = \bigcup_{r \in P_R} (\|\|I_r\| \setminus \{r\})$
7. N is a strongly connected net.

In [11] the following Theorem was proved:

Theorem 2 [11]: Let (N, M_0) be a marked S⁴R net. N is live under M_0 iff it satisfies *max-cs property*.

Consider that r is a resource place, S is a strict minimal siphon and $H(r) = I_r \setminus \{r\}$ in a S⁴R net. We can define $Th(S) = \sum_{r \in SR} H(r) \setminus S$. It can be seen that $\|\|Th(S)\| \subseteq P_S$ is true. Use $\sum_{p \in \|\|Th(S)\|} hs(p)p$ to denote $Th(S)$. $hs(p)$ indicates that if the number of tokens in p increases by one, the siphon S loses $hs(p)$ tokens

In [9] the following Lemma was proved:

Lemma 1 [9]: Let (N, M_0) be a marked net and S be a siphon of N . S is *max-controlled* if there exists a P -invariant I such that $\forall p \in (\|\|I\| \cap S)$, $max_{p \bullet} = 1$, $\|\|I\|^+ \subseteq S$, and $\sum_{p \in P} I(p)M_0(p) > \sum_{p \in S} I(p) (max_{p \bullet} - 1)$.

This Lemma will be applied below, within the context of Figure 2.

3.1. Siphons in Petri Nets Control

Siphons and traps were formally defined above at the end of the Section Introduction. Elementary and dependent siphons in OPN were proposed in [12]. Later, the terminology was more specified in [16]. In this section siphons and traps are defined in GPN.

With respect to [7] we have the following knowledge:

Definition 9 [7]: Let $S \subseteq P$ be a subset of places of $N = (P, T, F, W)$. P -vector λ_S is called the characteristic P -vector of S iff $\forall p \in S$, $\lambda_S(p) = 1$; otherwise $\lambda_S(p) = 0$.

Definition 10 [7]: Let $S \subseteq P$ be a subset of places of $N = (P, T, F, W)$. η_S is called the characteristic T -vector of S iff $\eta_S = \lambda_S \cdot [N]$.

Definition 11 [7]: Let $N = (P, T, F, W)$ be a net with $|P| = n$, which has k siphons S_1, \dots, S_k , m , $k \in \mathbb{N}^+$. We define $[\lambda]^{k \times n} = [\lambda_{S1} | \lambda_{S2} | \dots | \lambda_{Sk}]^T$ and $[\eta]^{k \times m} = [\lambda]^{k \times n} \cdot [N]_{n \times m} = [\eta_{S1} | \eta_{S2} | \dots | \eta_{Sk}]^T$ is called the characteristic matrix of the siphons in N .

Definition 12 [7]: Let $\eta_{S\alpha}, \eta_{S\beta}, \dots$, and $\eta_{S\gamma}$ ($\{\alpha, \beta, \dots, \gamma\} \subseteq \{1, 2, \dots, k\}$) be a linearly independent maximal set of the matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \dots, S_\gamma\}$ is called a set of elementary siphons in N .

Definition 13 [7]: $S \notin \Pi_E$ is called a strongly dependent siphon if $\eta_S = \sum_{S \in \Pi_E} a_i \cdot \eta_{Si}$, where $a_i \geq 0$.

Definition 14 [7]: $S \notin \Pi_E$ is called a weakly dependent siphon if \exists non-empty $A, B \subset \Pi_E$, such that $A \cap B = \emptyset$ and $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_i \in B} a_i \eta_{S_i}$, where $a_i > 0$.

Lemma 2 [7]: The number of elements in any set of elementary siphons in net N equals to $rank([\eta])$.

Verbally said, this important Lemma shows that the $rank([\eta])$ directly determines the number of elementary siphons, However, it simultaneously determines also the number of dependent siphons because $\Pi = \Pi_E \cup \Pi_D$.

Theorem 3 [7]: Let N_{ES} be the number of elementary siphons in net $N = (P, T, F, W)$. Then $N_{ES} \leq \min\{|P|, |T|\}$.

Corollary 1 [7]: Let $N = \bigcirc_{i \in \{1, \dots, m\}} N_i$, $N_i = (P_i, T_i, F_i, W_i)$ be an ES^3PR . Then, $N_{ES} \leq |P_S|$.

Verbally said, this important Corollary indicates that the maximal number of elementary siphons in an ES^3PR net is not greater than the number of its operation places $p \in P_S$.

