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The Distribution of Prime Numbers Based on the Proof of Riemann's Hypothesis and the Properties of the Numbers 3, 6, and 9

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Abstract: In this paper, the distribution of prime numbers is expressed based on proving the Riemann hypothesis. The relationship between three numbers, three, six, and nine, and the modality to the distribution of prime numbers, is one of the results of Riemann's zeta function. Prime numbers are classified into six groups of single-digit numbers. There are no prime numbers in groups of three, six, and nine. The groups are made based on the sum of the internal digits. And for each set, there is an angle in the complex plane. The distance between the prime numbers in each group has a regular pattern. This pattern is a multiple of the numbers three, six, and nine. According to Euler's number, for an angle of 60 degrees, the real part of the cosine is 1/2. Accordingly, all prime numbers are related to angles greater than 60 degrees to 90 degrees. As a result, based on the relationship between the golden spiral and the complex conjugate of the zeta function, the function in The 1/2 point becomes zero based on the prime numbers.

Keywords: prime numbers; Riemann's hypothesis; three numbers3,6,9

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1. Introduction

Prime and composite numbers have been recognized for thousands of years. [1] There have been numerous methods described for studying the distribution of prime numbers. [2–4] Most of these studies show the complexity of the problem of the distribution of prime numbers. The distribution of prime numbers plays an important role in various scientific fields. [5] Also, the proof of the Riemann hypothesis is directly related to the distribution of prime numbers. The higher-dimensional proof of the Riemann hypothesis presents an effective method for prime number distribution. based on geometric and topological methods, defines the complex structure of prime numbers across dimensions, revealing a connection between primes and the underlying number fields. [6] The geometric approach to physics and mathematics problems, particularly in higher dimensions like general relativity, is a conventional method for explaining and solving various issues. [7–9]

Accordingly, in this study, we identify different groups of prime numbers. These groups have simple repeating patterns. The distribution patterns of prime numbers in each group establish a direct relationship with three numbers, three, six, and nine. Based on this, different groups of prime numbers can be identified. For example, the number 11 belongs to the number 2 group. Because the center of the field and the properties of the number are determined by the sum of its internal digits. Of course, it seems that there is no prime number in the group of six, three and nine. This paper delves into investigating the connection between the golden spiral and complex numbers within the complex plane. Furthermore, it examines distinct sets of prime numbers in the complex number plane by utilizing geometric relationships. Based on this, a method for the proof of Riemann's hypothesis is given.

2. Golden Spiral

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The equation for the eccentricity of an ellipse can be written based on trigonometric functions. (2.1)

$\sin(\theta) = sin\left(cos^{-1}\frac{a}{b}\right) = \sqrt{1-\frac{a^2}{b^2}}$ 2.1

The Fibonacci sequence causes the eccentricity. The different ratios of the sequence are directly related to the slope and growth rate of the golden spiral. (2.2) Figure1

$sin\left(cos^{-1}\frac{8}{13}\right) = \frac{\sqrt{105}}{13}$ $sin\left(cos^{-1}\frac{13}{21}\right) = \frac{4\sqrt{17}}{21}$ $sin\left(cos^{-1}\frac{21}{34}\right) = \frac{\sqrt{715}}{34}$ ... 2.2

A Circular sector equal to the radius rotates around the circumference of the ellipse along with the spiral and ellipse rotation. (2.3)

$\left(\frac{1}{2}\right)^2 2\pi r$ $\left(\frac{1}{2}\right)^3 4\pi r^2$ $\left(\frac{1}{2}\right)^4 2\pi^2 r^3$ $\left(\frac{1}{2}\right)^5 \frac{8}{3}\pi^2 r^4$ $\left(\frac{1}{2}\right)^6 \pi^3 r^5$

$\left(\frac{1}{2}\right)^6 \pi^3 r^5 \cong \ln(\varphi)$ 2.3

Different angles form right triangles in the area of the golden spiral. The hypotenuses of these triangles have certain ratios with each other. Figure1

