Article

Not peer-reviewed version

A Structural Approach for the Concurrent Teaching of Introductory Propositional Calculus and Set Theory

Osvaldo Skliar * , Sherry E. Gapper , Ricardo E. Monge

Posted Date: 15 August 2024

doi: 10.20944/preprints202408.0700.v2

Keywords: logic and mathematics instruction, classical propositional logic, set theory, isomorphism

Article

A Structural Approach for the Concurrent Teaching of Introductory Propositional Calculus and Set Theory

Osvaldo Skliar [1,*], Sherry Gapper [1] and Ricardo E. Monge [2]

1 Universidad Nacional, Costa Rica; sherry.gapper.morrow@una.ac.cr
2 Universidad CENFOTEC, Costa Rica; rmonge@ucenfotec.ac.cr.
* Correspondence: osvaldoskliar@gmail.com

Abstract: A presentation is provided of a structural approach for the concurrent teaching of introductory propositional calculus and set theory. The existence of isomorphism is shown between each law (or tautology) of propositional calculus and the form of a corresponding set which is equal to the universal set $\mathbb{U}$ considered.

Keywords: logic and mathematics instruction, classical propositional logic, set theory, isomorphism

MSC: 03B05; 97D20

---

1. Introduction

The objective of this article is to present a systemic-structural approach [1] – or simply, a "structural approach" – for the concurrent teaching of the basic notions of 1) propositional calculus and 2) set theory at an introductory level. The term "systemic" refers to the presentation of isomorphisms existing between the "parallel disciplines" considered (in this case, propositional calculus and set theory). For this purpose, from these disciplines the pairs of corresponding operations will be discussed in section 2, and then, the pairs of corresponding laws, or theorems, in section 3.

This instructional method is based mainly on the structure of the subject matter presented, and not on the diverse possible ways of communicating it, such as presential or virtual classes, using different amounts of audiovisual resources, or having a majority of lectures or discussion groups. The basic hypothesis of this instructional approach is the following: The structure itself is what facilitates learning. For this reason, the term "structural" is used for the approach considered.

It is supposed that those reading this article teach in high school or college and are already familiar with the notions of logic and mathematics to be considered here. For this reason, they will not be covered too formally or exhaustively; a simple review of some of their main characteristics will be provided instead. In the remainder of this section, brief consideration will be given to some of these notions which are important for this article.

A proposition – or statement – is a linguistic expression to which a truth value may be assigned: According to classical bivalent logic, each proposition is either true or false. Thus, for example, "Caballito is a neighborhood of the Argentine capital, Buenos Aires" can be considered a proposition. That does not occur with "What is your name?" or "Sit down!".

In this article, when considering a sole proposition, it will be called "q". When considering more than one proposition, the denominations $q_1, q_2, q_3, \ldots$ will be used. The negation of a proposition will be symbolized by a horizontal bar above that proposition. Thus the negation of the proposition q – that is, not q – will be symbolized as $\overline{q}$. Of course, any negated proposition, such as $\overline{q}$, is also a proposition.

In set theory, the universal set $\mathbb{U}$ is the set to which all the elements that can belong to a set – according to the topic covered – belong. If within the framework of that $\mathbb{U}$ only one other set is considered – that is, a set to which some of the elements in that $\mathbb{U}$ belong – that sole set will be denominated C. If more than one set of that type is considered, those sets will be denominated C_1, C_2, $C_3, \ldots$.

The operator of complementation of a set will be symbolized by the symbol $+$ placed above the denomination of the set whose complement is sought. Thus, the complement of C is symbolized as $\overset{+}{C}$.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

All of the elements belonging to the universal set $\mathbb{U}$ considered that do not to belong to C belong to the set $\overset{+}{C}$. The complement sets of $C_1, C_2, C_3, \ldots$ will be symbolized as $\overset{+}{C_1}, \overset{+}{C_2}, \overset{+}{C_3}, \ldots$ respectively.

Given that all the elements which can be considered when covering a particular topic belong to the universal set $\mathbb{U}$ symbolized as $\mathbb{U}$, no element will belong to its complement, which will be symbolized as $\overset{+}{\mathbb{U}}$. For this reason, $\overset{+}{\mathbb{U}}$ will be called the empty set and will be symbolized as $\varnothing$; that is, $\overset{+}{\mathbb{U}} = \varnothing$.

