P-cages from Mozaic Graphs

B. Piette and Árpad Lukács

1 Graphs

In this supplementary material we consider the p-cages generated from 3 graphs which lead only to very deformed p-cages. The graphs considered are

p-cage name	graph name	Description
`TTM3`	**56_F44_4-0-3_4-3-0_12-2-1_24-1-2_V12_12**	Truncated tetrahedron where the hexagons become 3-mosaic.
`TOM3`	**56_F86_6-0-4_8-3-0_24-2-1_48-1-2_V24_24**	Truncated octahedron where the hexagons are 3-mosaic.
`TCM4`	**56_F86_6-4-0_8-0-3_24-2-1_48-1-2_V24_24**	Truncated cube where the octagons are 4-mosaic.

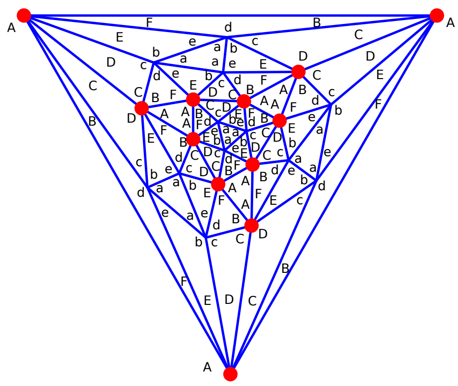
Table 1: Bi-symmetric hole polyhedron graphs with valency 5 and 6 nodes as well as triangular and square faces and which include mosaics subgraphs[Piette2024]. The first column is the label describing the p-cages derived from the graph, the second column is the label for the graph used in [Piette2024] and the third column describes a solid for which the planar graph corresponds to the hole polyhedron graph.

The hole-edge mappings for these graphs are presented on Figure 1 for the `TTM3` p-cages and on Figure 2.a and 2.b for respectively the `TOM3` and `TCM4` p-cages.

2 Parametrisation

In what follows we use the same notations as in the main paper: $R_{\boldsymbol{w}}(\theta)$ denotes a rotation of angle θ around the vector $\boldsymbol{w}$ while $R_x(\theta)$, $R_y(\theta)$ and $R_z(\theta)$ correspond to rotations of an angle θ around respectively the x, y and z axes.

$\boldsymbol{u}_{j_1,i;j_2,k}$ is a unit length vector parallel to the intersection line between face i of type j_1 and face k of type j_2 . $\boldsymbol{Q}_{j_1,i;j_2,k}$ is the intersection point between the reference face of type 2, face i of type j_1 and face k of type j_2 . $\boldsymbol{P}_{j_1,i;j_2,k}$ is the intersection point between the reference face of type 1, the type j_1 face i and type j_2 face k.



a b

Figure 1: Hole edges mapping for the hole polyhedron graphs: a) **56_F44_4-0-3_4-3-0_12-2-1_24-1-2_V12_12**