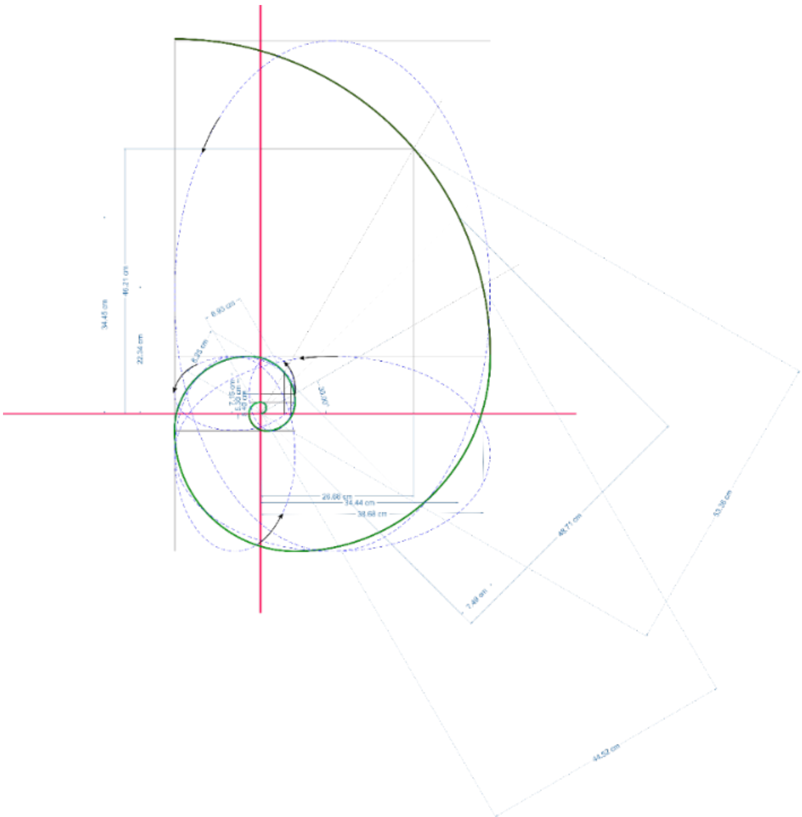

Figure 1. Golden spiral rotation and tangent ellipse, Also, the difference in degree in the second stage of the rotation is visible.

Based on eccentricity, simulations of the golden ratio can be tested. (2.4) The imaginary part of the new equation does not align with geometric rules. Naturally, this suggests the presence of higher dimensions. For instance, the eccentricity of an ellipse within a circle transforms an angle of 45 degrees into 26. Figure 2

$$\frac{1+\sqrt{5}}{2} = \varphi \Rightarrow \frac{1+\sqrt{-5}}{2} = 0.5 + 1.11803398875i$$

$$\frac{1+\sin\left(cos^{-1}\left(-\frac{8}{2}\right)\right)}{2} = 0.5 + 1.93649167i \Rightarrow cos^{-1}(0.5) = 60 \Rightarrow$$

$\sin(60) = \frac{\cot(45)}{\csc(60)} \Rightarrow sin^{-1}(1.9364..) = 90 - 73.309347i$ $\sin(73.309347) = 0.95786949627$

$1.11803398875i, 1.93649167i \not\equiv \theta, \phi$ $r = ae^{b\theta}$ $\tan(\alpha) = b \Rightarrow \alpha = 17.032..$