In this article it will be admitted that whenever an operation involving sets is carried out, the corresponding universal set $\mathbb{U}$ considered will be known.

For an introduction to topics of logic and set theory, one may consult [2]. For a more detailed treatment of topics of logic, [3] and [4] may be consulted. For a more advanced coverage of set theory, one may consult, for example [5].

2. Pairs of Operations Corresponding to 1) Propositional Calculus and 2) Set Theory

The presence of a zero – 0 – in the truth table of a proposition is equivalent to the truth value "false" of that proposition. The presence of a zero – 0 – in the membership table of a set C (or rather, C_1, C_2, C_3, ...) means that an element belonging to the universal set $\mathbb{U}$ considered does not belong to that set C (that is, to $C_1, C_2, C_3, \ldots$).

The presence of a one – 1 – in the truth table of a proposition is equivalent to the truth value "true" of that proposition. The presence of a one – 1 – in the membership table of a set C (or rather, C_1, C_2, C_3, ...) means that an element belonging to the universal set $\mathbb{U}$ considered does belong to that set C (that is, to $C_1, C_2, C_3, \ldots$).

Figure 1 presents a) the truth table of the negation $\overline{q}$ (not q) of the proposition q and b) the membership table of the complement $\overset{+}{C}$ of a set C.

q	$\overline{q}$
0	1
1	0

(a) truth table for $\overline{q}$

C	$\overset{+}{C}$
0	1
1	0

(b) membership table for $\overset{+}{C}$

Figure 1. a) truth table for $\overline{q}$ and b) membership table for $\overset{+}{C}$

The first row of the truth table represented in Figure 1a is $0, 1$. The first digit – 0 – in that numerical sequence in column q means that it is accepted that q is false. The second digit – 1 – in that numerical sequence means that it is accepted that $\overline{q}$ is true. In other words, if the proposition q is false, then its negation (the proposition $\overline{q}$) is true.

The second row of the truth table represented in Figure 1a is $1, 0$. The first digit – 1 – in that numerical sequence in column q means that it is accepted that q is true. The second digit – 0 – in that numerical sequence means that it is accepted that $\overline{q}$ is false. In other words, if the proposition q is true, then its negation ($\overline{q}$) is false.

The first row of the membership table in Figure 1b is $0, 1$. The first digit – 0 – in that numerical sequence in column C means that a certain element belonging to the universal set $\mathbb{U}$ considered does not belong to the set C. The second digit – 1 – in that numerical sequence means that the element does belong to the complement set $\overset{+}{C}$ of C. In other words, if any element belonging to the universal set $\mathbb{U}$ considered does not belong to a set C, then it does belong to the complement set $\overset{+}{C}$ of that set.

The second row of the membership table in Figure 1b is $1, 0$. The first digit – 1 – in that numerical sequence in column C means that a certain element belonging to the universal set $\mathbb{U}$ considered does

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

belong to the set C. The second digit – 0 – in that numerical sequence means that the element does not belong to the complement set $\overset{+}{C}$ of C. In other words, if any element belonging to the universal set $\mathbb{U}$ considered belongs to the set C, then it does not belong to the complement set $\overset{+}{C}$ of that set.

The symbol corresponding to the operator of the conjunction of two propositions q_1 and q_2 will be $\wedge$. The symbol corresponding to the operator of the intersection of two sets C_1 and C_2 will be symbolized as $\cap$.

Figure 2 presents a) the truth table for the conjunction $q_1 \wedge q_2$ (q_1 and q_2) and b) the membership table for the intersection set ($C_1 \cap C_2$) of C_1 and C_2.

q_1	q_2	$q_1 \wedge q_2$
0	0	0
0	1	0
1	0	0
1	1	1

(a) truth table for the conjunction of q_1 and q_2 – $q_1 \wedge q_2$

C_1	C_2	$C_1 \cap C_2$
0	0	0
0	1	0
1	0	0
1	1	1

(b) membership table for the set that is an intersection of C_1 and C_2 – $C_1 \cap C_2$

Figure 2. a) truth table for $q_1 \wedge q_2$ and b) membership table for $C_1 \cap C_2$

As seen in Figure 2a, only in the fourth row of the above truth table (where the numerical sequence 1, 1, 1 appears) is there a 1 in the column corresponding to $q_1 \wedge q_2$. In other words, only if q_1 is true as indicated by the first 1 in that numerical sequence, and q_2 is also true as indicated by the second 1 in that numerical sequence, is $q_1 \wedge q_2$ also true, as indicated by the third 1 in the numerical sequence considered. In the other three possible cases, considered in the first, second and third rows of the truth table in Figure 2, there is a 0 in the column corresponding to $q_1 \wedge q_2$, indicating that the proposition is false.