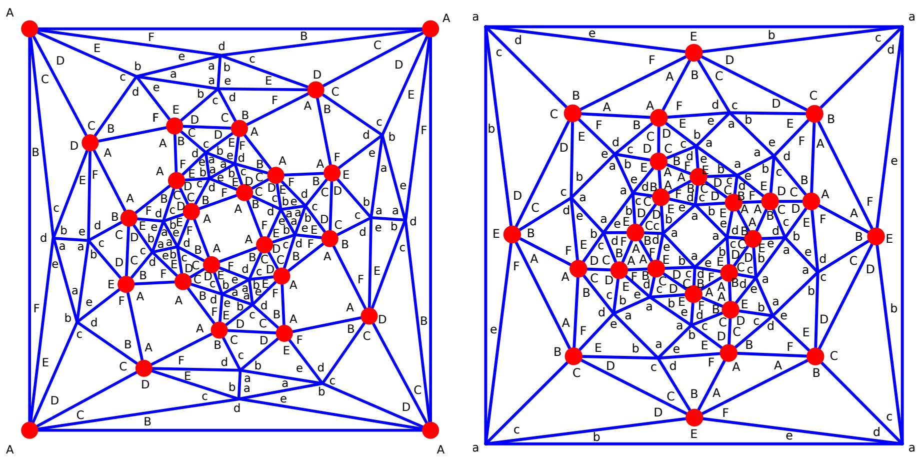


Figure 2: Hole edges mapping for the hole polyhedron graphs: a) **56_F86_6-0-4_8-3-0_24-2-1_48-1-2_V24_24** b) **56_F86_6-4-0_8-0-3_24-2-1_48-1-2_V24_24**

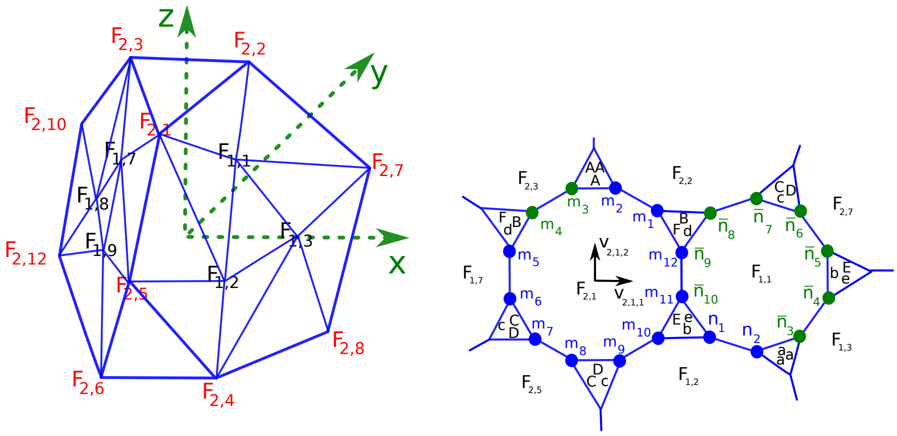


Figure 3: Parametrisation of the truncated tetrahedron with a 3-mosaic p-cage. a) Truncated tetrahedron with a 3-mosaic vectors. b) Mapping of vertices.

2.1 TTM3

The underlying symmetry of these p-cages is that of the truncated tetrahedron. We orient the truncated tetrahedron so that the centre of the top triangle face is in the direction $(0, 0, 1)$ and the centre of the side hexagon is in the direction $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$. This is illustrated in figure 3.

The vertices of the underlying tetrahedron are

$$\begin{aligned} \boldsymbol{G}_1 &= (0,0,1), & \boldsymbol{G}_2 &= \left(0, -\frac{\sqrt{8}}{3}, -\frac{1}{3}\right) \\ \boldsymbol{G}_3 &= \left(\sqrt{\frac{2}{3}}, \frac{\sqrt{2}}{3}, -\frac{1}{3}\right), & \boldsymbol{G}_4 &= \left(-\sqrt{\frac{2}{3}}, \frac{\sqrt{2}}{3}, -\frac{1}{3}\right) \end{aligned} \tag{1}$$

The centre of the top three faces of the tetrahedron, also the centre of the top three hexagons, are

$$\boldsymbol{H}_1 \left(\sqrt{\frac{2}{27}}, -\frac{\sqrt{2}}{9}, \frac{1}{9}\right),, \qquad \boldsymbol{H}_2 = R_z\left(\frac{4\pi}{3}\right)\boldsymbol{H}_1, \qquad \boldsymbol{H}_3 = R_z\left(\frac{2\pi}{3}\right)\boldsymbol{H}_1. \tag{2}$$

