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Quantum Measurement Problems, Decoherence, and Spontaneous Symmetry Breaking

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Quantum Measurement Problems, Decoherence, and Spontaneous Symmetry Breaking

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Abstract: In this paper, we propose a mechanism analogous to the spontaneous symmetry breaking combined with the quantum decoherence theory to explain the collapse of the wave function after the quantum measurement. We show that a wave function in a superposition of several eigenstates reduces to a single eigenstate due to the spontaneous-symmetry-breaking-like kinetic effect.

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1. Introduction

In quantum mechanics, the measurement problem is the problem of definite outcomes: quantum systems have superpositions as predicted by the Schrödinger equation but quantum measurements always give definite outcomes. To resolve the measurement problem and more generally explain the quantum-to-classical transition behavior, many objective collapse theories, including the Ghirardi–Rimini–Weber (GRW) model [1] and the continuous spontaneous localization (CSL) model [2], have been proposed. The GRW theory proposes that each constituent of a physical system independently undergoes a random "hit" on the order of once every hundred million years. In the CSL theory, the Schrödinger equation is supplemented with additional nonlinear and stochastic terms and the nonlinear modification induces the collapse of the wave function. Instead of introducing some vague concept of the unknown environmental degrees of freedom, Penrose (and Diósi, independently) suggested that the wave function collapse is induced by the gravity, the so-called DP model [3–6]. The wave function describing the state of a quantum system progressively loses its validity when the mass of the system becomes large enough. Although the DP model is the most influential model including gravity, it appears to have been ruled out in recent experiments [7].

All models listed so far cannot be described within quantum mechanics. On the other hand, quantum decoherence has been studied to understand how the classical world emerges from the quantum theory. In quantum mechanics, systems can exist in a superposition of states, described by a wave function that represents the probability amplitude of finding the system in each possible state. As long as there exists a definite phase relation between the components of the superposition, the system is said to be coherent and exhibits interference effects, as seen in the famous double-slit experiment. An isolated system always evolves according to unitary evolution and maintain coherence. But as soon as a system becomes entangled with its surroundings, the information about the relative phases between the quantum states leaks into the environment. This loss of information results in the destruction of quantum coherence, which in turn suppresses interference effects, the so-called environment-induced decoherence proposed by Zeh [8] (see Ref. [9–11] for a review). Quantum decoherence is a fundamental process that plays a crucial role in the transition from quantum to classical behavior. However, quantum decoherence does not describe the actual collapse of the wave function, but it explains the conversion of the quantum probabilities that exhibit interference effects to the ordinary classical probabilities.

Quantum phase transitions and spontaneous symmetry breaking (SSB) are fundamental concepts in quantum condensed matter physics, connecting microscopic quantum mechanics with macroscopic properties of materials. When the temperature approaches the quantum critical point, the states of the system might be effectively decoupled by a large energy barrier separating them. To interconvert between the different states and hence sample them in the ensemble average, we would need require

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to quantum mechanically tunnel through this large barrier. The wider and the higher the potential energy barrier separating two states, the longer it takes to quantum mechanically tunnel between them. The time scale for the tunneling is typically extremely long. It will take an infinitely long time to get to a different region of the phase space. Therefore, SSB occurs as the system gets stuck in a certain state.

In this paper, we argue that the basic origin of the wave function collapse is the same as the SSB. We propose a mechanism as a supplement to the decoherence theory to provide a solution to the measurement problem of quantum mechanics. Thus, quantum mechanics is itself applicable to the description of measurements.

This paper is organized as follows. In Section 2, we make a brief review of the decoherence theory. In Section 3, we demonstrate how the wave function collapses due to the spontaneous-symmetry-breaking-like kinetic effect. Finally, in Section 4 we summarize the main results obtained.

2. A Brief Review of Quantum Decoherence Theory

Let us first make a brief review of the decoherence theory. Consider the double-slit experiment and denote the quantum states of the particle (call it S, for "system") corresponding to passage through slit 1 and 2 by $|s_1\rangle$ and $|s_2\rangle$, respectively. Suppose that the particle interacts with a detector or an environment E such that if the quantum state of the particle before the interaction is $|s_1\rangle$, then the quantum state of E will become $|E_1\rangle$ (and similarly for $|s_2\rangle$). Then for an initial superposition state $\alpha|s_1\rangle + \beta|s_2\rangle$ the final composite state $|\Psi\rangle$ will be entangled. That is

$$|\Psi\rangle = \alpha\,|s_1\rangle\,|E_1\rangle + \beta\,|s_2\rangle\,|E_2\rangle\,. \tag{1}$$

For the composite state vector described by Eq. (1), the reduced density matrix of the particle can be written as

$$\rho_S = \mathrm{Tr}_E\,|\Psi\rangle\langle\Psi| = |\alpha|^2\,|s_1\rangle\,\langle s_1| + |\beta|^2\,|s_2\rangle\,\langle s_2| + \alpha\beta^*\,|s_1\rangle\,\langle s_2|\,\langle E_2 \mid E_1\rangle + \alpha^*\beta\,|s_2\rangle\,\langle s_1|\,\langle E_1 \mid E_2\rangle \tag{2}$$