As seen in Figure 2b, only in the fourth row of the membership table (where the numerical sequence 1, 1, 1 appears) is there a 1 in the column corresponding to $C_1 \cap C_2$. In other words, only if any element belonging to the $\mathbb{U}$ considered belongs to C_1, as indicated by the first 1 in that numerical sequence, and also belongs to C_2, as indicated by the second 1 in that numerical sequence, does that element belong to $C_1 \cap C_2$ (the intersection set of C_1 and C_2).

If q_1 is made to correspond to C_1, q_2 to C_2, the operator of conjunction $\wedge$ in propositional calculus to the operator of intersection $\cap$ in set theory (and as a result the correspondence between $q_1 \wedge q_2$ and $C_1 \cap C_2$ is established), the isomorphism existing between the truth table in Figure 2a and the membership table in 2b can be observed. In effect, for every 0 in the first table there is a corresponding 0 in the second table, and for every 1 in the first table there is a corresponding 1 in the second table.

The symbol of the inclusive disjunction (inclusive or) of the two propositions q_1 and q_2 will be $\vee$. The symbol of the exclusive disjunction (exclusive or) of two propositions q_1 and q_2 will be $\dot{\vee}$. The symbol corresponding to the inclusive union of two sets C_1 and C_2 will be $\cup$. The symbol corresponding to the exclusive union of two sets C_1 and C_2 will be $\dot{\cup}$.

Figure 3 presents a) the truth tables corresponding to the inclusive disjunction of q_1 and q_2 ($q_1 \vee q_2$) and to the exclusive disjunction of q_1 and q_2 ($q_1 \dot{\vee} q_2$), along with b) the membership tables for the inclusive union set of C_1 and C_2 ($C_1 \cup C_2$) and of the exclusive union set of C_1 and C_2 ($C_1 \dot{\cup} C_2$).

q_1	q_2	$q_1 \vee q_2$	$q_1 \dot{\vee} q_2$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	0

(a) truth table for the inclusive disjunction of q_1 and q_2 ($q_1 \vee q_2$) and the truth table for the exclusive disjunction of q_1 and q_2 ($q_1 \dot{\vee} q_2$)

C_1	C_2	$C_1 \cup C_2$	$C_1 \dot{\cup} C_2$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	0

(b) membership table for the inclusive union of the sets C_1 and C_2 ($C_1 \cup C_2$) and the membership table for the exclusive union of the sets C_1 and C_2 ($C_1 \dot{\cup} C_2$)

Figure 3. a) truth tables for $q_1 \vee q_2$ and of $q_1 \dot{\vee} q_2$, and b) membership tables for the sets $C_1 \cup C_2$ and $C_1 \dot{\cup} C_2$

If q_1 is made to correspond to C_1, q_2 to C_2, the operator $\vee$ in propositional calculus to the operator $\cup$ in set theory, and the operator $\dot{\vee}$ in propositional calculus to the operator $\dot{\cup}$ in set theory, note may be taken of 1) the isomorphism between the truth table corresponding to $q_1 \vee q_2$ and the membership table for $C_1 \cup C_2$ (given that for every 0 in this truth table, there is a 0 in the membership table), and 2) the isomorphism between the truth table for $q_1 \dot{\vee} q_2$ and the membership table for $C_1 \dot{\cup} C_2$, for the same reason mentioned in the above isomorphism.

Note that in the column corresponding to $q_1 \vee q_2$ there is a sole 0 indicating that the inclusive disjunction is false only when both q_1 and q_2 are false, as shown by the zeros in the first row both in the q_1 column and in the q_2 column. Likewise, note that in the column corresponding to $C_1 \cup C_2$ there is a sole 0 indicating that any element belonging to the universal set $\mathbb{U}$ considered does not belong to $C_1 \cup C_2$, only when that element belongs neither to C_1 nor to C_2, as shown by the zeros present in the first row both in the C_1 column and in the C_2 column.