$\beta = 90 - \alpha = 73$ 2.4

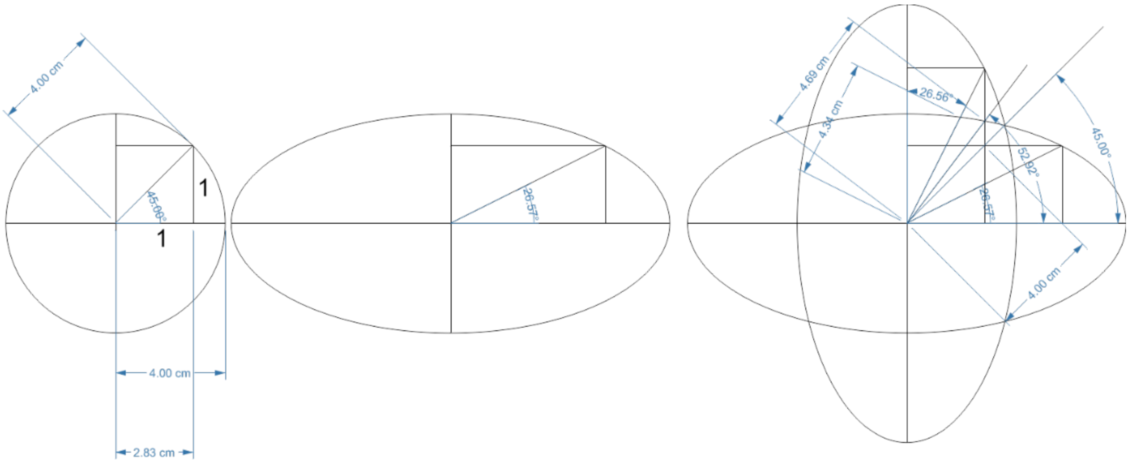

Figure 2. Creating symmetry after rotation and angle contraction in ellipse eccentricity.

3. Prime Numbers

We Define each prime number with an angle according to the definition of prime numbers. (3.1)

$11 \times 3 = 33, 3 \times 13 = 39 \Rightarrow (cos^{-1}(\frac{3}{33}))$- $(cos^{-1}\left(\frac{3}{39}\right) = 0.804$

$11 \times 5 = 55, 5 \times 13 = 65 \Rightarrow (cos^{-1}(\frac{5}{55}))$- $(cos^{-1}\left(\frac{5}{65}\right) = 0.804$

$(cos^{-1}\left(\frac{7}{77}\right) + i(sin^{-1}\left(\frac{7}{77}\right))) = 84.78 + 5.215i \Rightarrow 11(\cos(5.215) = 10.95 = \tan(84.78)$

$\Rightarrow sec^{-1}(2) = 60$, $(cos^{-1}\left(\frac{1}{2}\right) = 60$, $(cos^{-1}\left(\frac{1}{3}\right) = 70.528779..$ $(cos^{-1}\left(\frac{1}{11}\right) = 84.784091$ ...

$sec^{-1}(17) = 86.62771331657$ $sec^{-1}(11) = 84.78409142955$

$sec^{-1}(7) = 81.78678929826$, $sec^{-1}\left(\frac{180}{\pi}\right) = 1.85081571768$

$\sec\left((cos^{-1}\left(\frac{7}{77}\right)\right) = 11 \Rightarrow \boldsymbol{sec^{-1}(P_\mu) = \theta_{P_\mu}}$ $\boldsymbol{\mu = 1, 2, 3, 4, \dots, n}$ 3.1

Any angle can be expressed by Euler's number in the plane of complex numbers. (3.2)

$\Rightarrow e^{i\theta} = \cos(\theta) + i\left(\frac{tan\left(cos^{-1}\left(\frac{a}{b}\right)\right)}{\frac{b}{a}}\right) \Rightarrow e^{i\theta} = cos(\theta) + i(\frac{tan(\theta)}{sec(\theta)})$ 3.2

Based on this, according to the rotation angle of the golden spiral, and the definition of the golden ratio, we simulate the equation of the golden spiral. (3.3) This study is to investigate the connection of the eccentricity of the ellipse with the golden ratio. Figure3

$\cos(60) = 0.5 \Rightarrow e^{\tan\left(\frac{\ln(\varphi)}{\pi/3}\right)i} = 0.89626396199 + 0.4435210371i$

$\frac{1+\sqrt{-5}}{2} = 0.5 + 1.11803383i$

$e^{tan\left(\frac{ln\left(\frac{1+sin\,(cos^{-1}(-\frac{8}{2})}{2}\right)}{\pi/3}\right)i\phi} = \phi(0.9782063..+0.0112962342i)$ 3.3

