

Additional materials to the article

On the issue of assessing the accuracy of frequency standards

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Abstract. Additional materials present the results of modeling the calculation of the Allan function in the VisSim program .

Key words: accuracy, stability, error, frequency standard, Allan variance, Allan function, reproducibility

Simulation results

Modeling in the *VisSim program* verified some theoretical assumptions that only a function changing monotonically in one direction, for example, a linear drift, can be characterized by a graph of the Allan function increasing with growth τ . It was experimentally demonstrated that all other types of noise give a decrease in the Allan function with growth τ .

In modeling, the average frequency value over an interval τ can be calculated as the result of integration over that interval, divided by the value τ .

The average value on the subsequent section, separated by a time interval equal to τ , can be calculated in the same way. To obtain the difference between two average values on two intervals separated by an interval τ , it is sufficient to have two channels, to one of which the input signal is fed without delay, and to the other channel it can be calculated by delay, and to the other this signal is fed with a delay of τ . Then this difference can be integrated over the interval τ , or you can first perform integration in each channel and then calculate the difference.

For squaring, multiplication and addition, you can use the corresponding blocks that implement these functions.

To store the result, a sample-and-hold device should be used, which stores the signal in the form of a series of awl-shaped pulses. The interval between pulses and the initial phase can be set by adjusting the corresponding parameters of these elements.

The calculation result is displayed on a digital display. Two oscilloscope windows are also used, one of which displays the input signal of the calculator, and the other displays the result of the Allan function calculation. At first, the measurement result is unreliable, since at least a time equal to the doubled averaging interval must pass to calculate the average frequency increment, i.e. 2τ .

As statistics accumulate, the calculation result tends to some fixed value. While the result on the second graph (bottom left window) is changing, this indicates that the statistical data is insufficient. After the graph in this window has stopped changing significantly, we can conclude that the statistical data is sufficient to indicate the Allan function value quite accurately. The amplitude of the oscillations of this function demonstrates the error in its calculation at a given point in time. The results are shown in the following figures.

Figure A1 shows a project in the *VisSim program* for automatic calculation of the Allan function for a signal of any shape, including a signal from the noise generator output. In this project, the signal is averaged by integrating it over a given time interval with subsequent division of the result by the duration of the interval. Integrators are designated by blocks marked with the integration symbol in the Laplace transform domain, that is, the symbol $1/s$. The value is stored using a sample-and-hold device designated in the diagram by a block with the symbols S&H. The signal switches are designated by the symbols merge, the project also has adders designated by circles with the symbol Σ , squaring blocks designated by the symbols pow, a division block designated by the symbol $/$, and scaling factor blocks with the coefficient value indicated inside the pentagonal symbol, there are also signal delay devices, that is, pure delay links, which are designated by the symbol e^{-sT^d} and bus marks in the form of rectangles with a mark symbol in the form of a Latin letter with a number. All buses with the same marks are considered to be connected to each other. The right side of the graph shows the signal under study, and the bottom left side shows the result of calculating the Allan function. It is clear that while the number of measurements is insufficient, this function undergoes changes, but then, with an increase in the number of accumulated readings involved in averaging, the graph converges to its final value, the right side of this graph undergoes very insignificant deviations from the established value.