Note that in the truth table for the proposition $q_1 \dot{\vee} q_2$ this proposition is true if only one of the two propositions q_1 and q_2 is true. Likewise, in the membership table corresponding to $C_1 \dot{\cup} C_2$ it can be seen that only the elements belonging to the universal set $\mathbb{U}$ considered that belong only to one of the two sets C_1 and C_2 belong to that set ($C_1 \dot{\cup} C_2$).

The operator of material implication in propositional calculus is symbolized as $\rightarrow$. The proposition $q_1 \rightarrow q_2$ (that is, q_1 materially implies q_2) can be read as "If q_1, then q_2". The proposition $q_2 \rightarrow q_1$ (that is, q_2 materially implies q_1) can be read as "If q_2, then q_1". In the proposition $q_1 \rightarrow q_2$, q_1 is known as the antecedent of that proposition and q_2 is its consequent. In the proposition $q_2 \rightarrow q_1$, q_2 is the antecedent of that proposition and q_1 is its consequent.

Given two sets C_1 and C_2, the use of the operator of membership implication $\twoheadrightarrow$ makes it possible to generate the sets $C_1 \twoheadrightarrow C_2$ and $C_2 \twoheadrightarrow C_1$.

Figure 4 presents 1) the truth table for the proposition $q_1 \rightarrow q_2$ and the truth table for the proposition $q_2 \rightarrow q_1$, and 2) the membership table for the set $C_1 \twoheadrightarrow C_2$ and the membership table for the set $C_2 \twoheadrightarrow C_1$.

 doi:10.20944/preprints202408.0700.v2

q_1	q_2	$q_1 \to q_2$	$q_2 \to q_1$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

(a) truth table for the proposition $q_1 \to q_2$ and truth table for the proposition $q_2 \to q_1$

C_1	C_2	$C_1 \twoheadrightarrow C_2$	$C_2 \twoheadrightarrow C_1$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

(b) membership table for the set $C_1 \twoheadrightarrow C_2$ and membership table for the set $C_2 \twoheadrightarrow C_1$

Figure 4. a) Truth tables for the propositions $q_1 \to q_2$ and $q_2 \to q_1$, and membership tables for the sets $C_1 \twoheadrightarrow C_2$ and $C_2 \twoheadrightarrow C_1$.

Note that the proposition $q_1 \to q_2$ is false only when q_1 is true and q_2 is false. Likewise, any element belonging to the universal set $\mathbb{U}$ considered does not belong to the set $C_1 \twoheadrightarrow C_2$ only if that element belongs to the set C_1 but not to the set C_2.

Note also that the proposition $q_2 \to q_1$ is false only when q_2 is true and q_1 is false. Likewise, any element belonging to the universal set $\mathbb{U}$ considered does not belong to the set $C_2 \twoheadrightarrow C_1$ only if that element belongs to the set C_2 but not to the set C_1.

The truth table for $q_1 \to q_2$ is isomorphic to the membership table for $C_1 \twoheadrightarrow C_2$. (Both tables are the same from a purely numerical viewpoint.) This also occurs with the truth table for $q_2 \to q_1$ and the membership table for $C_2 \twoheadrightarrow C_1$.

The operator of logical equivalence (or of material bi-implication) in propositional calculus will be symbolized as $\longleftrightarrow$. The operator of membership bi-implication in set theory will be symbolized as $\twoheadleftarrow\!\!\twoheadrightarrow$. Figure 5 presents a) the truth table for the proposition $q_1 \longleftrightarrow q_2$ (that is, "q_1 is logically equivalent to q_2") and b) the membership table for the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$.

q_1	q_2	$q_1 \longleftrightarrow q_2$
0	0	1
0	1	0
1	0	0
1	1	1

(a) truth table for the proposition $q_1 \longleftrightarrow q_2$

C_1	C_2	$C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$
0	0	1
0	1	0
1	0	0
1	1	1

(b) membership table for the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$

Figure 5. a) truth table for the proposition $q_1 \longleftrightarrow q_2$ and b) membership table for the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$

In the truth table for $q_1 \longleftrightarrow q_2$ it is seen that the proposition is true in only two of the four possible cases: the cases in which q_1 and q_2 have the same truth value (that is, if both propositions are false or if both propositions are true). These cases are considered in rows 1 and 4 of the truth table. In the membership table for $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$ it is seen that any element belonging to the universal set $\mathbb{U}$ considered belongs to the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$ only in two of the four possible cases: if that element belongs

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

neither to the set C_1 nor to the set C_2, or if that element belongs both to the set C_1 and to the set C_2. These cases are considered in rows 1 and 4 of the membership table.