3.2. Procedure of Setting the Supervisor for ES^3PR

In [6,7] it can be found the following definition:

Definition 15 [6,7]: Let S be a strict minimal siphon in an ES^3PR net model of a plant $(N_{\mu 0}, M_{\mu 0})$ where $N_{\mu 0} = \bigcirc_{i \in \{1, \dots, n\}} N_i = (P_0 \cup P_S \cup P_R, T, F_{\mu 0}, W_{\mu 0})$. Let $\{\alpha, \beta, \dots, \gamma\} \subseteq \{1, 2, \dots, n\}$ such that $i \in \{\alpha, \beta, \dots, \gamma\}$, $|Th(S) \cap p \in P_{S_i} \neq \emptyset$ and $\forall j \in \{1, 2, \dots, n\} \setminus \{\alpha, \beta, \dots, \gamma\}$, $|Th(S) \cap p \in P_{S_j} \neq \emptyset$. For S , a nonnegative P -vector k_S is constructed as follows:

Step 1: $\forall p \in P_0 \cup P_S \cup P_R$, $k_S(p) = 0$;

Step 2: $\forall p \in |Th(S)|$, $k_S(p) = hs(p)$, where $Th(S) = \sum_{p \in |Th(S)|} hs(p)p$;

Step 3: $\forall i \in \{\alpha, \beta, \dots, \gamma\}$, let $p_s \in |Th(S) \cap P_{S_i}$ be such a place that $\forall p_t \in SP(p_u, p_{oi})$, $p_u \in p_s^{**}$, $p_t \notin |Th(S)|$. Suppose that there are m such places $p_{1s}, p_{2s}, \dots, p_{ms}$. Assuredly, we have $\{p_{is} \mid i = 1, 2, \dots, m\} \subseteq |Th(S) \cap P_{S_i}$. $\forall p_{is}$ let $p_{iv} \in SP(p_{oi}, p_{is})$ be such a place that $hs(p_{iv}) > hs(p_w)$, $\forall p_w \in SP(p_{oi}, p_{is})$. $\forall p_x \in SP(p_{oi}, p_{is})$, $k_S(p_x) = hs(p_{iv})$.

Step 4: $\forall I_m \subseteq \{1, 2, \dots, m\}$. $\forall p_y \in \bigcap_{i \in I_m} SP(p_{oi}, p_{is})$, $k_S(p_y) = hs(p_{iz})$, where $p_{iz} \in |Th(S) \cap P_{S_i}$, and $\nexists p \in Th(S) \cap P_{S_i}$, s.t. $hs(p) > hs(p_{iz})$.

In general, such procedure makes possible to finalize the supervisor synthesis, especially to set markings of the supervisor monitors V_{S_i} , $i = 1, 2, \dots$

3.3. Illustrative Example of ES^3PR – Siphons and P -Invariants

Let us apply the theory of siphon-based control, introduced above, on the example of ES^3PR . Consider the GPN in Figure 1 modelling an AMS. Here, the set of operation places is $P_S = \{p_1, p_2, p_3, p_4, p_5, p_7, p_8, p_9\}$, the set of resource places is $P_R = \{p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$, and the set of idle places is $P_0 = \{p_6, p_{10}\}$.

As we can see in Figure 1, there exists one arc with the weight equal to 2—notice the arc from t_6 to p_{13} . Consequently, in Figure 1 is GPN. The state space of the net N expressed by the reachability tree (RT) has 308 nodes (including the initial state). Because of such large amount of nodes, the RT cannot be displayed here. The minimal siphons are introduced in Table 1.

Table 1. Siphons and traps of the PN model.

No.	Siphons	Traps	Notice
1.	$S_1 = \{p_4, p_{12}\}$	$Tr_1 = \{p_1, p_{12}\}$	Eliminate S_1 , because $S_1 = Tr_1$
2.	$S_2 = \{p_2, p_{11}\}$	$Tr_2 = \{p_2, p_{11}\}$	Eliminate S_2 because $S_2 = Tr_2$
3.	$S_3 = \{p_4, p_{13}\}$	$Tr_3 = \{p_3, p_4, p_{13}\}$	
4.	$S_4 = \{p_5, p_7, p_{16}\}$	$Tr_4 = \{p_5, p_7, p_{16}\}$	Eliminate S_4 because $S_4 = Tr_4$
5.	$S_5 = \{p_4, p_8, p_{15}\}$	$Tr_5 = \{p_4, p_8, p_{15}\}$	Eliminate S_5 because $S_5 = Tr_5$