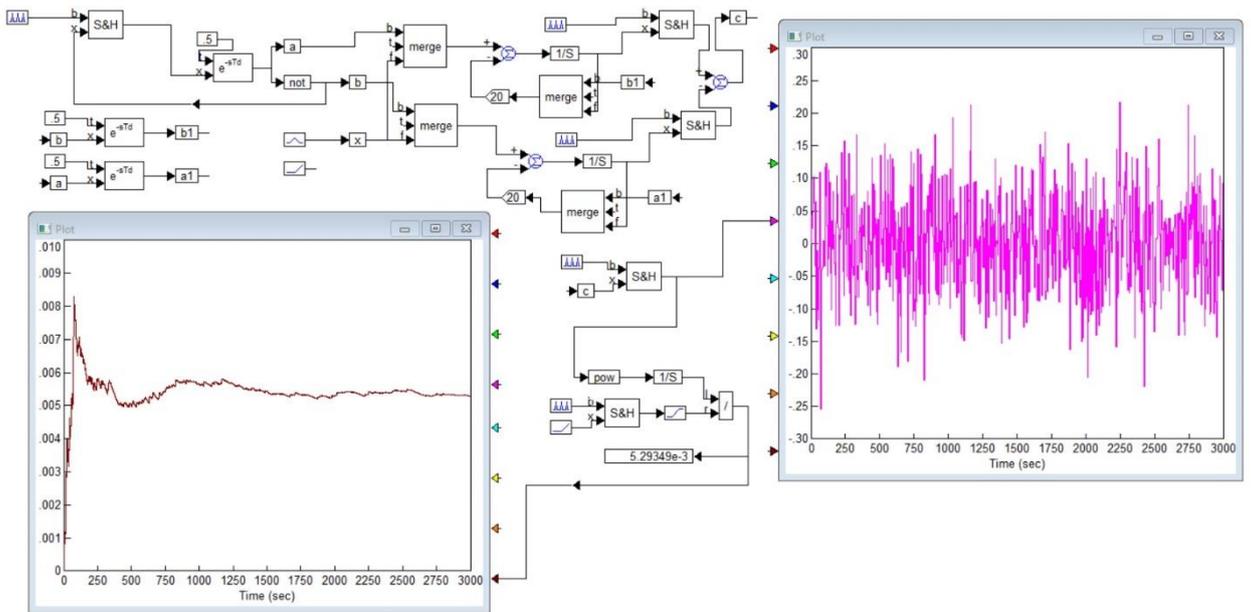


Figure A1. Project for calculating the Allan function of various types of time function representing the frequency changes of the estimated oscillator

In Figure A1, the right window shows not the original signal, which is Gaussian noise, but the result of calculating the differences between the subsequent and previous values of the average frequency at two successive measurement intervals. In all cases, the duration of the measurement interval is taken to be 0.5 seconds. As we can see, the Gaussian noise, in which subsequent measurement results differ by up to 0.25 units (the spread of this value can be easily estimated from the graph on the right), the Allan function tends to the value $F_A(0.5) = 5.29 \cdot 10^{-3}$. The change in this function is visible in the graph on the left, and the final value can be read from the corresponding display at the bottom in the center of the figure.

Figure A2 shows the design for computing the Allan function and the results of computing the Allen variance of a linearly increasing function. The rate of increase is 1, i.e. $y(t) = t \cdot F_A(0.5) = 2.49 \cdot 10^{-3}$.

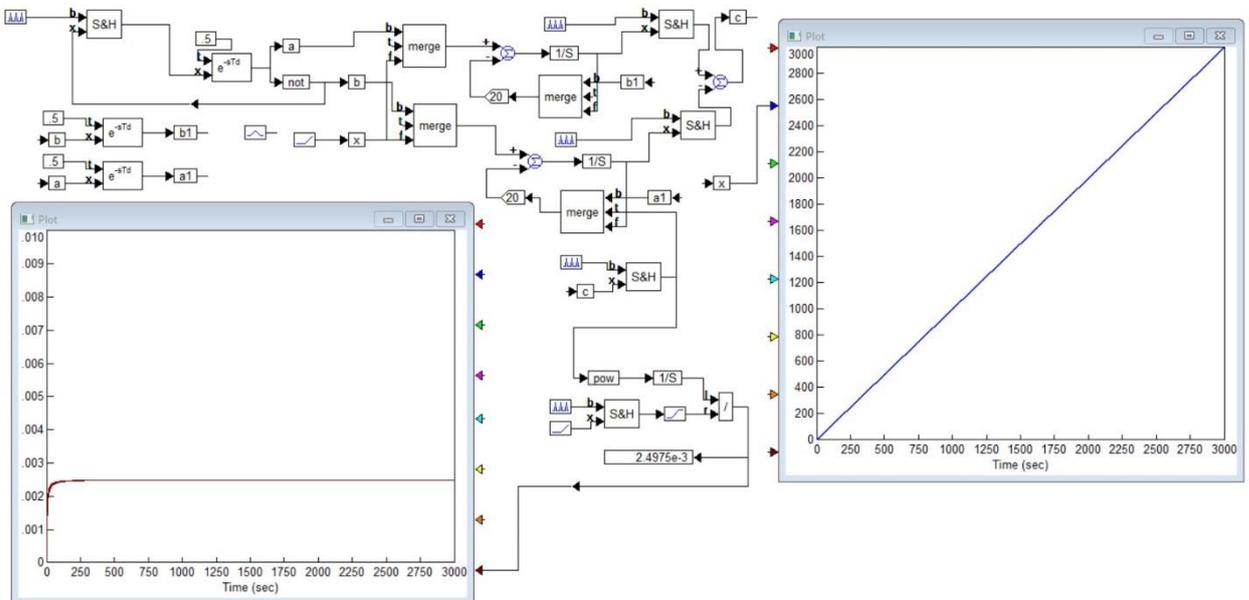


Figure A2. Project for calculating the Allan function: results of calculating the Allen variance of a linearly increasing function

Figure A3 shows the project for calculating the Allan function and the results of calculating the Allen variance of the harmonic function. The duration of the oscillation period is 64 seconds. The result is: $F_A(0.5) = 1.29 \cdot 10^{-5}$.

