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Mach's Principle Versus Dark Matter

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Highlights:
Mach's principle is linked with inertia and the mass around. Nowadays, physicists believe its effect is infinitesimal, so Mach's principle is simply ignored. This paper looks at the consequence of applying Mach's principle. It could replace dark matter.

Abstract: After seeing the history of Mach's principle, we will work out a new model of galaxies inspired by Mach's principle, which is more satisfactory than current dark matter models. This new model is just maths and needs an explanation. One possible explanation is obviously that Mach's principle is at work in a galaxy.

Keywords: Kepler's law; inertia; Mach's principle; dark matter

N.B.: This paper is mainly a logical paper and requires only a basic knowledge of physics

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History of Mach's Principle

Mach had a big influence on Einstein and he had a problem with the inertial reference. Absolute space as the inertial reference was justified by Newton because of the famous Newton's bucket. Behind this reasoning, Newton wanted to reach 'universality' (of his law of gravity); Newton's bucket experiment fitted very well.

Mach found Newton's absolute inertial reference troubling. An interesting question posed by Mach is this: "Newton's experiment with the rotating vessel of water simply informs us that the relative rotation of the bucket with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces produced by its relative rotation with respect to the mass of the Earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass, until they were ultimately several leagues thick" [1]. Mach had a point; Newton's argument is incomplete and not rigorous.

In 1916, Einstein wrote a 3-page-long article with a strong message about ignoring inertia, a "*complacency [which] will appear incomprehensible to a later generation*"[1]. In parallel with that warning, in 1918, he wrote: "*... in a consistent relativity theory there cannot be inertia relative to "space" but only inertia of masses relative to each other*". He did not expound on it but that was the beginning [2]. Later in life, he retracted his support of Mach's principle because he had linked it to general relativity, which aroused criticism.

There is an excellent review here [3] containing an important quote from James Isenberg: "…the distribution of matter and field energy-momentum everywhere at a particular moment in the Universe determines the inertial frame at each point in the Universe".

Mach's principle implies a local inertial frame of reference, which is contrary to Newton's absolute inertial frame of reference. From Isenberg's and Einstein's quotes we could conclude that the inertial frame of reference is defined by the masses around; from Mach's quote, we could conclude that the inertial frame of reference is the wall of Newton's bucket, if that wall is large enough to counteract the mass of the Earth.

One main objective of this paper is to present a model of a galaxy inspired by Mach's principle. The output of any model should fit two observations: the stellar density and the rotational curve.

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1 From German translated by Gill Bankowsky.

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With the rotational curve, current dark matter models (DM models) are marginally better than the model of this work, but current DM models don't fit with the observed stellar density; whereas our model does.

Application of Mach's Principle to a Galaxy

What does it mean for a galaxy if "… *the distribution of matter … determines the inertial reference...*" The stars are the matter and determine the inertial reference. As the stars are rotating, the inertial reference is rotating with the stars. As the inertial reference is rotating, individual stars have no relative speed (relative to the inertial reference), so no centrifugal force, no reason for the stars to escape the galaxy and no need for dark matter.

Hubble's Formula

At the time of Hubble, only the stellar density was available. From many pictures, Hubble had been able to deduce that the stellar distribution was respecting a law. At that time, that law was called Hubble's law. Nowadays, Hubble's law is about the expansion of the universe so I will call it Hubble's formula. In the case of Andromeda, that formula fits the observation very well as you can see with Figure 1.

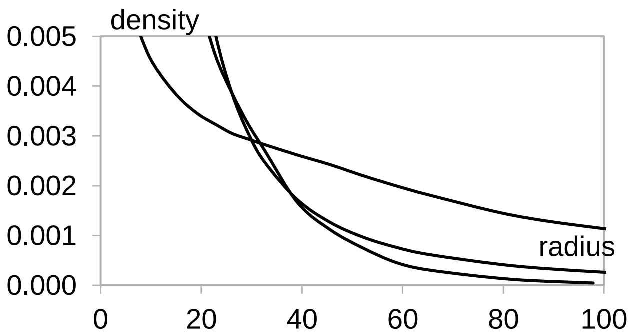

Figure 1. Matter distribution as a function of the distance from the galactic centre in Andromeda. The density is in (10^6M⊙ arcsec^{-2}), the radius is in (kly). The solid line and the dots close together are from Tamm *et al* [4] and represent respectively the observed distribution of stars and the stellar matter output from the best model (Einasto's model). The dash-dot line has been added and is the result of Hubble's formula.