6.	$S_6 = \{p_3, p_9, p_{14}\}$	$Tr_6 = \{p_3, p_9, p_{14}\}$	Eliminate S_6 because $S_6 = Tr_6$
7.	$S_7 = \{p_5, p_8, p_{15}, p_{16}\}$	$Tr_7 = \{p_4, p_7, p_{15}, p_{16}\}$	
8.	$S_8 = \{p_4, p_9, p_{14}, p_{15}\}$	$Tr_8 = \{p_3, p_8, p_{14}, p_{15}\}$	
9.	$S_9 = \{p_7, p_8, p_9, p_{10}\}$	$Tr_9 = \{p_7, p_8, p_9, p_{10}\}$	Eliminate S_9 because $S_9 = Tr_9$
10.	$S_{10} = \{p_5, p_9, p_{14}, p_{15}, p_{16}\}$	$Tr_{10} = \{p_3, p_7, p_{14}, p_{15}, p_{16}\}$	
11.	$S_{11} = \{p_1, p_2, p_3, p_4, p_5, p_6\}$	$Tr_{11} = \{p_1, p_2, p_3, p_4, p_5, p_6\}$	Eliminate S_{11} because $S_{11} = Tr_{11}$

As we can see in Table 1, there exist 11 minimal siphons in this GPN model of AMS. They can be enumerated e.g., by the tool GPenSIM [13–15] or by another PN tools. Because the traps corresponding to the crossed siphons are marked, crossed siphons may be omitted from the list of siphons. Hence, only 4 of them are strict minimal siphons— S_3 , S_7 , S_8 and S_{10} . Other 7 siphons are not relevant because they are equal to corresponding traps.

Let us renumber siphons S_3 , S_7 , S_8 and S_{10} to siphons S_3 , S_1 , S_2 and S_4 , respectively. Consequently, $S_1 = \{p_5, p_8, p_{15}, p_{16}\}$, $S_2 = \{p_4, p_9, p_{14}, p_{15}\}$, $S_3 = \{p_4, p_{13}\}$ and $S_4 = \{p_5, p_9, p_{14}, p_{15}, p_{16}\}$. In the form of row P -vectors they are as follows:

$$S_1 = (0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1)$$

$$S_2 = (0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0)$$

$$S_3 = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)$$

$$S_4 = (0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1)$$

Corresponding matrix $[\lambda] = [S_1^T, S_2^T, S_3^T, S_4^T]^T$ can be utilized (analogically to [1] in the case of S^3PR paradigm) for finding potential interconnections with the supervisor eliminating the deadlocks. Namely, when we multiply matrix $[\lambda]$ by $[N]$, being (16×11) dimensional incidence matrix of GPN in Figure 1, we obtain the matrix $[\eta] = [\lambda] \cdot [N]$. $[N] = [\eta_1^T, \eta_2^T, \eta_3^T, \eta_4^T]^T$ consisting of the following T -vectors

$$\eta_1 = (0, 0, 0, 0, -1, 1, 0, -1, 1, 0, 0)$$

$$\eta_2 = (0, 0, 0, -1, 1, 0, 0, 0, -1, 1, 0)$$

$$\eta_3 = (0, 0, 0, -1, 0, 1, 0, 0, 0, 0, 0)$$

$$\eta_4 = (0, 0, 0, -1, 0, 1, 0, -1, 0, 1, 0).$$

Hence, we have $\lambda S_1 = p_5 + p_8 + p_{15} + p_{16}$, $\lambda S_2 = p_4 + p_9 + p_{14} + p_{15}$, $\lambda S_3 = p_4 + p_{13}$, $\lambda S_4 = p_5 + p_9 + p_{14} + p_{15} + p_{16}$ and $\eta S_1 = -t_5 + t_6 - t_8 + t_9$, $\eta S_2 = -t_4 + t_5 - t_9 + t_{10}$, $\eta S_3 = -t_4 + t_6$, and $\eta S_4 = -t_4 + t_6 - t_8 + t_{10}$. It is easy to verify that $\eta S_4 = \eta S_1 + \eta S_2$. However, this set of vectors $\eta_{S_i}, i = 1, \dots, 4$ is not linearly independent, because $\eta_4 = \eta_1 + \eta_2$, i.e., $\text{rank}([\eta]) = 3$, and $\text{rank}([\eta]) = N_{ES} = 3 < \text{rank}([N]) = 8$. It means that there are three elementary siphons and one strongly dependent siphon. Thus, $\Pi_E = \{S_1, S_2, S_3\}$ and $\Pi_D = \{S_4\}$.