Defining

$$\boldsymbol{e}_t = (\sin\left(\frac{\pi}{3}\right), \cos\left(\frac{\pi}{3}\right), 0) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right). \tag{3}$$

the rotations linking the faces are then as follows:

$$
\begin{aligned}
&\mathcal{R}_{2,2} = R_z(\frac{2\pi}{3}), &&\mathcal{R}_{2,3} = R_z(\frac{4\pi}{3}), && \\
&\mathcal{R}_{2,3+i} = R_{\boldsymbol{H}_1}(\frac{2\pi}{3}), &&\mathcal{R}_{2,6+i} = R_{\boldsymbol{H}_1}(\frac{4\pi}{3}), &&\mathcal{R}_{2,9+i} = R_{\boldsymbol{H}_2}(\frac{2\pi}{3}), \quad i = 1\dots 3, \\
&\mathcal{R}_{1,1} = R_{\boldsymbol{e}_t}(\sigma), &&\mathcal{R}_{1,2} = R_{\boldsymbol{H}_1}\left(\frac{2\pi}{3}\right), &&\mathcal{R}_{1,3} = R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right), \\
&\mathcal{R}_{1,3+i} = R_z\left(\frac{2\pi}{3}\right), &&\mathcal{R}_{1,6+i} = R_z\left(\frac{4\pi}{3}\right), &&\mathcal{R}_{1,9+i} = R_{\boldsymbol{G}_2}\left(\frac{4\pi}{3}\right), \quad i = 1\dots 3.
\end{aligned} \tag{4}
$$

We then take as basis vectors of the reference faces

$$
\begin{aligned}
&\boldsymbol{V}_{1,1} = S_1\, R_{\boldsymbol{H}_1}(\tau)\, R_{\boldsymbol{e}_t}(\sigma)\, \boldsymbol{H}_1, && \\
&\boldsymbol{v}_{1,1,1} = R_{\boldsymbol{H}_1}(\tau)\, R_{\boldsymbol{e}_t}(\sigma)\, \boldsymbol{e}_t, &&\boldsymbol{v}_{1,1,2} = R_{\boldsymbol{H}_1}(\tau)\, R_{\boldsymbol{e}_t}(\sigma)\left(-\frac{1}{2\sqrt{3}}, \frac{1}{6}, \frac{2\sqrt{2}}{3}\right), \\
&\boldsymbol{V}_{2,1} = S_2\, R_z(\phi)\,(0, -\cos(\theta), \sin(\theta)), && \\
&\boldsymbol{v}_{2,1,1} = R_z(\phi)(1,0,0), &&\boldsymbol{v}_{2,1,2} = R_z(\phi)\,(0, \sin(\theta), \cos(\theta))\,.
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&\boldsymbol{m}_1 = Q_{1,1;2,2} + k_1 \boldsymbol{u}_{2,1;2,2}, &&\boldsymbol{m}_2 = Q_{1,1;2,2} + k_2 \boldsymbol{u}_{2,1;2,2}, \\
&\boldsymbol{m}_3 = R_{\boldsymbol{G}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_2, &&\boldsymbol{m}_4 = R_{\boldsymbol{G}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_1, \\
&\boldsymbol{m}_5 = Q_{2,3;1,7} + k_5 \boldsymbol{u}_{2,1;1,7}, &&\boldsymbol{m}_6 = Q_{2,3;1,7} + k_6 \boldsymbol{u}_{2,1;1,7}, \\
&\boldsymbol{m}_7 = Q_{1,7;2,5} + k_7 \boldsymbol{u}_{2,1;2,5}, &&\boldsymbol{m}_8 = Q_{1,7;2,5} + k_8 \boldsymbol{u}_{2,1;2,5}, \\
&\boldsymbol{m}_9 = Q_{2,5;1,2} + k_9 \boldsymbol{u}_{2,1;1,2}, &&\boldsymbol{m}_{10} = Q_{2,5;1,2} + k_{10} \boldsymbol{u}_{2,1;1,2}, \\
&\boldsymbol{m}_{11} = Q_{1,2;1,1} + k_{11} \boldsymbol{u}_{2,1;1,1}, &&\boldsymbol{m}_{12} = Q_{1,2;1,1} + k_{12} \boldsymbol{u}_{2,1;1,1}, \\
&\boldsymbol{n}_1 = P_{2,1;1,2} + k_{13} \boldsymbol{u}_{1,1;1,2}, &&\boldsymbol{m}_2 = P_{2,1;1,2} + k_{14} \boldsymbol{u}_{1,1;1,2}, \\
&\boldsymbol{n}_3 = R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{n}_2, &&\boldsymbol{n}_4 = R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{n}_1, \\
&\boldsymbol{n}_5 = R_{\boldsymbol{H}_1}\left(-\frac{2\pi}{3}\right) R_z \boldsymbol{m}_{10}, &&\boldsymbol{n}_6 = R_{\boldsymbol{H}_1}\left(-\frac{2\pi}{3}\right) R_z \boldsymbol{m}_9, \\
&\boldsymbol{n}_7 = R_z\left(\frac{2\pi}{3}\right)\boldsymbol{m}_6, &&\boldsymbol{n}_8 = R_z\left(\frac{2\pi}{3}\right)\boldsymbol{m}_5, \\
&\boldsymbol{n}_9 = \boldsymbol{m}_{12}, &&\boldsymbol{n}_{10} = \boldsymbol{m}_{11},
\end{aligned} \tag{6}
$$

