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The CMB Luminosity Distance and Cosmological Redshifts

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The CMB Luminosity Distance and Cosmological Redshifts

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Abstract: We will outline the relationship between luminosity distance and cosmological redshifts, demonstrating that it is consistent with a new cosmological model recently proposed by Haug and Tatum [1], which appears to resolve the Hubble tension within the $R_h = ct$ cosmology.

Keywords: luminosity distance; angular distance and co-moving distance

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1. Luminosity Distance Consistent with the Haug and Tatum Cosmological Model

According to the Stefan-Boltzmann law [2,3], the luminosity of a spherical black body is given by:

$$L = 4\pi R^2 \sigma T^4 \tag{1}$$

This must also hold true for a black hole, where $R = R_s$, meaning the radius is equal to the Schwarzschild radius. Solving for the radius, one gets:

$$D_L = R = \sqrt{\frac{L}{4\pi\sigma T^4}} \tag{2}$$

Furthermore, the flux density is given by:

$$F = \frac{L}{4\pi R^2} \tag{3}$$

This means we also have:

$$D_L = R = \sqrt{\frac{L}{4\pi F}} \tag{4}$$

This is also called the luminosity distance. Moreover, we must have:

$$\frac{F_t}{F_H} = \frac{R_t^2}{R_H^2} \tag{5}$$

where F_t is the flux density in a growing black hole in the $R_h = ct$ cosmological model at time t and at radius R_t, and F_H is the flux density.

Haug and Tatum have demonstrated that if $T_t = T_0(1+z)$, then in their type of $R_h = ct$ cosmology, one must have:

$$z = \sqrt{\frac{R_h}{R_t}} - 1 \tag{6}$$

Solved for R_h, this gives:

$$R_h = R_t(1+z)^2 \tag{7}$$

where R_t is the comoving transverse luminosity distance. This also means the luminosity distance must follow accordingly. We can rearrange this to:

$$\begin{aligned}
R_h &= R_t(1+z)^2 \\
R_h &= R_t(1+z)^2 \\
R_h &= R_t + 2R_t z + R_t z^2 \\
R_h - R_t &= 2R_h z + R_h z^2 \\
R_h - R_t &= 2\frac{c}{H_0}z + \frac{c}{H_0}z^2 \\
R_h - R_t &= \frac{2cz + cz^2}{H_0}
\end{aligned} \tag{8}$$

We define $R_h - R_t = D$, which is the distance to the object emitting photons toward us, so we have:

$$D = \frac{2cz + cz^2}{H_0} \tag{9}$$

This is the same formula that Haug and Tatum have presented, namely:

$$D = \frac{c}{H_0}\left(1 + \frac{1}{(1+z)^2}\right) = \frac{2cz + cz^2}{H_0} \tag{10}$$

The well-known Etherington equation gives the relationship between the luminosity distance of standard candles and the angular diameter distance, which is given by:

$$D_L = (1+z)^2 D_A \tag{11}$$

Here, D_A represents the angular-diameter distance. We can see that this bears close similarities to our equation (7). We have already proven that the luminosity distance in our $R_h = ct$ cosmology is identical to R_h (at present), and that before the present time it corresponds to R_t. This implies that we have essentially demonstrated the angular diameter distance is identical to the luminosity distance in the Haug and Tatum cosmology model, which assumes $z = \sqrt{\frac{R_h}{R_t}} - 1$. However, this should naturally be studied carefully over time. It is also worth mentioning that there is a different variation of the Haug and Tatum cosmology, where they propose $z = \frac{R_h}{R_t} - 1$, which would yield different results.

2. The Mattig Formula

The Mattig [4] formula (see also [5]) is used to calculate the luminosity distance in terms of redshift and is given by:

$$R = \frac{c}{H_0}\frac{q_0 z + (q_0 - 1)(\sqrt{1 + 2q_0 z} - 1)}{q_0^2(1+z)} \tag{12}$$

If we set $q_0 = 1$, we obtain:

$$d = R = \frac{cz}{H_0(1+z)} \tag{13}$$

This is the same distance as given by the Haug and Tatum model when they assume $z = \frac{R_h}{R_t} - 1$. The first-term Taylor series expansion of this is the well-known distance formula often used in the Λ-CDM model:

$$d \approx \frac{cz}{H_0} \tag{14}$$

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3. Speculative Equivalent Mattig Formula for $z = \sqrt{\frac{R_h}{R_t}} - 1$ Scaling

We are suggesting a speculative modification to the Mattig [4] formula in an attempt to make it consistent with $z = \sqrt{\frac{R_h}{R_t}} - 1$. This is the only formula in this paper that is not strictly based on derivation, so it should simply be regarded as an initial guess. The idea is that the Mattig formula may require modification to accommodate the redshift scaling $z = \sqrt{\frac{R_h}{R_t}} - 1$. Based on a qualified assumption, we propose the following:

$$d = \frac{c}{H_0} \frac{2q_0 z + z^2 q_0^2 + (q_0 - 1)(\sqrt{1 + 2q_0 z} - 1)}{q_0^2 (1+z)^2} \tag{15}$$

When $q_0 = 1$, we obtain:

$$d = \frac{2cz + cz^2}{H_0(1+z)^2} \tag{16}$$

This is equivalent to the distance Haug and Tatum [1] presented when they assume $z = \sqrt{\frac{R_h}{R_t}} - 1$. Naturally, someone should derive an equivalent Mattig formula from scratch as our speculative suggestion in this section could be wrong as it not is based on derivations from first principles.

4. Conclusions

The Haug-Tatum cosmological model, in which they propose the redshift scaling $z = \sqrt{\frac{R_h}{R_t}} - 1$, leads to a comoving luminosity distance relationship of $R_h = R_t(1+z)^2$, as was already indirectly suggested in their paper. We have pointed out that the angular diameter distance in their model appears to be identical to this. In our $R_h = ct$ cosmological model, it seems that many types of cosmological distances converge to the same value. We suspect that some of the many different types of cosmological distances in the Λ-CDM model may result from an overly complex framework that does not represent the cosmos as accurately.

Conflicts of Interest: The author declare no conflict of interest.

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