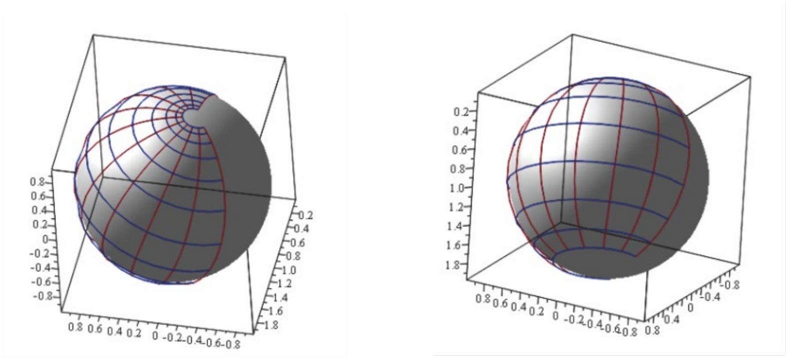

Figure 3. plot of a golden spiral in the 3D plane of the complex number.

4. Result

As a result of the introduction of prime numbers on the page of complex numbers, we group the numbers according to the definition of prime numbers based on different angles. First, we write each prime number based on five prime numbers in base two. (4.1) μ and υ are only counters, and also d represents the digits of prime numbers.

$sec^{-1}(2^{\upsilon}2^{\upsilon}..)\ \upsilon = 1,2,3,5,7 \Rightarrow P_{\mu} = d_1d_2d_3 \ldots$

$(7 + 4 = 11) \rightarrow (2^7 2^3 2^1) \Rightarrow sec^{-1}(2^2 2^9) - sec^{-1}(2^8 2^3) = 0$

$17\,,233 \Rightarrow\ 1 + 7 = 8, 2 + 3 + 3 = 8, \Rightarrow\ cos^{-1}(sec^{-1}(2^2 2^3 2^3)) = cos^{-1}(sec^{-1}(2^1 2^7))$

$61\,,151 \Rightarrow\ 6 + 1 = 7, 1 + 5 + 1 = 7, \Rightarrow\ cos^{-1}(sec^{-1}(2^1 2^5 2^1)) = cos^{-1}(sec^{-1}(2^6 2^1))$

$sec^{-1}(x) + csc^{-1}(x) = 90$ 4.1

Accordingly, every prime number belongs to a group. The sum of the internal digits of a number shows the number's dependence on the group. None of the prime numbers belongs to the group three six and nine. Also, the distance between the first two numbers in each group is an integer multiple of the numbers three, six and nine. (4.2) Table 1.

Table 1. Different groups of prime numbers, no number is a member of three, six and nine groups.

1	2	3	4	5	6	7	8	9
19	11		13	23		43	17	
37	29		31	41		61	53	
73	47		67	59		79	71	
109	83		103	113		97	89	
127	101		139	131		151	107	
163	137		157	149		223	179	
181	173		193	167		241	197	
199	191		211	239		277	233	
271	227		229	257		313	251	
307	263		283	293		331	269	
379	281		337	311		349	359	
397	317		373	347		367	431	
433	353		409	383			449	
487	389-443-461-479		463-499	401-419-491		421-439-457	467	

In groups, the distance between numbers always jumps, but all groups follow regular patterns. (4.2)

$1 \Rightarrow 37 - 19 = 18\,, 73 - 37 = 36\,, 109 - 73 = 36\,, 127 - 109 = 18$

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$2 \Rightarrow 29 - 11 = 18, 47 - 29 = 18, 83 - 47 = 36, 101 - 83 = 18$
$4 \Rightarrow 31 - 13 = 18, 67 - 31 = 36, 103 - 67 = 36,$
$5 \Rightarrow 41 - 23 = 18, 59 - 41 = 18, 113 - 59 = 54,$ 4.2
$(2^3 2^3 2^1) = 256 \Rightarrow P_{52} = 233 \rightarrow \Rightarrow 233 + 36 = 269 \ , 36 = 6.6 \rightarrow 233 + (6)239,$
$233 + (3 \times 6) = 251, 233 + (18 + 6) = 257, 233 + (18 + 12) = 263, \dots$

The sum of internal figures may be one stage or several stages. (4.3) Therefore, the distance between the numbers may be different. Table 2

$61 \rightarrow 6 + 1 = 7, 79 \longrightarrow 7 + 9 = 16 \Rightarrow 1 + 6 = 7 \Rightarrow 79 - 61 = 18$
$(2^3 2^3 2^1) = (2^6 2^1) = 128 \ (2^7 2^9) = 65536 \ \Rightarrow 128 \times 512 = 65536$
$\Rightarrow (2^7 2^3 2^3 2^3) \div (2^3 2^3 2^3) = 128$ 4.3

Table 2. The internal summation of the digits of a number to determine the group is one or more steps.