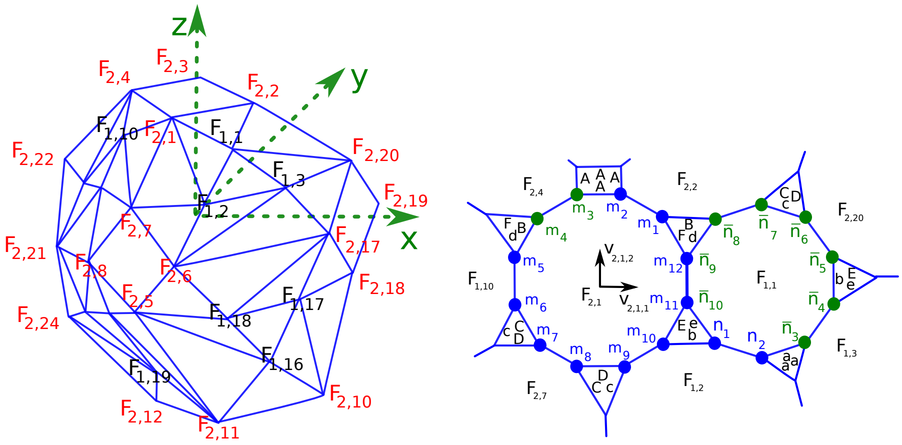


Figure 4: Parametrisation of the truncated octahedron with a 3 mosaic p-cage. a) truncated octahedron vectors. b) Mapping of vertices.

The optimization parameters are θ, ϕ, σ, τ, S_1, S_2, k_1, k_2, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14} as well as the planar coordinates of the non-shared vertices for both reference faces.

2.2 TOM3

We orient the truncated octahedron so that the centre of the top square face is in the direction $(0, 0, 1)$ and the centre of the side hexagon in the direction $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$. This is illustrated in figure 4.

The symmetry axes of the truncated octahedron are the centre of the face of the underlying cube

$$\begin{aligned} \boldsymbol{G}_1 &= (0,0,1), & \boldsymbol{G}_2 &= (0,0,-1) & \boldsymbol{G}_3 &= (1,0,0), \\ \boldsymbol{G}_4 &= (-1,0,0) & \boldsymbol{G}_5 &= (0,1,0), & \boldsymbol{G}_6 &= (0,-1,0). \end{aligned} \tag{7}$$