Andromeda is a spiral galaxy. The observation of stellar density doesn't allow us to differentiate between inside the spiral and outside; so the observation at about 60kly is an average which could explain the difference with Hubble's formula. This is an ad hoc explanation and would need more studies.

Hubble's formula is about fluid mechanics and it is an equilibrium between weight and pressure (or energy density) that gives an isothermal density distribution. As soon as the rotational speed of stars around the galaxy was observed, a centrifugal force was expected and the physics behind the formula lost validity so it was abandoned. With Mach's principle, the formula makes sense and can be justified.

There is an important point: Kepler's simulation (the new model) doesn't depend on the validity of Mach's principle. For Kepler's simulation, the density of stars is needed. Instead of doing interpolation between observed density of stars, it is easier to use Hubble's formula. If the scientific community subsequently were to establish that Mach's principle shouldn't be applied to a galaxy, Hubble's formula wouldn't make sense on physics ground but it fits the observation as on Figure 1. So Hubble's formula is an easy way to calculate the density of stars. It is used only because its fits; not because it is justified by Mach's principle.

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Kepler's Law

To recover the observed rotational curve, we will use the third law of planetary motion of Kepler that I simply call Kepler's law. That law is famous because it recovers the rotational speed of planets around the sun. With a galaxy, Keplerian decrease was expected and not observed; that is one reason for dark matter. Kepler's law is simply about the rotation of an object in space: rotation around the sun or rotation around the centre of a galaxy.

Kepler's law is using the mass of the sun (or of the galaxy) and the distance of the object rotating, and predicts the rotational speed of that object. Mach's principle suggests a local inertial reference so, instead of applying Kepler's law at a global level (taking the galaxy as a whole), I decided to apply Kepler's law at a local level (from star to star). From a mass and a distance, the result of Kepler's law is a speed. In Kepler's simulation, Kepler's law is used to calculate a speed between two stars side by side as if one star is rotating around the other. Most stars in a galaxy are not rotating around each other but only the speed is used, not the rotation. The origin of this idea is explained in detail in a book [5]. What is important is the result of Kepler's simulation, not Mach's principle.

Kepler's Simulation

From the stellar density obtained with Hubble's formula, we obtain the mass and the distance between two stars side by side so we can apply Kepler's law. The result is a speed of one star compared to the other; that will be Kepler's speed. If the two stars are side by side along the radius of the galaxy, Kepler's speed will be interpreted as the difference of rotational speed between those two stars. The fact that they don't rotate around each other is explained in [5]. Moving along the radius by choosing two more stars etc. we can observe a change of the rotational speed as in Figure 2. Kepler simulation is simply that idea expressed mathematically in the appendix.

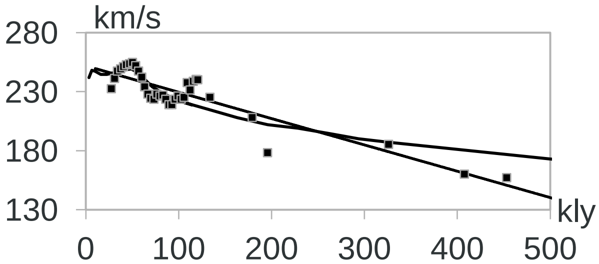

Figure 2. The rotational speed is plotted against the distance from the galactic centre. The dashed line is Kepler's simulation, the asterisks are the observed values from Tamm et al. [4]. The plain line is the output of the best dark matter model presented by Tamm et al.

Astronomers will assert that there are data up to 2,000kly and that Kepler's simulation won't fit those data. That is correct. In fact there is a change of mechanism at around 350kly, the other mechanism involves a new speed-force as briefly described in [6]. A proper description of the other mechanism can be found in [5]. Data up to 2,000kly will fit with the other mechanism.