The Sheffer stroke operator – or NAND, the negation of the conjunction of two propositions q_1 and q_2 – will be symbolized as $\uparrow$. The Sheffer stroke operator in set theory will be symbolized as ‡. Figure 6 represents a) the truth table for the proposition $q_1 \uparrow q_2$, and b) the membership table for the set C_1‡C_2.

q_1	q_2	$q_1 \uparrow q_2$
0	0	1
0	1	1
1	0	1
1	1	0

(a) truth table for the proposition $q_1 \uparrow q_2$

C_1	C_2	C_1‡C_2
0	0	1
0	1	1
1	0	1
1	1	0

(b) membership table for the set C_1‡C_2

Figure 6. a) truth table for the proposition $q_1 \uparrow q_2$; and b) membership table for the set C_1‡C_2

In the truth table for the proposition $q_1 \uparrow q_2$ it is seen that in only one of four possible cases is the proposition false: that in which both q_1 and q_2 are true. In the membership table for C_1‡C_2 it is seen that in only one of four possible cases, does any element whatsoever belonging to the universal set $\mathbb{U}$ considered not belong to the set C_1‡C_2: that in which that element belongs both to C_1 and to C_2.

If the proposition q_1 is made to correspond to the set C_1, the proposition q_2 is made to correspond to the set C_2, the operator $\uparrow$ in propositional calculus to the operator‡ in set theory – and therefore, $q_1 \uparrow q_2$ to C_1‡C_2 – it can be seen that the truth table for $q_1 \uparrow q_2$ is isomorphic to the membership table for C_1‡ C_2. (Note that both tables are the same from a purely numerical viewpoint.)

The operator Peirce's arrow – or NOR, the negation of the inclusive disjunction of two propositions q_1 and q_2 – will be symbolized as $\downarrow$. The operator Peirce's arrow in set theory will be symbolized as ↨. Figure 7 presents a) the truth table for the proposition $q_1 \downarrow q_2$ and b) the membership table for the set C_1 ↨ C_2.

q_1	q_2	$q_1 \downarrow q_2$
0	0	1
0	1	0
1	0	0
1	1	0

(a) truth table for the proposition $q_1 \downarrow q_2$

C_1	C_2	C_1 ↨ C_2
0	0	1
0	1	0
1	0	0
1	1	0

(b) membership table for the set C_1 ↨ C_2

Figure 7. a) truth table for the proposition $q_1 \downarrow q_2$ and b) membership table for the set C_1 ↨ C_2

In the truth table for the proposition $q_1 \downarrow q_2$ it is seen that in only one of four possible cases is the proposition true: that in which both q_1 and q_2 are false. In the membership table for C_1 ↨ C_2 it is seen

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

that in only one of four possible cases does any element whatsoever belonging to the universal set $\mathbb{U}$ belong to the set $C_1 \updownarrow C_2$: that in which that element belongs neither to the set C_1 nor to the set C_2.

If the proposition q_1 is made to correspond to the set C_1, the proposition q_2 to correspond to the set C_2, the operator $\downarrow$ in propositional calculus to the operator $\updownarrow$in set theory – and therefore, $q_1 \downarrow q_2$ to $C_1 \updownarrow C_2$ – it can be seen that the truth table for $q_1 \downarrow q_2$ is isomorphic to the membership table $C_1 \updownarrow C_2$. (Note that both tables are the same from a purely numerical viewpoint.)

3. Each Law – or Tautology – of Propositional Calculus Is Isomorphic to an Expression Corresponding to a Set Equal to the Universal Set Considered

Any proposition resulting from operations between n propositions, for $n = 1, 2, 3, \ldots$, which is true because of its logical form, regardless of the truth value of each of those n propositions, is known as a law (or tautology) in propositional calculus.