In Figure 1 we have $p_{01} = p_6$, $p_{02} = p_{10}$, $P_{S1} = \{p_1, \dots, p_5\}$, $P_{S2} = \{p_7, \dots, p_9\}$, $P_{R1} = \{p_{11}, \dots, p_{16}\}$, and $P_{R2} = \{p_{14}, \dots, p_{16}\}$. Unfortunately, the system is deadlocked.

GPN in Figure 1 has eight minimal P -invariants (see Table 2). They may also be computed e.g., by PN tools [13–15]. Namely, $I_1 = p_2 + p_{11}$, $I_2 = p_1 + p_{12}$, $I_3 = p_3 + 2p_4 + p_{13}$, $I_4 = p_3 + p_9 + p_{14}$, $I_5 = p_4 + p_8 + p_{15}$, $I_6 = p_5 + p_7 + p_{16}$, $I_7 = p_1 + p_2 + p_3 + p_4 + p_5 + p_6$ and $I_8 = p_7 + p_8 + p_9 + p_{10}$. P -invariants in the form of row vectors we can see in Table 2. Negative elements in the vectors η_i in S^3PR paradigm (see [1]) mean that directed arcs are emerging from the GPN model and enter through corresponding transition t_i (index i depends on the position of the transition in a vector η_j) to the supervisor, while positive elements represent directed arcs in opposite direction, i.e., from the supervisor to GPN model through corresponding transitions. However in case of ES^3PR as well as S^4PR paradigms the supervisor synthesis is not so simple like in S^3PR paradigm. The structural properties presented here will be applied in the continuation of this Example introduced in the Section 4.3.

Table 2. P -invariants in the form of row vectors.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
I_1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
I_2	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

I_3	0	0	1	2	0	0	0	0	0	0	0	0	1	0	0	0
I_4	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0
I_5	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
I_6	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1
I_7	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
I_8	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0

If we applied these parameters (which are suitable for S^3PR) to ES^3PR , we would find that the responding supervisor does not comply for such paradigm of the AMS model, because it does not prevent deadlocks.

4. Controllability of Siphons in Generalized Petri Nets

The deadlock prevention policy for GPN consists in the request that all siphons (elementary as well as dependent) must satisfy maximal cs-property. The elementary siphons in the GPN model are properly supervised by means of explicitly adding monitors for them with appropriate initial markings.

Therefore, the controllability of both the elementary siphons and the dependent ones will be analysed.

For the siphon control the facts from the Section 3.1 will be utilized. In [8,9] the following three important definitions are introduced:

Definition 16 [8,9]: Let the net (N, M_0) be a marked net and S be a siphon of N . S is said to be *max-marked* (*min-marked*) at a marking M iff $\exists p \in S$ such that $M(p) \geq \max_{p^*} (M(p) \geq \min_{p^*})$;

Definition 17 [8,9]: Let the net (N, M_0) be a marked net and S be a siphon of N . S is said to be *max-controlled* iff S is *max-marked* at any reachable marking;

Definition 18 [8,9]: A net (N, M_0) is said to be satisfying the *max cs-property* (controlled-siphon property) iff each minimal siphon of N is *max-controlled*.

Thus, in our case (see Example in the Section 4.2), we have four strict minimal siphons. All of them have to be *max-controlled*. Each siphon satisfying the *cs-property* can be always sufficiently marked. Consequently, it can allow firing a transition once at least.

Namely, in [8] the following Lemma is proved:

Lemma 3 [8,9]: If a net (N, M_0) satisfies the max cs-property, it is deadlock-free.

As it was explained above, strict minimal siphons in an ES^3PR consist of both elementary siphons and dependent ones. Monitors are explicitly added only to the elementary siphons. However, so that they satisfy *max cs-property*. The question arises as to how to ensure *max-cs property* in case of dependent siphons. This is ensured by means of *max cs-property* of elementary siphons. Namely, this is performed by properly selection of the control depth variables ξ_{S_i} of its elementary siphons S_i . Below, in the next Sections 4.1 and 4.2, results relevant not only to the controllability of elementary siphons but also to the controllability of dependent siphons are presented too.

4.1. Controllability of Elementary Siphons

Here, for elementary siphon control especially the knowledge from the Section 3.1 will be utilized. Moreover, the following definition for settings the markings to monitors V_{S_i} creating the supervisor will be applied.