The result obtained in Figure 2 has nothing to do with Mach's principle. Kepler's law existed before Mach's principle. Here also Mach's principle will justify the use of Kepler's law with the local inertial reference, but the fit with the observation has nothing to do with the justification. Mach's principle is one possible explanation why Kepler's simulation works so well; scientists are welcome to look for another explanation to justify Kepler's simulation.

A Case for Kepler's Simulation

I have some very strong arguments against dark matter models; I have been told, as an excuse to reject my submission, that many models do seem to work, so I have to compare Kepler's simulation to DM models.

No DM models seem to fit the data: look at Figure 1. How can you reconcile the output of DM models with the observation? In addition, the distribution of DM is ad hoc [7] to fit the rotational curve.

Kepler's simulation has three parameters: the initial distance from the centre and its corresponding rotational speed plus a third one. The first two inputs are from observation. A linear regression is calculated with the observation and any point on that linear regression provides two inputs. The third input is the stellar density at the centre of the galaxy. That is an input that cannot be measured accurately enough and is therefore ad hoc and adjusted to obtain Figure 2. But once Figure 2 is obtained, that same input is the input to obtain Figure 1 and there is only one input for Figure 1. If the fit were incorrect, it would have implied a mistake. This is what is missing for DM models. In principle, the third input could be measured from observation as it would be dictated by the fit to the data of Figure 1. Therefore there is no ad hoc parameter, all inputs to obtain Figure 2 are from observation. But if we start with the fit to Figure 1, the uncertainty on the third input would be too large, indeed Figure 2 is very sensitive to that input as explained in the appendix; that is why it is adjusted to fit Figure 2 then is used for Figure 1.

The conclusion is that the result of this model is an improvement on DM models for two reasons: no ad hoc parameters are used, and the output is a better fit with Figure 1.

Discussion

Kepler's simulation is a result on its own; Mach's principle is not involved. But Kepler's simulation requires an explanation: why does it work? Hubble's formula is reinstated only if there is no centrifugal force; something justified by Mach's principle. Using Kepler's law for motion in space as used in Kepler's simulation makes sense only if the inertial reference is local, as stated by Mach's principle.

Mach's principle is the only explanation I can think of; it doesn't necessarily mean that it is correct. There are more studies on Mach's principle in [5] but the most convincing case is Kepler's simulation. Mach's principle could also explain galaxy clusters.

Another discussion should be about the existence of dark matter. Stars oscillate around the galactic plane. If dark matter exists, for a given thickness of a galaxy, the speed of stars perpendicular to the plane should be affected because that speed gives the thickness of a galaxy. Near the sun, without dark matter, that perpendicular speed should be around 15m/s; with 20% of dark matter (and 80% of baryonic matter) the speed should be around 16m/s. Are the observations of speeds and thickness accurate enough to reach a conclusion?

For over 40 years, there have been non-stop experiments trying to detect dark matter. Surely a meta study could tell us whether there is any credible evidence for dark matter.

However, dark matter distribution needed for the fit of Figure 2 doesn't follow the distribution of baryonic matter (it is ad hoc as stated in [7]). To explain that change of distribution would mean that there is some kind of repulsion (very weak) between dark matter and baryonic matter. Baryonic matter occupies mainly the space near the centre, then dark matter occupies the space further away. That repulsion is the reason why we can't detect it on earth (full of baryonic matter). That would make dark matter an elusive matter with the exceptional property of repulsion; if such a matter can exist then it is a possibility.

One strong objection to Mach's principle will be the Foucault pendulum. But it is a gravitational instrument and not an inertial instrument as suggested in [5]; it doesn't show anything about inertia.

Conclusion

This work is a call to the scientific community to explain why Kepler's simulation works and to reconsider Mach's principle. It is a work in progress: please let me know if you cite this paper in your work. Comments on Preprint are more than welcomed. A public exchange should be found (hopefully) on https://physicsoverflow.org/.

Acknowledgements: Professor G. Gilmore told me the story of Hubble's formula.

Appendix A

Appendix A.1. The maths of Kepler's simulation

First, the mass of the sun is assumed to be the mass of all stars mainly because the mass of the sun is a unit (M⊙).