Now suppose that we measure the particle's position by letting the particle impinge on a distant detection screen. Then the resulting particle probability density $P(x)$ is given by

$$P(x) = \mathrm{Tr}_S\,(\rho_S\hat{x}) = |\alpha|^2\,|\psi_1(x)|^2 + |\beta|^2\,|\psi_2(x)|^2 + 2\,\mathrm{Re}\,\{\alpha\beta^*\psi_1(x)\psi_2^*(x)\,\langle E_2 \mid E_1\rangle\} \tag{3}$$

where $\psi_a(x) \equiv \langle x|s_a\rangle$. The last off-diagonal term contains information about the relative phase of the particle and represents the interference contribution. Thus, the visibility of the interference pattern is quantified by the overlap $\langle E_2|E_1\rangle$, i.e., by the distinguishability of $|E_1\rangle$ and $|E_2\rangle$. Perfect distinguishability ($\langle E_2|E_1\rangle = 0$) leads to the loss of quantum coherence and the interference pattern vanishes. This describes the usual decoherence process. Physically, we interpret this as a flow of information from the system to the environment and hence information becomes delocalized. On the contrary, if the interaction between S and E is such that E is completely unable to resolve the path of the particle, then $|E_1\rangle$ and $|E_2\rangle$ are indistinguishable and full coherence is retained for the system S.

We arrive at the central result of the quantum decoherence that open systems are effectively described by diagonal reduced density matrices, generally called an "improper mixture". The "improper mixture" refers to that we view the pure state of the total entangled system as an effective mixed state for the subsystem while the "proper mixture" is an ensemble of pure states. Although they are defined differently, no physically realizable measurements can distinguish between a "proper mixture" ensemble and an "improper mixture" ensemble described by the same density matrix. Therefore, the time evolutions of the corresponding ensemble statistics are also indistinguishable.

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3. Wave Function Collapse and Spontaneous Symmetry Breaking

For simplicity, we still consider a two-level quantum system with eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$. According to decoherence theory, as the system becomes entangled with the environment, such as a measuring device, the off-diagonal elements of the reduced density matrix of the system vanish. The diagonal reduced density matrix is given by

$$\rho_S = |\alpha|^2 |\phi_1\rangle \langle\phi_1| + |\beta|^2 |\phi_2\rangle \langle\phi_2| . \tag{4}$$

Thus we are led to an expression for the expectation value of an observable (e.g., the position $\hat{x}$) of the two-level system:

$$\langle\hat{x}\rangle = \sum_{i=1}^{2} p_i \langle\phi_i|\hat{x}|\phi_i\rangle, \tag{5}$$

where $p_1 = |\alpha|^2$ and $p_2 = |\beta|^2$.

Indeed, to derive the expression for the expectation value of the observable, we have used the fact that the behavior averaged over time is the same as the behavior averaged over states in phase space at a given instant in time (known as the ensemble average). However, this approach is reasonable based on the validity of the Ergodic Principle, which states that all states of the system are accessible and eventually explored in the dynamical evolution of the system. The validity of the Ergodic Principle requires that different states in phase space are allowed to interconvert, and therefore we can sample all states in our ensemble average. However, the Ergodic Principle does not hold and the formula (5) of the ensemble average is incorrect for the case of the SSB. In quantum condensed matter physics, when the temperature is below the critical point, the states of the system might be effectively decoupled by a large energy barrier separating them. To interconvert between the different states and hence sample them in the ensemble average, we would need require to quantum mechanically tunnel through this large barrier. But it will take an infinitely long time to get to a different region of the phase space due to the lack of energy at low temperatures. The averages over a finite amount of time and therefore not necessarily equal to the averages over all states in phase space at an instant in time. In this case, the phase space becomes fragmented and we should compute our ensemble expectation values using only a part of the phase space.

Thus, the key point is whether different eigenstates of a quantum system can interconvert after quantum decoherence due to a measurement. Since the interference is suppressed as the system undergoes decoherence, the system becomes a classical object. As a result, it is impossible to get to a different eigenstate at the classical level. This means that for our two-level system, the phase space becomes fragmented and the particle in a mixed state gets stuck in a certain eigenstate $|\phi_1\rangle$ or $|\phi_2\rangle$ with the corresponding probability. Therefore, actual measurements always find the physical system in a definite state. In this sense, the "improper mixture" and usual "proper mixture" are fundamentally different.

4. Conclusions

In this paper, we model a quantum measurement within quantum theory and propose a mechanism similar to the SSB to provide possible explanations of how quantum measurements always give definite outcomes. It is actually a new supplementation of the quantum decoherence theory. Based on the results, this axiom, which states that the measurement of an observable yields one of its eigenvalues as the result, will be removed from the list of the "axioms of quantum mechanics". An important point about our results is whether the eigenstates of a system with continuous eigenvalues can interconvert. Assume that we measure the position of a free quantum particle, we cannot precisely locate it due to Heisenberg's uncertainty principle. This means that it still retains weak quantum coherence effects and has some nonzero probability of passing through the "potential barrier". If the wave function of this particle is a Gaussian wave packet after measurement, it would instantly spread out.

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