Definition 19 [6]: Let S be a strict minimal siphon in an ES^3PR model (N_{μ_0}, M_{μ_0}) , where $N = \bigcirc_{i \in \{1, \dots, n\}} N_i = (P_0 \cup P_S \cup P_R, T, F_{\mu_0}, W_{\mu_0})$. Let $\{\alpha, \beta, \dots, \gamma\} \subseteq \{1, 2, \dots, n\}$ such that $\forall i \in \{\alpha, \beta, \dots, \gamma\}, |\text{Th}(S)| \cap P_{S_i} \neq \emptyset$ and $\forall j \in \{1, 2, \dots, n\} \setminus \{\alpha, \beta, \dots, \gamma\}, |\text{Th}(S)| \cap P_{S_j} = \emptyset$. For S , a nonnegative P-vector k_S is constructed by 4 steps introduced above in the Definition 15.

Besides, almost every fact introduced below in the Section 4.2 touch both elementary siphons and dependent ones.

4.2. Controllability of Dependent Siphons

In addition to the control of elementary siphons, the control of dependent siphons is also very important.

Lemma 4 [8]: Let (N, M_0) be a marked net and S be a siphon of N . S is max-controlled if there exists a P -invariant I such that $\forall p \in (||I||^- \cap S)$, $\max_{p \bullet} = 1$, $||I||^+ \subseteq S$, and $\sum_{p \in P} I(p)M_0(p) > \sum_{p \in S} I(p)(\max_{p \bullet} - 1)$.

When in an S^4R net r is a resource place, S is a strict minimal siphon and $H(r) = I_r \setminus \{r\}$. Let $Th(S) = \sum_{r \in SR} H(r) \setminus S$. We can see that $||Th(S)|| \subseteq P_S$ is true. Let us use $\sum_{p \in |Th(S)|} h_S(p)p$ to denote $Th(S)$. With respect to the Theorem 2 in the beginning of the Section 3, $h_S(p)$ shows that siphon S loses just $h_S(p)$ tokens if the number of tokens in p increases by one.

In [6] the following Theorem (such Theorem is also introduced in [7]) dealing with strongly dependent siphons is introduced:

Theorem 4 [6,7]: Let (N, M_0) , $N = (P, T, F, W)$, be a marked net and S be a strongly dependent siphon with $\eta_S = \sum_{i \in \{1, n\}} a_i \eta_{S_i}$, where S_1, \dots, S_n are elementary siphons of S . S is max-controlled if

1. $\forall i \in \{1, 2, \dots, n\}$, I_i is a P -invariant of N , $||I_i||^+ = S_i$, and $\forall p \in S_i$, $I_i(p) = 1$;
2. $M_0(S) > \sum_{i \in \{1, n\}} \sum_{p \in ||I_i||^-} (a_i |I_i(p)| M_0(p)) + \sum_{p \in S} (\max_{p \bullet} - 1)$.

In our Example 3.3 of ES^3PR the siphon $S_4 = \{p_5, p_9, p_{14}, p_{15}, p_{16}\}$ is, due to $\eta_{S_4} = \eta_{S_1} + \eta_{S_2}$, a strongly dependent siphon and S_1 and S_2 are its elementary siphons. Using previous Theorem 4 we can verify the controllability of S_4 . As to Figure 2, h_1 and h_2 are P -invariants satisfying the condition 1 of the Theorem 4. Now, we only have to check the condition 2 of the Theorem 4: $\sum_{i \in \{1, 2\}} \sum_{p \in ||I_i||^-} (a_i |h_i(p)| M_{\mu 1}(p)) + \sum_{p \in S_4} (\max_{p \bullet} - 1)$. After computing the supervisor consisting of monitors V_{S_i} , $i = 1, \dots, 3$, we may denote the controlled net as $(N_{\mu 1}, M_{\mu 1})$.

4.3. Illustrative Example of ES^3PR —Control

Let us continue in the illustrative example introduced in the Section 3.3. While there rather the GPN model and its structural properties were presented, here the supervisor synthesis will be performed.

In our Example of ES^3PR we have $P_0 = \{p_6, p_{10}\}$ where $p_{01} = p_6$, $p_{02} = p_{10}$, $P_{S_1} = \{p_1, \dots, p_3\}$, $P_{S_2} = \{p_7, \dots, p_9\}$, $P_{R_1} = \{p_{11}, \dots, p_{16}\}$, $P_{R_2} = \{p_{14}, \dots, p_{16}\}$.