The density of Figure 1 is in the unit of M⊙ arcsec^{-2}. With the distance from the Sun to Andromeda, and using the Pythagorean theorem, one can transform an angle (arcsec) into a length (ly). So, the new unit of density is M⊙ ly^{-2}. The star density ϱ_r (in M⊙ ly^{-2}) at radius r is calculated by Hubble's formula (E1); the third input ϱ_0 (adjusted for Figure 2 to 4.34×10^9 M⊙ ly^{-2}) is the density at the galactic centre, and r is the distance from the centre of the galaxy (in ly).

$$\varrho_r = \varrho_0/(1+r^2) \quad (A1)$$

To obtain Hubble's formula from maths and physics is not so easy. As the formula already exists, the expression of Hubble's formula found in ref [A8] has been used and (E1) is a simplified version of it. (E1) is directly used to obtain Figure 1.

We need a change from a density per surface to a density per volume. Andromeda's thickness is assumed to be equal to the thickness of the Milky Way (1000 ly). The factor of 1000 in formula (E2) is simply the assumed thickness of Andromeda, and used this way, the density is back to a mass per volume. As we need the distance between two stars, its cubic root is taken.

$$\delta r = (1000/\varrho_r)^{1/3} \quad (A2)$$

We saw that the value of ϱ_0 in (E1) is adjusted. What happens if there is an error in the thickness of Andromeda? Because of how formulae (E1) and (E2) are written, an eventual thickness error on the 1000 factor would be automatically corrected by adjusting ϱ_0. So it is not simply ϱ_0 which is adjusted but the thickness is adjusted as well. It is a little trick but now you can forget it and consider that only ϱ_0 is adjusted.

The two other inputs for the simulation, the first r_0 (distance from the centre of the galaxy: 10kly) and v_0 (rotational speed at r_0: 249km/s) are introduced at this point. Those two inputs come from the linear regression of the observed data.

At the start of the simulation, the first ϱ_r and δr are calculated for r_0. Then the second ϱ_r and δr are calculated with r_1 ($r_1 = r_0+\delta r$) etc. So:

$$r_{n+1} = r_n+\delta r. \quad (A3)$$

Kepler's speed vk between a star at r_n and a star at $r_n+\delta r$ is calculated using Kepler's law (E4); G is the gravitational constant, M⊙ is the solar mass, and δr is calculated from ϱ_r at $r=r_n$ in formula (E2).

$$vk = (M_\odot G/\delta r)^{1/2} \quad (A4)$$

The core of Kepler's simulation is that the difference in speed between stars is Kepler's speed, vk. We need vncte, the exact speed of the star s_{n+1} if there was no difference in speed (therefore the indice "cte"). If there is no difference in angular speed between s_n and s_{n+1}, the linear speed of s_{n+1} would be vncte which is different to v_n because s_{n+1} is slightly further from the centre and so turns slightly faster.

$$vncte = v_n\,(r_n+\delta r)/r_n \quad (A5)$$

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Therefore, vk should be subtracted to the speed vncte. The vncte is just for calculation, and does not correspond to an observation; it is just geometry.

The final result v_{n+1} is the rotational speed in linear value of the stars s_{n+1}, and is calculated with formula (E6):

$$v_{n+1} = \text{vncte} - \text{vk} \quad (A6)$$

Then, v_{n+1} becomes the new v_n, and r_{n+1} becomes the new r_n, and we restart from formula (E1). From a first star (the first r_0 and v_0), by iteration, the model can calculate the apparent rotational speed up to the edge of the galaxy.

The number of iterations required is large (>10,000). A small error on ϱ_0 will be added 10,000 times. That is why ϱ_0 is a very sensitive parameter.

To build your own model without being a software expert, insert formulae (E1 to E6) into a simple spreadsheet with the three inputs; the simulation yields the result presented in Figure 2.

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As the result is a straight line, it means there is a simple law: the speed at any r_n is the linear value (at r_n) of the angular speed of the first star (r_0) minus the sum of all vk from r_0 to r_n. It is more complex to mathematically find the straight line because δr is continuously changing but that is the law.

The addition of vncte step seems superfluous but without it, the result is not a straight line.

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