1	2	3	4	5	6	7	8	9
10	2		4	5		7	8	
10	11		4	5		7	8	
10	11		13	14		16	8	
10	11		4	5		16	17	
10	2		13	5		7	8	
10	11		13	14		7	17	
10	11		13	14		7	17	
19	11		4	14		16	8	
10	11		13	14		7	8	
10	11		13	14		7	17	
19	11		13	5		16	17	
19	11		13	14		16	8	
10	11		13	14		7	17	

On this basis, to study the gap between numbers, in a group, we eliminate different steps for internal summation. (4.4)

$1777 \rightarrow 1 + 7 + 7 + 7 = 22 \rightarrow sec^{-1}(2^1 2^7 2^7 2^7) \neq sec^{-1}(2^1 2^3)$
$\Rightarrow sec^{-1}(2^2 2^2) = sec^{-1}(2^1 2^3)$ 4.4

Due to the existence of two imaginary vectors, there is an equation with internal conjugate to construct prime numbers based on five prime numbers. (4.5)

$$\left(tan\left(cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right)^2 + \left(cot\left(cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right)^2 = 2 \Rightarrow (\sqrt{2} + i)(\sqrt{2} - i) = 3$$
$$\Rightarrow \big(bx + (a(y + iz)(y - iz))i\big)\big(bx - (a(y + iz)(y - iz))i\big) = P_n$$
$$x, y, z = \sqrt{0}, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, a, b = 1,2, \dots, n \quad 4.5$$

Mirror numbers are specified in each group. These numbers have special properties in a group. Table1

Based on this, it is suggested to identify and classify prime numbers in each group. There is a simple equation for each group. And also an equation to identify an N prime number is predictable.

According to the Riemann hypothesis, the zeta function is zero for the real part of 0.5. First, we rewrite the zeta function based on the defined angles. (4.6)

$$\zeta(s) = \frac{1}{1^{e^{\pm i\theta}}} + \frac{1}{2^{e^{\pm i\theta}}} + \frac{1}{3^{e^{\pm i\theta}}} + .. \quad 4.6$$

Two types of zeta functions are introduced due to right and left rounded angles. (4.7)

$$\zeta(e^{-i\theta}), \zeta(e^{i\theta}) \Rightarrow \zeta, \zeta^* \quad 4.7$$

Regarding the golden spiral and the golden ratio, it should be noted that the ratios are related to the ellipse's eccentricity. (4.8)

$$\zeta\zeta^* = sin^2\left(sec^{-1}(P_\mu)\right) \Rightarrow sec^{-1}(2^0) = sec^{-1}(e^0)$$
$$\Rightarrow sec^{-1}(2^0 2) = 60 \Rightarrow \cos(60) = \frac{1}{2} \quad 4.8$$

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$$\left(\sum_{n=1}^{\infty}\frac{1}{n^{(\frac{\pi}{3}+i\frac{2\pi}{3})}}\right)\left(\sum_{n=1}^{\infty}\frac{1}{n^{(\frac{\pi}{3}-i\frac{2\pi}{3})}}\right)=\sum_{n=1}^{\infty}\frac{1}{n(\zeta((\frac{1}{3}-\frac{2}{3}i)\pi))^{\frac{1}{3}\pi\left(\left(\zeta\left(\left(\frac{1}{3}-\frac{2}{3}i\right)\pi\right)\right)\right)+\frac{2}{3}i\pi\left(\left(\zeta\left(\left(\frac{1}{3}-\frac{2}{3}i\right)\pi\right)\right)\right)}}=0$$

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