$I_{p_{15}} = p_4 + p_8 + p_{15}$ and $I_{p_{16}} = p_5 + p_7 + p_{16}$ are minimal P -semiflows associated with resource places p_{15} , p_{16} , respectively. Hence, $Th(S_1) = (H(p_{15}) + H(p_{16})) \setminus S_1 = ((p_4 + p_8) + (p_5 + p_7)) \setminus \{p_5, p_8\} = p_4 + p_7$. $I_{p_{14}} = p_3 + p_9 + p_{14}$ and $I_{p_{15}} = p_4 + p_8 + p_{15}$ are minimal P -semiflows associated with resource places p_{14} , p_{15} , respectively. Hence, $Th(S_2) \sum_{p \in P} h_2(p)M_{\mu 1}(p) = M_{\mu 1}(p_{14}) + M_{\mu 1}(p_{15}) - M_{\mu 1}(V_{S_2}) = 2 + 1 - 2 = 1 > \sum_{p \in S_2} h_2(p)(\max_{p \bullet} - 1) = 0$. Thus, we can say that S_2 is max-controlled by P -invariant h_2 .

P -invariants h_i , $i = 1, 2, 3$ indicate how the structure of interconnections of monitors V_{S_i} (in Figure 2 right) with the net model representing the original uncontrolled plant will look like. Marking of particular monitors V_{S_i} needs to be computed as it can be seen below: h_1 and h_2 are P -invariants satisfying the condition 1 of the above introduced Theorem 4.

To check the condition 2 of the Theorem 4, $\sum_{i \in \{1, n\}} \sum_{p \in ||I_i||^-} (a_i |h_i(p)| M_{\mu 1}(p)) + \sum_{p \in S_4} (\max_{p \bullet} - 1) = M_{\mu 1}(p_1) + M_{\mu 1}(p_3) + M_{\mu 1}(p_7) + M_{\mu 1}(V_{S_1}) + M_{\mu 1}(V_{S_2}) + (1-1) = (0 + 0 + 0 + 2 + 2 + 0) = 4$. While $M_{\mu 1}(S_4) = M_{\mu 1}(p_{14}) + M_{\mu 1}(p_{15}) + M_{\mu 1}(p_{16}) = 5$. Thus, S_4 is max-controlled. When we consider V_{S_1} , V_{S_2} we see that based on them we have $\sum_{i \in \{1, 2\}} \sum_{p \in ||I_i||^-} (a_i |h_i(p)| M_{\mu 1}(p)) + \sum_{p \in S_4} (\max_{p \bullet} - 1) = M_{\mu 1}(p_1) + M_{\mu 1}(p_3) + M_{\mu 1}(p_7) + M_{\mu 1}(V_{S_1}) + M_{\mu 1}(V_{S_2}) + (1-1) = (0 + 0 + 0 + 2 + 2 + 0) = 4$. While $M_{\mu 1}(S_4) = M_{\mu 0}(S_4) = M_{\mu 1}(p_{14}) + M_{\mu 1}(p_{15}) + M_{\mu 1}(p_{16}) = 5$. Thus, S_4 is max-controlled. Because $g_S = k_S + V_S$ is a P -invariant of the resultant net system $(N_{\mu 1}, M_{\mu 1})$, a monitor V_S is added to the original net model $(N_{\mu 0}, M_{\mu 0})$. Thus, $N_{\mu 1} = (P_0 \cup P_S \cup P_R \cup V_S, T, F_{\mu 1}, W_{\mu 1})$, $\forall p \in P_0 \cup P_S \cup P_R$, $M_{\mu 1}(p) = M_{\mu 0}(p)$. Let $h_S = \sum_{r \in SR} I_r - g_S$ and $N_{\mu 1}(V_S) = M_{\mu 0}(S) - \xi_S$ ($\xi_S \in \mathbb{N}$). Then S is max-controlled if $\xi_S > \sum_{p \in S} h_S(p) (\max_{p \bullet} - 1)$.

As we can see in Figure 2, $(N_{\mu 0}, M_{\mu 0})$ is an ES^3PR being the model of plant net, $(N_{\mu 1}, M_{\mu 1})$ is its liveness-enforcing supervisor. Monitors are added to control elementary siphons S_1 – S_3 only. They enforce liveness for $(N_{\mu 0}, M_{\mu 0})$,

The number N_{ES} of elementary siphons is bounded—see Theorem 3 in the Section 3.1 (it is smaller than minimum of the pair the place count and the transition count). Therefore, dependent

siphons have to be implicitly controlled by fittingly setting the initial number of tokens in monitors. Thus, we can get a structurally simple liveness-enforcing PN supervisor for an ES³PR paradigm of the plant net model.