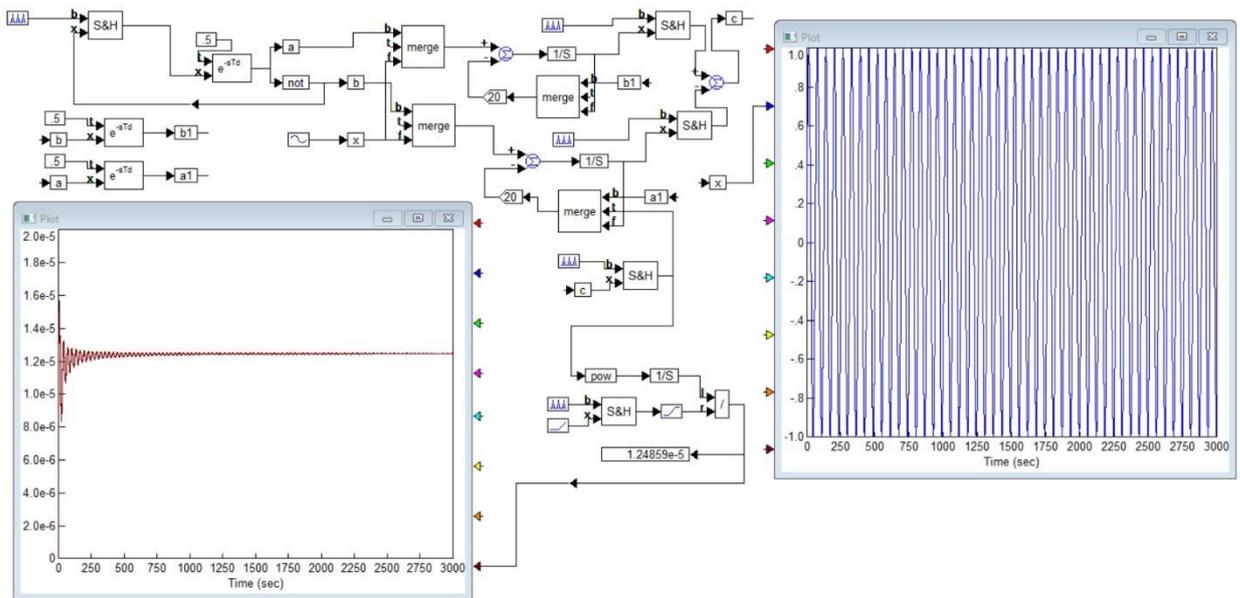


Figure A3. Results of calculating the Allen variance of a harmonic function with a period of 64 seconds.

Figure A4 shows the project for calculating the Allan function and the results of calculating the Allen variance of the harmonic function. The duration of the oscillation period is 6400 seconds. The result is: $F_A(0.5) = 1.19 \cdot 10^{-9}$.