The centres of the hexagonal faces are then

$$\begin{aligned} &\boldsymbol{H}_1 \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), && \boldsymbol{H}_2 = R_z\left(\frac{\pi}{2}\right)\boldsymbol{H}_1 && \boldsymbol{H}_3 = R_z(\pi)\,\boldsymbol{H}_1 \\ &\boldsymbol{H}_4 = R_z\left(\frac{3\pi}{2}\right)\boldsymbol{H}_1 && \boldsymbol{H}_{4+i} = R_x(\pi)\,\boldsymbol{H}_i \quad i = 1\ldots 4. \end{aligned} \tag{8}$$

Defining

$$\boldsymbol{e}_{xy} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \tag{9}$$

the rotations linking the faces are then as follows:

$$\begin{aligned}
\mathcal{R}_{2,2} &= R_z(\frac{\pi}{2}), & \mathcal{R}_{2,3} &= R_z(\pi), & \mathcal{R}_{2,4} &= R_z(\frac{3\pi}{2}), \\
\mathcal{R}_{2,4+i} &= R_x(\frac{\pi}{2}), & \mathcal{R}_{2,8+i} &= R_x(\pi), & \mathcal{R}_{2,12+i} &= R_x(\frac{3\pi}{2}), \quad i = 1\ldots 4, \\
\mathcal{R}_{2,16+i} &= R_y(\frac{\pi}{2}), & \mathcal{R}_{2,20+i} &= R_y(\frac{3\pi}{2}), & i &= 1\ldots 4, \\
\mathcal{R}_{1,2} &= R_{\boldsymbol{H}_1}\left(\frac{2\pi}{3}\right), & \mathcal{R}_{1,3} &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right), \\
\mathcal{R}_{1,3+i} &= R_z\left(\frac{\pi}{2}\right), & \mathcal{R}_{1,6+i} &= R_z\left(\pi\right), & i &= 1\ldots 4, \\
\mathcal{R}_{1,9+i} &= R_z\left(\frac{3\pi}{2}\right), & \mathcal{R}_{1,12+i} &= R_x\left(\pi\right), & i &= 1\ldots 12.
\end{aligned} \tag{10}$$

We then take

$$\begin{aligned}
\boldsymbol{V}_{1,1} &= S_1\, R_{\boldsymbol{H}_1}\left(\tau\right) R_{\boldsymbol{e}_{xy}}\left(\sigma\right) \boldsymbol{H}_1, & \boldsymbol{v}_{1,1,1} &= R_{\boldsymbol{H}_1}\left(\tau\right) R_{\boldsymbol{e}_{xy}}\left(\sigma\right) \boldsymbol{e}_{xy}, \\
\boldsymbol{v}_{1,1,2} &= R_{\boldsymbol{H}_1}\left(\tau\right) R_{\boldsymbol{e}_{xy}}\left(\sigma\right) \left(-\frac{\sqrt{2}}{\sqrt{3}}, -\frac{1}{\sqrt{(6)}}, \frac{1}{\sqrt{(6)}}\right), \\
\boldsymbol{V}_{2,1} &= S_2\, R_z(\phi)\left(0, -\cos(\theta), \sin(\theta)\right), & \boldsymbol{v}_{2,1,1} &= R_z(\phi)(1,0,0), \\
\boldsymbol{v}_{2,1,2} &= R_z(\phi)\left(0, \sin(\theta), \cos(\theta)\right).
\end{aligned} \tag{11}$$