In Figure 1 we have $P_{S_1} = \{p_1, \dots, p_5\}$, $P_{S_2} = \{p_7, \dots, p_9\}$. For $S_1 = \{p_5, p_8, p_{15}, p_{16}\}$ we have $Th(S_1) = p_4 + p_7$. With respect to the procedure described in the Section 3.1: For P_{S_1} we have $p^{1_s} = p_4$, $p^{1_v} = p_4$, $p^{1_z} = p_4$. For P_{S_2} we have $p^{2_s} = p^{2_v} = p^{2_z} = 7$. Finally, $k_{S_1}(p_1) = k_{S_1}(p_3) = k_{S_1}(p_4) = k_{S_1}(p_7) = 1$. $\forall p \in P_S \setminus \{p_1, p_3, p_4, p_7\}$, $k_{S_1}(p) = 0$. Naturally, we obtain $k_{S_1}(p_6) = k_{S_1}(p_{10}) = k_{S_1}(p_{11}) = k_{S_1}(p_{12}) = k_{S_1}(p_{13}) = k_{S_1}(p_{14}) = k_{S_1}(p_{15}) = k_{S_1}(p_{16}) = 0$ since p_6 and p_{10} are idle places and p_{11}, \dots, p_{16} are resource places. Let $K_S = \{p \mid k_S(p) \neq 0, p \notin Th(S) \}$. In Figure 1 we have $K_{S_1} = \{p_1, p_3\}$.

In Figure 2 we have the following results:

1. $g_1 = p_1 + p_3 + p_4 + p_7 + V_{S_1}$ and $h_1 = I_{p_{15}} + I_{p_{16}} = p_4 + p_8 + p_{15} + p_5 + p_7 + p_{16}$ are P-semiflows. Let $h_1 = h_1 - g_1$. $h_1 = p_5 + p_8 + p_{15} + p_{16} - p_1 - p_3 - V_{S_1}$ is a P-invariant. When we notice that $||h_1||^- \cap S_1 = \emptyset$, $||h_1||^+ = S_1$, and $\sum_{p \in P} h_1(p) M_{\mu 1}(p) = M_{\mu 1}(p_{15}) + M_{\mu 1}(p_{16}) - M_{\mu 1}(V_{S_1}) = 1 + 2 - 2 = 1 > \sum_{p \in S_1} h_1(p)(\max_{p \bullet} - 1) = 0$ we can say that S_1 is max-controlled by P-invariant h_1 .

2. $g_2 = p_1 + p_3 + p_7 + p_8 + V_{S_2}$ and $h_2 = I_{p_{14}} + I_{p_{15}} = p_3 + p_9 + p_{14} + p_4 + p_8 + p_{15}$ are P-semiflows of the controlled net. Let $h_2 = h_2 - g_2$. $h_2 = p_4 + p_9 + p_{14} + p_{15} - p_1 - p_7 - V_{S_2}$ is hence a P-invariant. When we notice that $||h_2||^- \cap S_2 = \emptyset$, $||h_2||^+ = S_2$, and $\sum_{p \in P} h_2(p) M_{\mu 1}(p) = M_{\mu 1}(p_{14}) + M_{\mu 1}(p_{15}) - M_{\mu 1}(V_{S_2}) = 2 + 1 - 2 = 1 > \sum_{p \in S_2} h_2(p)(\max_{p \bullet} - 1) = 0$, we can say that S_2 is max-controlled by P-invariant h_2 .

3. $g_3 = p_1 + p_3 + V_{S_3}$ and $h_3 = I_{p_{13}} = p_3 + p_4 + p_{13}$ are P-semiflows. Let $h_3 = h_3 - g_3$. $h_3 = p_4 + p_{13} - p_1 - V_{S_3}$ is a P-invariant. When we notice that $||h_3||^- \cap S_3 = \emptyset$, $||h_3||^+ = S_3$ and $\sum_{p \in P} h_3(p) M_{\mu 1}(p) = M_{\mu 1}(p_{16}) - M_{\mu 1}(V_{S_3}) = 2 - 1 = 1 > \sum_{p \in S_3} h_3(p)(\max_{p \bullet} - 1) = 0$ which implies that S_3 is max-controlled by P-invariant h_3 .