In this section reference will be made to several laws (or tautologies) of propositional calculus. For each a specification will be given, according to section 2 above, of the corresponding set which is isomorphic to that law. A representation will be provided of 1) the truth table for that law and 2) the membership table for that set.

In each truth table it will be possible to note in the corresponding column that, regardless of the truth values of n propositions considered to construct it, the proposition that qualifies as a law is true. In each membership table it will be possible to note how, regardless of whether any element of the universal set $\mathbb{U}$ considered belongs or not to one of the n sets considered to construct the set corresponding to that law – which is isomorphic to it – that element belongs to this latter set. Therefore, that set is equal to the universal set $\mathbb{U}$ considered.

First of all consider in Figure 8 the "law of the excluded middle": $q \vee \overline{q}$. In this case $n = 1$.

The set corresponding to the law $q \vee \overline{q}$ – or which is isomorphic to it – is the following: $C \cup \overset{+}{C}$.

q	$\overline{q}$	$q \vee \overline{q}$
0	1	1
1	0	1

(a) truth table for the law $q_1 \vee \overline{q}$

C	$\overset{+}{C}$	$C \cup \overset{+}{C}$
0	1	1
1	0	1

(b) membership table for the set $C \cup \overset{+}{C}$; $(C \cup \overset{+}{C}) = \mathbb{U}$

Figure 8. a) truth table for the law $q \vee \overline{q}$ and b) membership table for the set $C \cup \overset{+}{C}$

Consider in Figure 9 one of the De Morgan's laws in propositional calculus: $\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$. In this case $n = 2$.

The set corresponding to the law $\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$ – or which is isomorphic to it – is the following: $\overset{+}{\overline{(C_1 \cap C_2)}} \leftrightarrow (\overset{+}{C}_1 \cup \overset{+}{C}_2)$.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

q_1	q_2	$q_1 \wedge q_2$	$\overline{(q_1 \wedge q_2)}$	$\overline{q}_1$	$\overline{q}_2$	$\overline{q}_1 \vee \overline{q}_2$	$\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2)$
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

(a) truth table for the law $\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2)$

C_1	C_2	$C_1 \cap C_2$	$\overline{(C_1 \cap C_2)}$	$\overset{+}{C}_1$	$\overset{+}{C}_2$	$\overset{+}{C}_1 \cup \overset{+}{C}_2$	$\overline{(C_1 \cap C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cup \overset{+}{C}_2)$
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

(b) membership table for the set $\overline{(C_1 \cap C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cup \overset{+}{C}_2)$; $(\overline{(C_1 \cap C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cup \overset{+}{C}_2)) = \mathbb{U}$

Figure 9. a) truth table for the law $\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2)$ and b) membership table for the set $\overline{(C_1 \cap C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cup \overset{+}{C}_2)$

Consider in Figure 10 another De Morgan's law in propositional calculus: $\overline{(q_1 \vee q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$. In this case $n = 2$.

The set corresponding to the law $\overline{(q_1 \vee q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$ – or which is isomorphic to it – is the following: $\overline{(C_1 \cup C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cap \overset{+}{C}_2)$.

q_1	q_2	$q_1 \vee q_2$	$\overline{(q_1 \vee q_2)}$	$\overline{q}_1$	$\overline{q}_2$	$\overline{q}_1 \wedge \overline{q}_2$	$\overline{(q_1 \vee q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$
0	0	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	1	0	0	1	0	1
1	1	1	0	0	0	0	1

(a) truth table for the law $\overline{(q_1 \vee q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$

C_1	C_2	$C_1 \cup C_2$	$\overline{(C_1 \cup C_2)}$	$\overset{+}{C}_1$	$\overset{+}{C}_2$	$\overset{+}{C}_1 \cap \overset{+}{C}_2$	$\overline{(C_1 \cup C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cap \overset{+}{C}_2)$
0	0	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	1	0	0	1	0	1
1	1	1	0	0	0	0	1

(b) membership table for the set $\overline{(C_1 \cup C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cap \overset{+}{C}_2)$; $(\overline{(C_1 \cup C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cap \overset{+}{C}_2)) = \mathbb{U}$

Figure 10. a) truth table for the law $\overline{(q_1 \vee q_2)} \longleftrightarrow (\overline{q}_1 \wedge \overline{q}_2)$ and b) membership table for the set $\overline{(C_1 \cup C_2)} \mathrel{\leftarrow\!\!\!+\!\!\!\rightarrow} (\overset{+}{C}_1 \cap \overset{+}{C}_2)$

Consider in Figure 11 the law of propositional calculus "modus ponendo ponens": $((q_1 \rightarrow q_2) \wedge q_1) \rightarrow q_2$. In this case $n = 2$.