$$\begin{aligned}
\boldsymbol{m}_1 &= Q_{1,1;2,2} + k_1 \boldsymbol{u}_{2,1;2,2}, & \boldsymbol{m}_2 &= Q_{1,1;2,2} + k_2 \boldsymbol{u}_{2,1;2,2}, \\
\boldsymbol{m}_3 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right) \boldsymbol{m}_2, & \boldsymbol{m}_4 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right) \boldsymbol{m}_1, \\
\boldsymbol{m}_5 &= Q_{2,4;1,10} + k_5 \boldsymbol{u}_{2,1;1,10}, & \boldsymbol{m}_6 &= Q_{2,4;1,10} + k_6 \boldsymbol{u}_{2,1;1,10}, \\
\boldsymbol{m}_7 &= Q_{1,10;2,7} + k_7 \boldsymbol{u}_{2,1;2,7}, & \boldsymbol{m}_8 &= Q_{1,10;2,7} + k_8 \boldsymbol{u}_{2,1;2,7}, \\
\boldsymbol{m}_9 &= Q_{2,7;1,2} + k_9 \boldsymbol{u}_{2,1;1,2}, & \boldsymbol{m}_{10} &= Q_{2,7;1,2} + k_{10} \boldsymbol{u}_{2,1;1,2}, \\
\boldsymbol{m}_{11} &= Q_{1,2;1,1} + k_{11} \boldsymbol{u}_{2,1;1,1}, & \boldsymbol{m}_{12} &= Q_{1,2;1,1} + k_{12} \boldsymbol{u}_{2,1;1,1}, \\
\boldsymbol{n}_1 &= P_{2,1;1,2} + k_{13} \boldsymbol{u}_{1,1;1,2}, & \boldsymbol{m}_2 &= P_{2,1;1,2} + k_{14} \boldsymbol{u}_{1,1;1,2}, \\
\boldsymbol{n}_3 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right) \boldsymbol{n}_2, & \boldsymbol{n}_4 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right) \boldsymbol{n}_1, \\
\boldsymbol{n}_5 &= R_y\left(\frac{\pi}{2}\right) R_z\left(\pi\right) \boldsymbol{m}_{10}, & \boldsymbol{n}_6 &= R_y\left(\frac{\pi}{2}\right) R_z\left(\pi\right) \boldsymbol{m}_9, \\
\boldsymbol{n}_7 &= R_y\left(\frac{\pi}{2}\right) \boldsymbol{m}_6, & \boldsymbol{n}_8 &= R_y\left(\frac{\pi}{2}\right) \boldsymbol{m}_5, \\
\boldsymbol{n}_9 &= \boldsymbol{m}_{12}, & \boldsymbol{n}_{10} &= \boldsymbol{m}_{11},
\end{aligned} \tag{12}$$

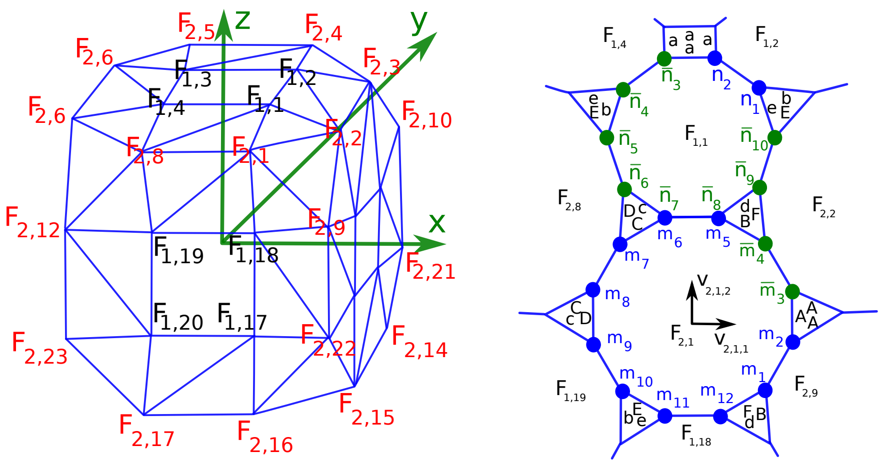


Figure 5: Parametrisation of the truncated cube with a 4 mosaic p-cage. a) Vectors. b) Mapping of vertices.

The optimization parameters are θ, ϕ, σ, τ, S_1, S_2, k_1, k_2, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14} as well as the coordinates in the plane of the non-shared vertices for both reference faces.