The supervised ES³PR is deadlock free. Left the original GPN model of plant is displayed, while right its supervisor is placed. Both pictures are shown separately to avoid complicated and non-transparent interconnections between both the plant and the supervisor. Namely, their cooperation is brighter when one observes insinuated transitions of the supervisor and corresponding real ones in the plant.

As we can see in Figure 2, $0 < \xi_{S_1} < 3$, $0 < \xi_{S_2} < 3$, $0 < \xi_{S_3} < 2$, and $\xi_{S_1} + \xi_{S_2} > 1$. Let $\xi_{S_1} = \xi_{S_2} = \xi_{S_3} = 1$. Hence, we have $M_{\mu 1}(V_{S_1}) = 2$, $M_{\mu 1}(V_{S_2}) = 2$, and $M_{\mu 1}(V_{S_3}) = 1$. It is easy to verify that all strict minimal siphons are max-controlled—see results 1–3 introduced above in this Section 3.5. Consequently, the net system in Figure 2 is live, i.e., deadlock free.

5. Conclusions

Liveness (deadlock-freeness) is a very important property of PN from behavioural point of view. Siphons, traps and invariants are structural objects of PN. Especially siphons are closely related to the PN liveness. There are many deadlock prevention control policies for AMS (alternatively named also flexible manufacturing systems—FMS) based on siphons. Namely, AMS are frequently modelled by PN possessing siphons. There are many paradigms of PN models of AMS like S³PR, ES³PR, S⁴R, GS³PR and many others.

To prevent deadlocks in AMS liveness-enforcing supervisors are used. Most of existing methods of their design are based on adding monitors (control places) for siphons. Monitors are based on exact controllability conditions for siphons. In such a way a liveness-enforcing supervisor with permissive behaviour is obtained. However, as we have found, conditions of max controllability of siphons, are overly restrictive and in general only sufficient.

The subject of this paper was to deal with the problem of deadlocks in ES³PR paradigm of GPN models of AMS. This paper may be seen as a free continuation of the author's paper [1], where similar problem was resolved for the S³PR paradigm of ordinary PN (OPN) models of AMS. A difference between OPN and GPN consists in the weights of directed arcs. While in OPN weights of directed arcs are only 1, in GPN may be greater than 1, i.e., there may occur multiplex arcs. Therefore, the approach used in [1] to deal with deadlocks in S³PR is insufficient for dealing with deadlocks in ES³PR and it was necessary to find another one.

An empty siphon in an OPN can cause some transitions to be disabled forever. Therefore, it is necessary to prevent emptying siphons. Such a case in GPN is much more complicated. In consequence of arcs with greater weights, an insufficiently marked siphon can lead to the occurrence of other deadlocks. Therefore, setting the marking of monitors V_{Si} (which create the supervisor), is very important as well as the selection of the control depth variables ξ_{Si} of elementary siphons S_i .

ES³PN models of APN can be understood to be a subset of S⁴PR models, which are even much more complicated.

The approach presented here is based on verification of the *max cs-property* of a net (N, M_0) . When such net N satisfies this property, we can say that it is deadlock-free. In case of GPN, a marked S⁴PR net is live if it satisfies the cs-property. This means that the cs-property is a sufficient but not necessary condition (unlike ES³PR) for the liveness of an S⁴R.

It was presented here, that the deadlock prevention in GPN models of AMS is much more complicated as that in OPN models of AMS. Simultaneously, the deadlock prevention of ES³PR (as a subclass of GPN models of AMS) was performed and illustrated on the example.

More complex analysis of the deadlock prevention problem depends on specific paradigms of GPN. Namely, there are many paradigms of GPN with specific properties and corresponding specific policies how to deal with deadlocks in them. For example, the specific paradigm GS³PR (generalised systems of simple sequential processes with resources) was analysed in [22] as concerns the problem of the deadlock prevention.

In future it will be necessary to concern in more and more complicated structures of AMS modelled by GPN.

A serious review of siphon-based approaches to control GPN are presented in [21]. An application on a specific paradigm GS³PR was presented in [22].

In this paper was shown that prevention of deadlocks in AMS with resources ensures a correct allocation of resources and thereby ensure the continuous and fluent operation of AMS. Thus, by means of controlling siphons of their PN-based models, the fluent and continuous operation of constituents in industrial production can be sustainable what is very important in social practice.

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