The set corresponding to the law $((q_1 \rightarrow q_2) \wedge q_1) \rightarrow q_2$ – or which is isomorphic to it – is the following: $((C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2) \cap C_1) \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

q_1	q_2	$q_1 \rightarrow q_2$	$(q_1 \rightarrow q_2) \wedge q_1$	$((q_1 \rightarrow q_2) \wedge q_1) \rightarrow q_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

(a) truth table for the law $((q_1 \rightarrow q_2) \wedge q_1) \rightarrow q_2$

C_1	C_2	$C_1 \mathrel{+\!\!\!\!\rightarrow} C_2$	$(C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap C_1$	$((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap C_1) \mathrel{+\!\!\!\!\rightarrow} C_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

(b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap C_2) \mathrel{+\!\!\!\!\rightarrow} C_1$; $(((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap C_1) \mathrel{+\!\!\!\!\rightarrow} C_2) = \mathbb{U}$

Figure 11. a) truth table for the law $((q_1 \rightarrow q_2) \wedge q_1) \rightarrow q_2$ and b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap C_1) \mathrel{+\!\!\!\!\rightarrow} C_2$

Consider in Figure 12 the law of propositional calculus "modus tollendo tollens": $((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$. In this case $n = 2$.

The set corresponding to the law $((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$ – or which is isomorphic to it – is the following: $((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_1$.

q_1	q_2	$q_1 \rightarrow q_2$	$\overline{q}_1$	$\overline{q}_2$	$(q_1 \rightarrow q_2) \wedge \overline{q}_2$	$((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$
0	0	1	1	1	1	1
0	1	1	1	0	0	1
1	0	0	0	1	0	1
1	1	1	0	0	0	1

(a) truth table for the law $((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$

C_1	C_2	$C_1 \rightarrow C_2$	$\overset{+}{C}_1$	$\overset{+}{C}_2$	$(C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2$	$((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_1$
0	0	1	1	1	1	1
0	1	1	1	0	0	1
1	0	0	0	1	0	1
1	1	1	0	0	0	1

(b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_1$; $(((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_1) = \mathbb{U}$

Figure 12. a) truth table for the law $((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$, and b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\rightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_1$

Consider in Figure 13 the law $(q_1 \dot{\vee} q_2) \longleftrightarrow \overline{(q_1 \longleftrightarrow q_2)}$ in propositional calculus. In this case $n = 2$.

The set corresponding to the law $(q_1 \dot{\vee} q_2) \longleftrightarrow \overline{(q_1 \longleftrightarrow q_2)}$ – or which is isomorphic to it – is the following: $(C_1 \dot{\cup} C_2) \mathrel{\leftarrow\!\!\!\!+\!\!\!\!\rightarrow} \overset{+}{\overline{(C_1 \mathrel{\leftarrow\!\!\!\!+\!\!\!\!\rightarrow} C_2)}}$.

q_1	q_2	$q_1 \dot{\vee} q_2$	$q_1 \longleftrightarrow q_2$	$\overline{(q_1 \longleftrightarrow q_2)}$	$(q_1 \dot{\vee} q_2) \longleftrightarrow \overline{(q_1 \longleftrightarrow q_2)}$
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	0	1	1
1	1	0	1	0	1

(a) truth table for the law $(q_1 \dot{\vee} q_2) \longleftrightarrow \overline{(q_1 \longleftrightarrow q_2)}$

C_1	C_2	$C_1 \dot{\cup} C_2$	$C_1 \leftarrow\!\!+\!\!\rightarrow C_2$	$\overline{(C_1 \leftarrow\!\!+\!\!\rightarrow C_2)}$	$(C_1 \dot{\cup} C_2) \leftarrow\!\!+\!\!\rightarrow \overline{(C_1 \leftarrow\!\!+\!\!\rightarrow C_2)}$
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	0	1	1
1	1	0	1	0	1