2.3 TCM4

We orient the truncated cube so that the octagons are in planes orthogonal to the x, y and z axes. This is illustrated in figure 5.

The symmetry axes of the truncated cube are the centres of the faces of the cube

$$\begin{aligned} &\boldsymbol{G}_1 = (0,0,1), &&\boldsymbol{G}_2 = (0,0,-1) &&\boldsymbol{G}_3 = (1,0,0), \\ &\boldsymbol{G}_4 = (-1,0,0), &&\boldsymbol{G}_5 = (0,1,0), &&\boldsymbol{G}_6 = (0,-1,0), \end{aligned} \tag{13}$$

as well as the corners of the cube

$$\begin{aligned} &\boldsymbol{H}_1 = (1,-1,1), &&\boldsymbol{H}_2 = (1,1,1), &&\boldsymbol{H}_3 = (-1,1,1), &&\boldsymbol{H}_4 = (-1,-1,1), \\ &\boldsymbol{H}_5 = (1,-1,-1), &&\boldsymbol{H}_6 = (1,1,-1), &&\boldsymbol{H}_7 = (-1,1,-1), &&\boldsymbol{H}_8 = (-1,-1,-1). \end{aligned} \tag{14}$$

The rotations linking the faces are then as follows:

$$\begin{aligned}
\mathcal{R}_{2,2} &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right) \quad \mathcal{R}_{2,3} = R_{\boldsymbol{G}_1}\left(\frac{\pi}{2}\right), \qquad \mathcal{R}_{2,4} = R_{\boldsymbol{G}_1}\left(\frac{\pi}{2}\right), \\
\mathcal{R}_{2,5} &= R_{\boldsymbol{G}_1}\left(\pi\right), \qquad \mathcal{R}_{2,6} = R_{\boldsymbol{G}_1}\left(\pi\right), \qquad \mathcal{R}_{2,7} = R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right), \\
\mathcal{R}_{2,8} &= R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right), \quad \mathcal{R}_{2,9} = R_{\boldsymbol{H}_1}\left(\frac{2\pi}{3}\right), \quad \mathcal{R}_{2,9+1} = R_z\left(i\frac{\pi}{2}\right) \quad i = 1, 2, 3, \\
\mathcal{R}_{2,12+i} &= R_x\left(\pi\right) \qquad\qquad i = 1 \ldots 12.
\end{aligned} \tag{15}$$

We then pick the following vectors for the pentavalent faces

$$\begin{aligned}
\mathcal{R}_{1,2} &= R_{\boldsymbol{G}_1}\left(\frac{\pi}{2}\right), \qquad \mathcal{R}_{1,3} = R_{\boldsymbol{G}_1}\left(\pi\right), \qquad \mathcal{R}_{1,4} = R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right), \\
\mathcal{R}_{1,4+i} &= R_y\left(\frac{\pi}{2}\right) \qquad \mathcal{R}_{1,8+i} = R_y\left(\pi\right) \qquad \mathcal{R}_{1,12+i} = R_y\left(\frac{3\pi}{2}\right) \quad i = 1, 2, 3, 4 \\
\mathcal{R}_{1,16+i} &= R_x\left(\frac{\pi}{2}\right) \qquad \mathcal{R}_{1,20+i} = R_x\left(\frac{3\pi}{2}\right) \qquad\qquad i = 1, 2, 3, 4.
\end{aligned} \tag{16}$$

Defining

$$\boldsymbol{e}_{xy} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right). \tag{17}$$