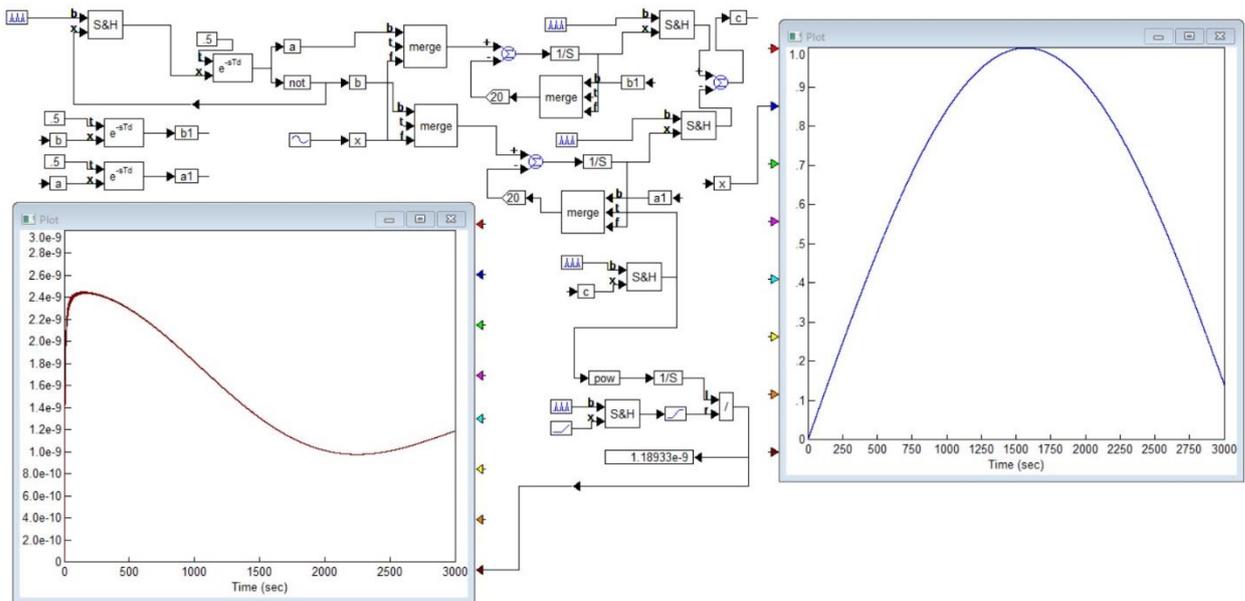


Figure A4. Results of calculating the Allen variance of a harmonic function with a period of 6400 seconds

Figure A5 shows the project for calculating the Allan function and the results of calculating the Allen variance of the harmonic function. The duration of the oscillation period is 625 seconds. The result is: $F_A(0.5) = 1.24 \cdot 10^{-7}$.

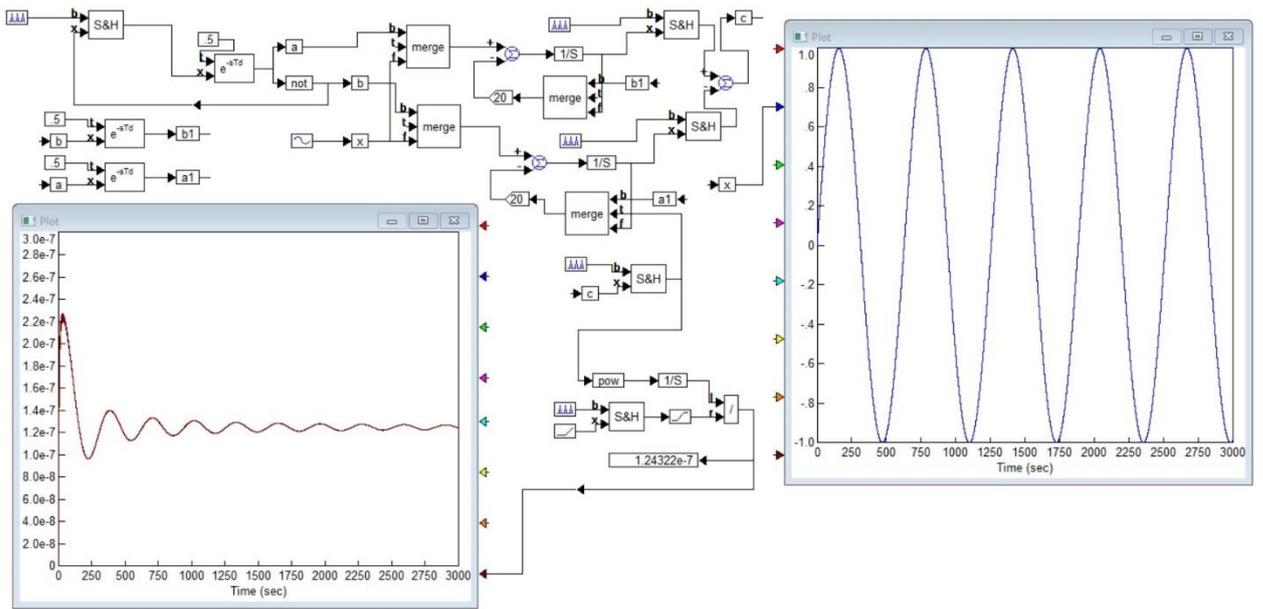


Figure A5. Results of calculating the Allen variance of a harmonic function with a period of 625 seconds