(b) membership table for the set $(C_1 \dot{\cup} C_2) \leftarrow\!\!+\!\!\rightarrow \overline{(C_1 \leftarrow\!\!+\!\!\rightarrow C_2)}$; $((C_1 \dot{\cup} C_2) \leftarrow\!\!+\!\!\rightarrow \overline{(C_1 \leftarrow\!\!+\!\!\rightarrow C_2)}) = \mathbb{U}$

Figure 13. a) truth table for the law $(q_1 \dot{\vee} q_2) \longleftrightarrow \overline{(q_1 \longleftrightarrow q_2)}$ and b) membership table for the set $(C_1 \dot{\cup} C_2) \leftarrow\!\!+\!\!\rightarrow \overline{(C_1 \leftarrow\!\!+\!\!\rightarrow C_2)}$

Consider in Figure 14 the law of transitivity of material implication in propositional calculus: $((q_1 \to q_2) \wedge (q_2 \to q_3)) \to (q_1 \to q_3)$. In this case $n = 3$.

The set corresponding to the law $((q_1 \to q_2) \wedge (q_2 \to q_3)) \to (q_1 \to q_3)$ – or which is isomorphic to it – is the following: $((C_1 -\!\!+\!\!\rightarrow C_2) \cap C_2 -\!\!+\!\!\rightarrow C_3)) -\!\!+\!\!\rightarrow (C_1 -\!\!+\!\!\rightarrow C_3)$.

q_1	q_2	q_3	$q_1 \to q_2$	$q_2 \to q_3$	$(q_1 \to q_2) \wedge (q_2 \to q_3)$	$q_1 \to q_3$	$((q_1 \to q_2) \wedge (q_2 \to q_3)) \to (q_1 \to q_3)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

(a) truth table for the law $((q_1 \to q_2) \wedge (q_2 \to q_3)) \to (q_1 \to q_3)$

Figure 14. *Cont.*

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 August 2024 doi:10.20944/preprints202408.0700.v2

C_1	C_2	C_3	$C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2$	$C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3$	$(C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

$C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_3$	$((C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)) \mathrel{+\!\!\!\!\!\longrightarrow} (C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)$
1	1
1	1
1	1
1	1
0	1
1	1
0	1
1	1

(b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)) \mathrel{+\!\!\!\!\!\longrightarrow} (C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)$; $(((C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)) \mathrel{+\!\!\!\!\!\longrightarrow} (C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)) = \mathbb{U}$

Figure 14. a) truth table for the law $((q_1 \to q_2) \wedge (q_2 \to q_3)) \to (q_1 \to q_3)$ and b) membership table for the set $((C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_2 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)) \mathrel{+\!\!\!\!\!\longrightarrow} (C_1 \mathrel{+\!\!\!\!\!\longrightarrow} C_3)$

4. Discussion and Perspectives

The importance given in the instructional approach proposed here to the structure of the subject matter presented, explains and justifies why reference is made to a contribution of the general systems theory of Ludwig von Bertalanffy [1], regarding the detection and use of isomorphisms between the laws and regularities of diverse areas of knowledge.

In a future article on this topic, a broader and more detailed characterization of this approach will be provided. In addition, in this and other papers clear examples of its usage will be given. This has been done here by pointing out certain correspondences between propositional calculus – the most basic level of logic – and set theory. In these articles, emphasis will be given to correspondences existing between propositional calculus, predicate calculus, set theory, Boole's algebra, and syllogistics, a historically and instructionally important discipline of logic.

Author Contributions:

Funding:

Conflicts of Interest:

References

1. Bertalanffy, L. (1968). *General System Theory: Foundations, Development, Applications*. George Braziller.
2. Grassmann, W. K. & J.-P. Tremblay (1996). *Logic and Discrete Mathematics*. Prentice Hall.
3. Quine, W. V. (1952; 1982). *Methods of Logic*, 4th ed. Harvard University Press.
4. Swaart, H. (2018). *Philosophical and Mathematical Logic*. Springer.
5. Jech, T. J. (2013). *Set Theory (The Third Millenium Edition)*. Springer.