We then take

$$\begin{aligned}
\boldsymbol{V}_{1,1} &= S_1\, R_z(\phi) R_{\boldsymbol{e}_{xy}}(\theta)(\frac{1}{2}, -\frac{1}{2}, 1), \\
\boldsymbol{v}_{1,1,1} &= R_z(\phi) R_{\boldsymbol{e}_{xy}}(\theta)\boldsymbol{e}_{xy}, \qquad \boldsymbol{v}_{1,1,2} = R_z(\phi) R_{\boldsymbol{e}_{xy}}(\theta)\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \\
\boldsymbol{V}_{2,1} &= S_2\, R_{\boldsymbol{H}_1}(\tau)\, R_z(\sigma)\left(\frac{1}{3}, -1, 1\right), \\
\boldsymbol{v}_{2,1,1} &= R_{\boldsymbol{H}_1}(\tau)\, R_z(\sigma)\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 0\right), \qquad \boldsymbol{v}_{2,1,2} = R_{\boldsymbol{H}_1}(\tau)\, R_z(\sigma)\left(-\frac{3}{\sqrt{190}}, \frac{9}{\sqrt{190}}, \frac{\sqrt{10}}{\sqrt{19}}\right).
\end{aligned} \tag{18}$$

$$
\begin{aligned}
\boldsymbol{m}_1 &= Q_{1,18;2,9} + k_1 \boldsymbol{u}_{2,1;2,9}, & \boldsymbol{m}_2 &= Q_{1,18;2,9} + k_2 \boldsymbol{u}_{2,1;2,9}, \\
\boldsymbol{m}_3 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_2, & \boldsymbol{m}_4 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_1, \\
\boldsymbol{m}_5 &= Q_{2,2;1,1} + k_5 \boldsymbol{u}_{2,1;1,1}, & \boldsymbol{m}_6 &= Q_{2,2;1,1} + k_6 \boldsymbol{u}_{2,1;1,1}, \\
\boldsymbol{m}_7 &= Q_{1,1;2,8} + k_7 \boldsymbol{u}_{2,1;2,8}, & \boldsymbol{m}_8 &= Q_{1,1;2,8} + k_8 \boldsymbol{u}_{2,1;2,8}, \\
\boldsymbol{m}_9 &= Q_{2,8;1,19} + k_9 \boldsymbol{u}_{2,1;1,19}, & \boldsymbol{m}_{10} &= Q_{2,8;1,19} + k_{10} \boldsymbol{u}_{2,1;1,19}, \\
\boldsymbol{m}_{11} &= Q_{1,19;1,18} + k_{11} \boldsymbol{u}_{2,1;1,18}, & \boldsymbol{m}_{12} &= Q_{1,19;1,18} + k_{12} \boldsymbol{u}_{2,1;1,18}, \\
\boldsymbol{n}_1 &= P_{2,2;1,2} + k_{13} \boldsymbol{u}_{1,1;1,2}, & \boldsymbol{m}_2 &= P_{2,2;1,2} + k_{14} \boldsymbol{u}_{1,1;1,2}, \\
\boldsymbol{n}_3 &= R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right)\boldsymbol{n}_2, & \boldsymbol{n}_4 &= R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right)\boldsymbol{n}_1, \\
\boldsymbol{n}_5 &= R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right)R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_{10}, & \boldsymbol{n}_6 &= R_{\boldsymbol{G}_1}\left(\frac{3\pi}{2}\right)R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_9, \\
\boldsymbol{n}_7 &= \boldsymbol{m}_6, & \boldsymbol{n}_8 &= \boldsymbol{m}_5, \\
\boldsymbol{n}_9 &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_{12}, & \boldsymbol{n}_{10} &= R_{\boldsymbol{H}_1}\left(\frac{4\pi}{3}\right)\boldsymbol{m}_{11},
\end{aligned}
\tag{19}
$$

3 Results

The p-cages **TTM3** and **TOM3** do not lead to any p-cages with deformation below 10% and the best **TCM3** p-cages have deformation exceeding 8% and are not good candidate for protein cages.

The **TCM3** p-cages are all presented graphically in the supplementary file `bi_symmetrix_full_list_56_34.pdf`.

References

[Piette2024] Piette BMAG. Biequivalent Planar Graphs. *Axioms* **2024**, *13*, 437 DOI: 10.3390/axioms13070437/.